CS 2104
Introduction to Problem Solving

Deductive and Hypothetical Reasoning

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Slides based on the “Problem Solving and Comprehension” book
Deductive and Hypothetical Thinking

The day before yesterday you did not get home until yesterday; yesterday you did not get home until today. If today you do not get home until tomorrow, you will find that I have left yesterday.

• The mental reasoning needed here is fundamental in solving many problems.
  – Comprehend verbal statements.
  – Move in some dimension.
  – Think backward through a sequence to see where a movement began.
Simple Problems

Making a diagram helps with these.

• Suppose Valentine’s Day is 3 days after Friday. What day is Valentine’s day?
• Suppose Lincoln’s birthday is 4 days before Thursday. What day is Lincoln’s Birthday?
• Suppose Christmas is 2 days before Wednesday?
  – What day is Christmas?
  – What day is 4 days before Christmas?
Slightly Harder

• Saturday is 5 days before Labor day. What day is Labor day?
• Suppose Christmas is 2 days after Thursday. What day is Christmas?
• Suppose Thursday is 2 days after Christmas. What day is Christmas?
Deduction

• Today is Thursday. What is 2 days after tomorrow?
• Yesterday was Monday. What is 4 days after tomorrow?
• Today is Saturday. What is the day after 4 days before tomorrow?
Break It Down

Many math problems are solved by breaking them into parts. Your brain can only hold so much at once.

• Today is Monday. What is 1 day after 3 days before yesterday?
  – Use a diagram, and take it one step at a time.
Examples

• Yesterday was Tuesday. What is 2 days before 4 days after tomorrow?
• Tomorrow is Sunday. What is 2 days after 3 days before yesterday?
• Yesterday was Saturday. What is 4 days before 7 days after 2 days before today?
• Today is Monday. What is 3 days after 2 days before 6 days after 5 days after tomorrow?
Subtle Variations

• What is the difference between these two questions?
  – Today is Sunday. What is 3 days after today?
  – Sunday is 3 days after today. What is today?
More Examples

• Friday is 3 days before yesterday. What is tomorrow?
• Monday is 5 days before 2 days after yesterday. What is yesterday?
• Wednesday is 6 days before 2 days after tomorrow. What is tomorrow?
Mixed Problems

• Yesterday was Friday. What is the third letter in the day after tomorrow?
• If 6 days ago was Wednesday, what is the second letter after the second letter in 2 days after tomorrow?
Math-like Problems

• A man divides $1622.50 among four persons so that the first has $40 more than the second, the second $60 more than the third, and the third $87.50 more than the fourth. How much did the fourth person receive?

• A man bequeathes to his wife 1/3 of his estate; to his daughter, 1/5 of it; to his son, ½ of the daughter’s share. He divides the remainder equally between a hospital and a public library. What part is received by the hospital?
Mathematical Word Problems

• A lot of word problems involve math.
  – That just means they involve (simple) numerical relationships.
  – It’s all about setting up the relationships, not about the arithmetic.

• Process:
  – Be concerned about accuracy
  – Proceed step-by-step
  – Restate and subvocalize
    – Repeat information, rephrase, weigh it, compare different facts
  – Clarify ideas for yourself
Old Problem

Sally loaned $7 to Betty. But Sally borrowed $15 from Estella and $32 from Joan. Moreover, Joan owes $3 to Estella and $7 to Betty. One day the women got together at Betty’s house to straighten out their accounts. Which woman left with $18 more than she came with?
Hint: Make a **diagram** and use arrows to show which person has to **return money** to another person. Show the direction in which the money must be returned.
A train can travel 10 miles in 4 minutes. How far will it travel in 14 minutes?
Alternative Solutions

• $14/4 = 3.5$, so there are $3.5$ ($4$-minute) units. The train goes 10 miles in each unit, so $3.5 \times 10 = 35$.

• Ratios: $10\text{mi}/4\text{min} = X \text{mi}/14\text{min}$ so $(14)(10)/4 = X$. $X = 35\text{mi}$.

• How many miles in one minute? $10\text{mi}/4\text{min} = 2.5\text{mi}/\text{min}$. So in 14 minutes, $14\text{min} \times 2.5\text{mi}/\text{min} = 35\text{mi}$.

• Write and check physical units!
Sample Problems

• Ted’s weekly income is $100.00 less than double Gary’s weekly income. If Ted makes $500.00 a week, what does Gary make?

• Paul makes $25.00 a week less than the sum of what Fred and Carl together make. Carl’s weekly income would be triple Steven’s if he made $50.00 more a week. Paul makes $285.00 a week and Steven makes $75.00 a week. How much does Fred make?
\[
\begin{align*}
\text{\$600} & \quad 2 \times \text{Gary's income} \\
\text{- \$100} & \\
\text{\$500} & \quad \text{Ted's income}
\end{align*}
\]

\[
\begin{align*}
\text{Fred + Carl} & \\
\text{- \$25} & \\
\text{Paul}
\end{align*}
\]

\[
\begin{align*}
3 \times \text{Steven's income} & \\
\text{+ \$50} & \\
\text{Carl}
\end{align*}
\]