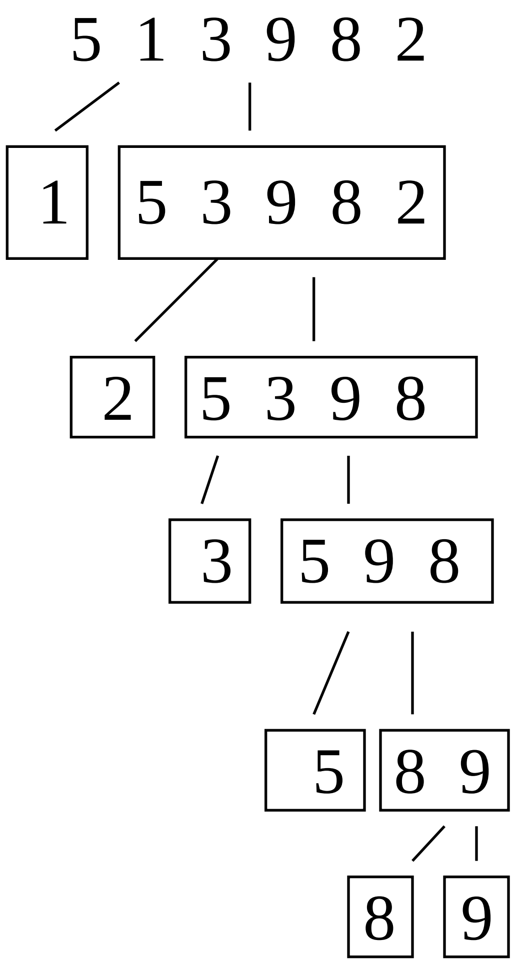
Sorting (2)

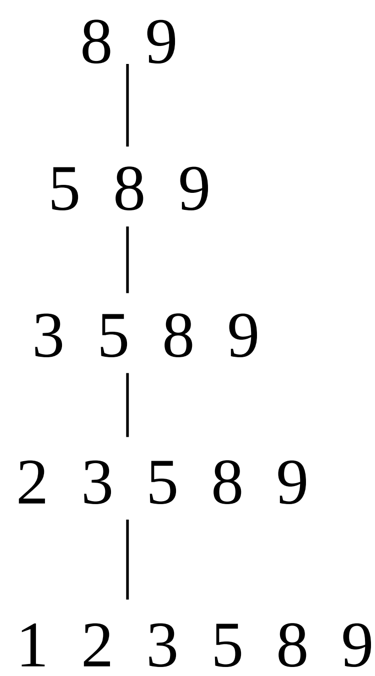
- **Sort examples**
- **Merge sort**
- **Complexity, Big O notation**

Selection Sort

Decompose:



Compose:



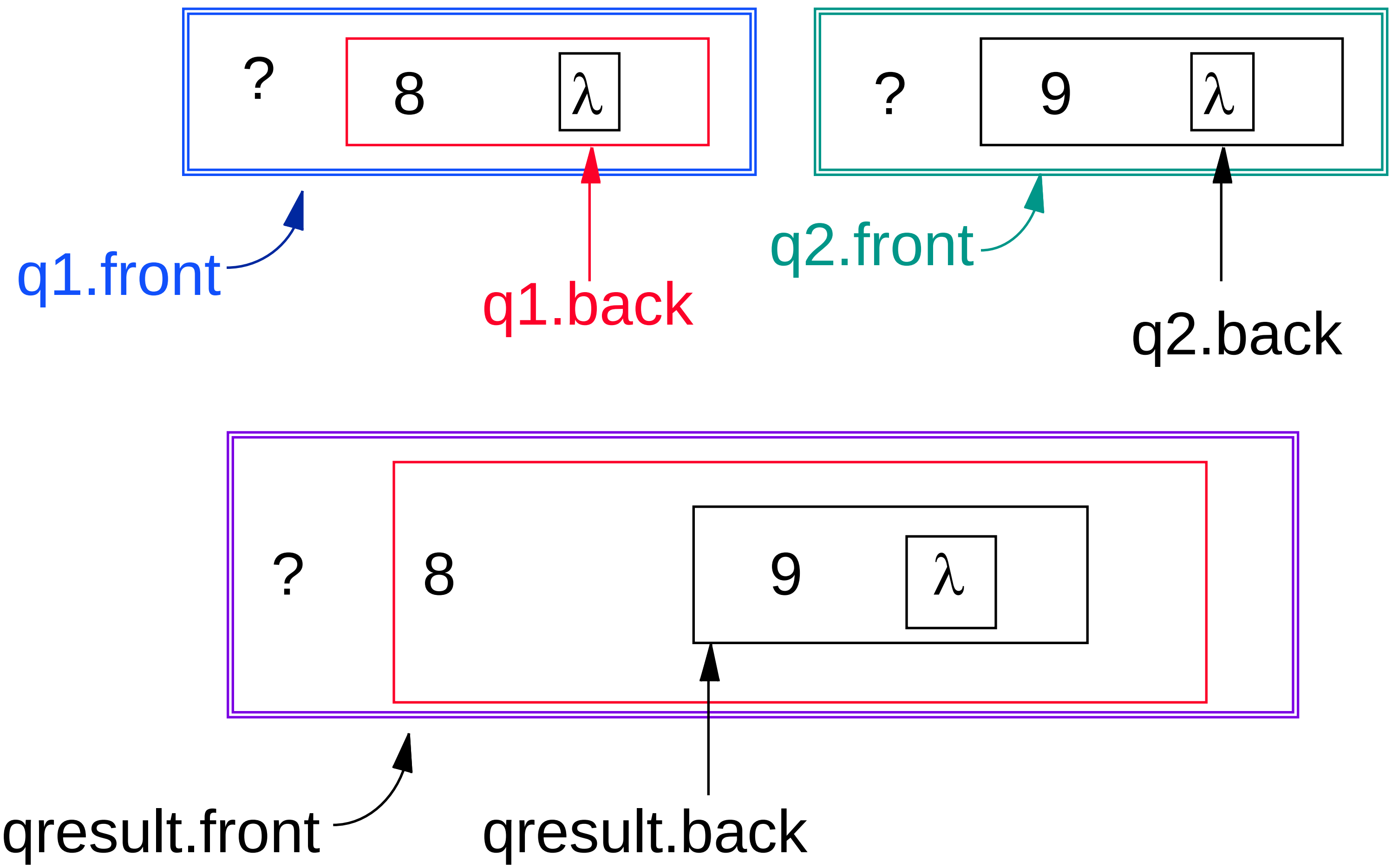
appending 2 queues, one in front of the other

