# Algorithm Design: GCD

- Problem solution through refinement
  - GCD Example of use of loops
  - Arguing the complexity of an algorithm
  - Greek mathematics achievement: Euclid’s Algorithm

# How to find the GCD of 2 ints?

- Greatest common divisor
  - Given two numbers: small , large
  - Write a class method to find their GCD
- Example of refining a solution procedure for a problem until you get it RIGHT! (cheap and elegant)
- First approach: try largest possible guess and try it; if doesn't work, decrement guess and try again. repeat.

# GCD - Algorithm 1

```
public static int GCD(int small, int large){
  int div;
  for (div=large; div>0; div--){
     if( ((large%div))==0) &&
         ((small%div))==0) ) return div;
  }
}
```

How many checks do we do in the loop in the worst case?   at most large if checks

But largest common factor can't be larger than small!

# GCD - Algorithm 2

```
public static int GCD(int small, int large){
  int div;
  for (div=small; div>0; div--){
     if ( ((large%div)==0) &&
          ((small%div)==0) ) return div;
  }
}
```

Do in worst case small if checks.

Is there a better way to do this?

# How GCD works?

- Take 15 and 18 in algorithm 1
  - when div is 15 - 15%15==0? yes, 18%15==0? no, 14 (no,no), 13 - (no,no)
  - 12, 11, 10, 9, 8, 7, 6, 5, 4 all fail to produce (yes,yes)
  - 3 succeeds
- Should only test the following (divisor,quotient) pairs: (1, 15), (2, 7.5), (3, 5)
  - Then (4, 3.75) will have been already tested
  - So when div > quotient means you have already checked this divisor, quotient pair if they are both integers

# GCD - Algorithm 3

```
public static int GCD(int small, int large){
  int div=0, best=1, quo;
  loopLabel: while (true){
     div++;
     quo = small/div;
     if (quo < div) break loopLabel;
     if ( ((large%quo) == 0) &&
          ((small%quo) == 0) ) return quo;
     if ( ((large%div) == 0) &&
          ((small%div) == 0) ) best = div;
  }
return best;
}
```

# Algorithm 3 - trace

small = 15, large = 18

| div | quo | %quo | %div | best |
|---|---|---|---|---|
| 0 | __ | __ | __ | 1 |
| 1 | 15 | false | true | 1 |
| 2 | 7 | false | false | 1 |
| 3 | 5 | false | true | 3 |
| 4 | 3 | | | |

loop exits and returns 3.

# Performance of Algm 3

- **How many if checks?** ***2*sqrt(small)***
- **Can we use another form of loop for this code?**
  - **Want no redundant statements or messy control flow**

# Algorithm 3 -with While

```
int div=1, best=1, quo=small/div;
  loopLabel: while (div<quo){
     if ( ((large%quo) == 0) &&
          ((small%quo) == 0) ) return quo;
     if ( ((large%div) == 0) &&
          ((small%div) == 0) ) best = div;
     div++;
     quo = small/div;
  }
return best;
```

# Algorithm 3 -with For

```
int best = 1, quo;
f1: for (int div = 1; true ; div++){
  quo = small/div;
  if (quo < div) break f1;
  .....
}
```

seems to add no code, but stopping condition harder to understand with check being true

# Algorithm 3 - with Do-while

```
int div=1, quo=small/div;
do1: do{
          if ( ((large%quo...
          ...
          div++;
          quo = small/div;
     }
     while (div < quo);
```

# Which loop to use?

- Most straight forward to understand code
- Can make them all work, but why?
- Redundant code or obscured control flow are not desirable
- Generalized loop construct is what was used in first code for algorithm

# General Loop Structure

```
public static int GCD(int small, int large){
  int div=0, best=1, quo;
  loopLabel: while (true){
     div++;
stmt1
     quo = small/div;
     if (quo < div) break loopLabel;
     if ( ((large%quo) == 0) &&
          ((small%quo) == 0) ) return quo;
stmt2
     if ( ((large%div) == 0) &&
          ((small%div) == 0) ) best = div;
  }
return best;
}
```

# Bishop’s Method for GCD

- **If large and small are both multiples of k, then large - small is a multiple of k**
  - **Note: large-small is smaller than large, so we have *reduced the problem* to one easier to solve**
  - **Need greatest multiple of large - small and small.**
- **Why? lg = k * n; sm = k * m;**

**So lg - sm = k* (n-m) = k * diff**

# Algorithm 4 - Bishop

```
public int static GCD (int small,int large){
  int smsave=small, lgsave=large;
  while (small != large) {
     if (large > small) large = large-small;
     else {
          int tmp = small;//swap large and
          small = large;  //small
          large = tmp;
          }
  }  return sm;
}//save of original arguments on entry is not
//necessary
```

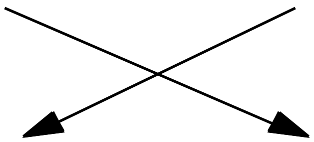
# How does this work?

- *large* is reduced by successive subtraction (i.e., division) until it is smaller than *small*
- *small* and *large* are then swapped
- Continues until *large* == *small*
  - then *large* is the GCD (it can be 1)
- Bishop's program is a variant of Algorithm 4

# Algorithm 4 Trace

*small* - 6, *large* - 21

| large | small |
|---|---|
| 21 | 6 |
| 15 | 6 |
| 9 | 6 |
| 3 | 6 |
| 6 | 3 |
| 3 | 3 |



3 is returned

How long does it take?

at least *large/small* steps

Which is better,

*large/small* or *sqrt(small)* ?

Neither.

large - 2000, small - 2

large - 2000, small - 1000

# Euclid's Method

- Recognize that repeated subtraction is division
- Exchange *small* and *large* when *large* is replaced by *large%small*
- Greek mathematician Euclid discovered this algorithm around 300 B.C.
  - Father of Geometry
- Moral: in mathematics we can't ignore the past

# Algm 5: Euclid’s Method

```
public static int GCD(int small, int large){
  while ((large%small) != 0){
     int tmp = large%small;
     large = small;
     small = tmp;
  }
  return small;
}
```

**small - 6, large 28**

| large | small | tmp |
|---|---|---|
| 28 | 6 | 4 |
| 6 | 4 | 2 |
| 4 | 2 | |

**2 is returned**

# Algm 6: Another Formulation

**in class A define:**

```
public static int GCD(int small,int large){
  int rem = large%small;
  if (rem == 0) return small;
  return A.GCD(rem, small);
}
```

**small-6, large-28**

**Bert: What is GCD of 28, 6?  ask Ernie 6,4**

**Ernie: What is GCD of 6,4?  ask Elmo 4,2**

**Elmo: What is GCD of 4, 2?  It’s 2!!**

# Algorithm 6

- **Problem decomposition in terms of function calls**
- **Particular usage called *recursion***
- **Succinct statement of solution of one problem in terms of another reduced problem**