Cascading Behavior in Networks

Anand Swaminathan, Liangzhe Chen
CS 6604
10/23/2013
Outline

- Diffusion in networks
- Modeling diffusion through a network
- Diffusion, Thresholds and role of Weak Ties
- Knowledge, Thresholds and Collective Action
- Advance Material: The Cascade Capacity
  - Cascade on infinite networks
  - How large can a cascade capacity be?
  - Capacity and its role in cascades
- Conclusion
Diffusion in networks

- Diffusion of innovations
  - *Informational effects*: People observed the decisions of their network neighbors and it provided indirect information that led them to try the innovation.
  - Initial Research: Hybrid Corn, Doctors.
  - The success of innovation depends on: *Observability, Trialability, and Compatibility*.
  - *Principle of Homophily*: Since new innovations tend to arrive from outside the system, it can be difficult for innovations to enter into a tightly-knit society.
Modeling diffusion through a network

- Direct-benefit effects
  - The benefits to you of adopting a new behavior increase as more and more of your neighbors adopt it.

- A Networked Coordination Game
  - Lets look at a game (presented in the text book)
A Networked Coordination Game

- There are two nodes v and w, and two strategies A and B.
- Node v and w are connected by an edge.
- Rules are
  - if v and w both adopt behavior A, they each get a payoff of $a > 0$.
  - if they both adopt B, they each get a payoff of $b > 0$
  - if they adopt opposite behaviors, they each get a payoff of 0.
A Networked Coordination Game

- If a fraction \( p \) of \( v \)'s neighbors choose A, and there are \( d \) neighbors. If \( v \) choose A then payoff is \( pda \), then A is a better choice if

\[
pda \geq (1-p)db, \quad \text{or} \quad p \geq \frac{b}{a+b}.
\]
Cascading Behavior - Example

(a) The underlying network

(b) Two nodes are the initial adopters

(c) After one step, two more nodes have adopted

(d) After a second step, everyone has adopted
Cascading Behavior - Example

(a) Two nodes are the initial adopters
Cascading Behavior - Example

(b) The process ends after three steps
Inference from the Example

- Tightly-knit communities can hinder diffusion.
- In order to improve diffusion, one has to
  - Either improve the payoff.
  - Or make some more small group (nodes) within the community adopt the strategy.
Diffusion, Thresholds, and the Role of Weak Ties

- Bridges in a social network are powerful ways to convey awareness of new things, but are weak at transmitting behaviors.
Knowledge, Thresholds, and Collective Action

- Collective Action and Pluralistic Ignorance
  - Consider a public demonstration or a riot.
  - Should you participate or not?
  - *Collective action*: an activity produces benefits only if enough people participate.
  - There might be a large fraction of the population, strong enough to oppose the government, but that most of these people believe they’re in a small minority - *pluralistic ignorance*
A Model for the Effect of Knowledge on Collective Action

- Each node knows the threshold of its neighbor but not of everyone in the network.
- We’ll also assume that each node knows what the social network looks like.
Common Knowledge and Social Institutions

- A widely-publicized speech, or an article in a popular newspaper, has the effect not just of transmitting a message, but of making the readers realize that many others have gotten the message as well.
- Super Bowl commercials are often used to advertise products where there are strong network effects — like cell-phone plans.
Advanced Material: The Cascade Capacity

- Given a network, what is the largest threshold at which any “small” set of initial adopters can cause a complete cascade?
- What does “small” mean?
- To formalize the question consider infinite networks in which each node has a finite number of neighbors.

**Cascades on Infinite Networks**
- A finite set $S$ of nodes has behavior $A$ (this is the small set of early adopters), and all other nodes adopt $B$.
- We say that the set $S$ causes a complete cascade if, starting from $S$ as the early adopters of $A$, every node in the network eventually switches permanently to $A$. 
The *cascade capacity* of the network is the largest value of the threshold $q$ for which some finite set of early adopters can cause a complete cascade.

What is the value of $q$ for which there can be complete cascade?
Cascade Capacity

Figure 19.16: An infinite grid with a set of early adopters of behavior A (shaded).
How Large Can the Cascade Capacity Be?

- **Claim:** There is no network in which the cascade capacity exceeds $\frac{1}{2}$.
- Approach this claim by tracking the “interface” where adopters of $A$ are linked to adopters of $B$.
- An interface is a set of A-B links.
- At each step the size of the interface must
Modeling the Bilingual Option

- Apart from strategies A and B, AB represents decision to adopt both.

![Figure 19.18: A Coordination Game with a bilingual option. Here the notation \((a, b)^+\) denotes the larger of \(a\) and \(b\).]
## Bilingual Option - Example

<table>
<thead>
<tr>
<th></th>
<th>z</th>
<th>x</th>
<th>v</th>
<th>r</th>
<th>s</th>
<th>u</th>
<th>w</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Step 1</td>
<td>B</td>
<td>B</td>
<td>AB</td>
<td>A</td>
<td>A</td>
<td>AB</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Step 2</td>
<td>B</td>
<td>AB</td>
<td>AB</td>
<td>A</td>
<td>A</td>
<td>AB</td>
<td>AB</td>
<td>B</td>
</tr>
<tr>
<td>Step 3</td>
<td>AB</td>
<td>AB</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>AB</td>
<td>AB</td>
</tr>
<tr>
<td>Step 4</td>
<td>AB</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>AB</td>
</tr>
</tbody>
</table>
When and How does Cascades Happen on an Infinite Path?

payoff from choosing A: a
payoff from choosing B: 1
payoff from choosing AB: a + 1 - c

(a) Lines showing break-even points between strategies.  
(b) Regions defining the best choice of strategy.
When and How does Cascades Happen on an Infinite Path?