How to achieve queue append in unit cost?

- **Each queue `q1, q2` has a front and a back**
- **Requires knowledge of the representation of Queue objects**
- **Use the following code:**

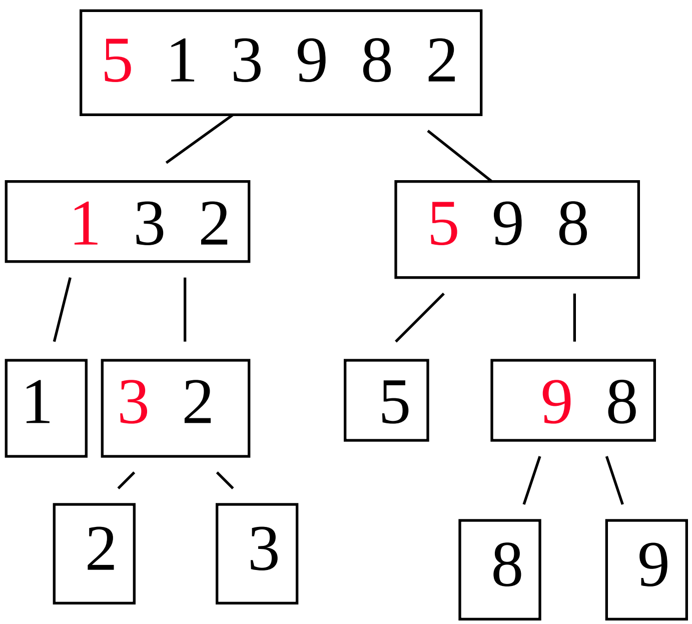
```
Queue qresult = new Queue();
qresult.front = q1.front;
(q1.back).subList = (q2.front).subList;
qresult.back = q2.back;
```

Append in unit cost

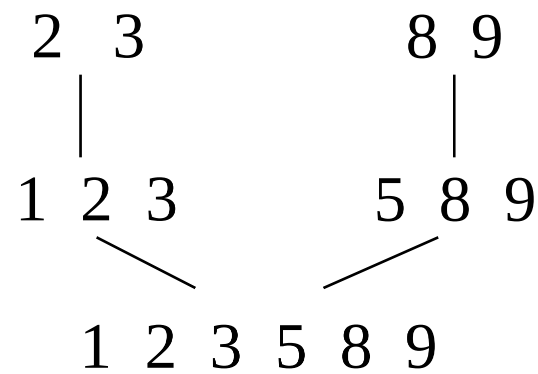


Quick Sort

Decompose:



Compose: (append)



Merge Sort

- Similar to selection and quick sorts
- Selection and quick use data value comparison; merge just splits the queue

```
public SortProblem mergeSort() throws QueueException
  {   if (getLength() == 1) return this;
      SortProblem sp1 = new SortProblem(),
                  sp2 = new SortProblem();
      SortProblem sp1sorted, sp2sorted;
      inHalf(sp1,sp2);
      sp1sorted = sp1.mergeSort();
      sp2sorted = sp2.mergeSort();
      SortProblem ret = sp1sorted.merge(sp2sorted);
      return ret;
  }
```