- AB
- ?
- B

payoff from choosing A: a
payoff from choosing B: 2
payoff from choosing AB: a + 1 - c (if A is better)

(a) Lines showing break-even points between strategies.
(b) Regions defining the best choice of strategy.
When and How does Cascades Happen on an Infinite Path?

- Neither A nor AB spreads.
- AB spreads but then stops.
- A spreads directly (no adoption of AB).
- AB spreads indefinitely, followed by A (B becomes vestigial).

Diagram:

- Axes: C (vertical) and a (horizontal).
- Thresholds at C=1 and a=1.
- Quadrants indicating different outcomes of AB and A spreading.
Maximize Product Adoption

- Maximizing Product Adoption in Social Networks

  by Smriti Bhagat, Amit Goyal, Laks V.S. Lakshmanan
Example

- Bob buys kindle, dislikes it, posts a blog about it. His friend Kali reads that and decide not to buy it.
- Bob watches ‘The Ring’, likes it, tell Kali that it’s a great movie. Kali didn’t watch it, but recommend her friend Charles about the movie.
Goal

- Distinguish product adoption from influence
- A model that factors in a user’s experience
- Maximize the number of product adoptions
Existed Propagation models

- Linear Threshold (LT)
- Independent Cascade (IC)
Figure 1: LT-C model with colored end states: adopt—green, promote—blue, inhibit—red
LT-C

- A: the set of active in-neighbors of node v
- $f_v(A)$: the activation function of v, that is, the total influence on v from A.

$$f_v(A) = \sum_{u \in A} w_{u,v} (r_{u,i} - r_{\min}) \frac{r_{\max} - r_{\min}}{r_{\max} - r_{\min}}$$
Dynamics of the propagation

- At t=0, a seed set is chosen to be active, each of them can choose any of the three states ADOPT, PROMOTE or INHIBIT.
- Each time step, calculate $f_v(A)$ for all nodes, and the influence exceeds the threshold, node $v$ becomes active.
- Activated nodes enter any of the three states with corresponding probability, and each of them provides a rating of the product.
- The process of activation stops if no new nodes can be activated.
Properties

- NP-hard
  - \( \lambda = 1, r_{v,i} = r_{\text{max}} \)
- Still monotone and submodular
- Use CELF algorithm to speed up
Learning the Parameters

- $w_{u,v}$: the fraction of times user $v$ rated an item after $u$ had done so, and normalize over all in-neighbors such that they sum up to 1.
- Rating matrix: collaborative filtering to predict the rating of a user.
- $\lambda$: the fraction of times a user provides an explicit rating for a product over the times the user provided any opinions.
- $\mu$: the fraction of times a user gave the rating ‘want to see it’ over the number of times any such special rating was given by that user.
Evaluation

- Compare the following models:
  - Classical LT: Linear threshold
  - LT-C: their proposed model
  - LT rating: LT-C without TATTLE state, or LT with rating
  - LT tattle: LT-C without rating

- Data divided into training and test, such that all the ratings of a movie fall in exactly one of training or test sets.
Evaluation

- Flixter, Movielens, Last.fm
Evaluation

- Parameters learnt for Flixter and Movielens

(b) Distribution of edge weights in Flixster

(c) Distribution of edge weights in Movielens
Evaluation

- Parameters learnt for Flixter and Movielens

(d) Distribution of ratings in Flixter
(e) Distribution of ratings in Movielens
Evaluation

- Parameters learnt for Flixter and Movielens

(f) Distribution of $\lambda$ in Flixter

(g) Distribution of $\mu$ in Flixter
Evaluation

(a) Coverage obtained with different models on Flixster
(b) Coverage obtained with different models on Movielens
Evaluation

(c) Actual vs predicted coverage on Flixster

(e) Actual vs predicted coverage on Movielens
Evaluation

(g) Consistency of chosen seeds across actions on Flixster

(h) Consistency of chosen seeds across actions on Movielens
Evaluation

(i) Coverage with $\lambda$, $\mu$ and ratings from data and distributions on Flixster
Evaluation

(a) Edge weight distribution  (b) Distribution of ratings  (c) Distribution of $\lambda$
Evaluation

(a) Actual vs predicted coverage

(c) Consistency of chosen seeds across actions
Conclusion

- Proposed a propagation model LT-C that distinguishes between influence and product adoption.
- Formalized the problem and show it’s NP-hard, the expected adoption spread function under LT-C is monotone and submodular, thus greedy algorithm can be used to get an approximate.
- Results show that LT-C model is consistently performing better than classical LT model.
Future work

- Users don’t express opinions after adoption
- Scalable heuristic algorithms needed in order to handle very large networks
- Validate LT-C model against many more real datasets from diverse domains