Decompose: halving the queue

```
private void inHalf(SortProblem halfA,
      SortProblem halfB) throws QueueException {
//halfA, halfB are input as empty Queues
    while (true) {
//alternate elements from this into each
//of halfA, halfB
      if (this.empty()) return;
      halfA.enter(this.remove());
      if (this.empty()) return;
      halfB.enter(this.remove());
    }
  }
```

Compose

- Former append doesn't work because need to interleave elements from both subproblems
- Must merge two ordered queues
  – First item in each queue is smallest
  – Smallest of two first items is smallest in both
- Always compare first item in each queue
  – Remove smaller and place on new queue
  – Repeat until one queue becomes empty
  – Append what remains

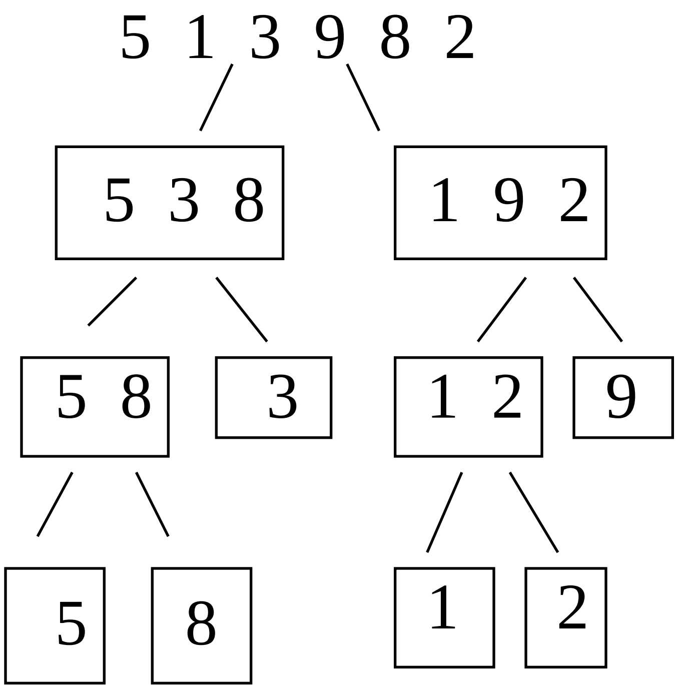
Compose: merge()

```
private SortProblem merge(SortProblem other) throws
   QueueException {
     SortProblem ret = new SortProblem();
     while ( !(this.empty()) && !(other.empty()) ) {
       Sortable vThis = (Sortable) this.peek(),
                vOther = (Sortable) other.peek();
       if (vThis.lessThan(vOther))
             ret.enter(this.remove());
       else  ret.enter(other.remove());
     }
     if (this.empty())
             while ( !(other.empty()) )
                   ret.enter(other.remove());
     else    while ( !(this.empty()) )
                   ret.enter(this.remove());
     return ret;
   }
```

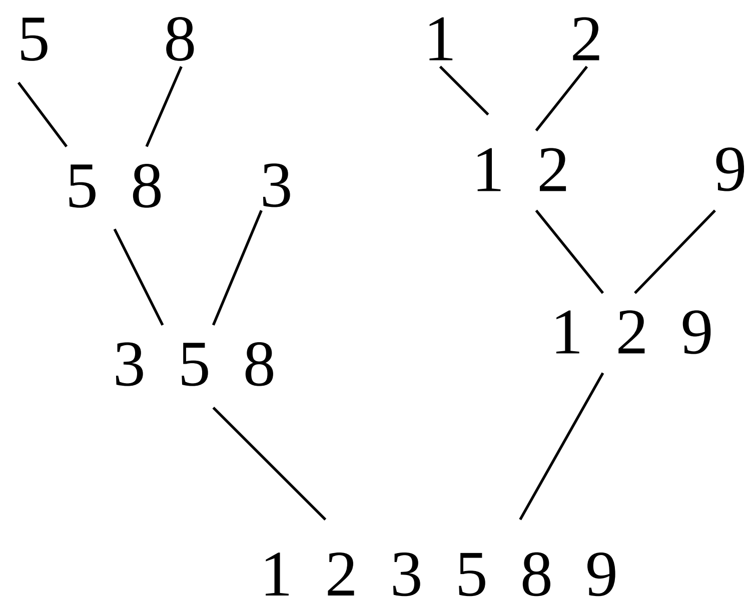
append() ops

Merge Sort

Decompose:



Compose: (merge)



Worst Case Cost - mergeSort()

- Costs for a queue of length n
  - Decomposition (halving) - 1
    - With creating new queues this is cost n
    - Can be reimplemented similarly to binary search
  - Compose - n
    - Fixed amount of work for every item in the result

mergeSort() - worst case cost

Length of Queue	Worst case cost of mergeSort
1	(1) = 1
2	(1+2) + C(1) + C(1) = 3 + 1 + 1 = 5
3	(1+3) + C(2) + C(1) = 4 + 5 + 1 = 10
4	(1+4) + C(2) + C(2) = 5 + 5 + 5 = 15
5	(1+5) + C(3) + C(2) = 6 + 10 + 5 = 21
6	(1+6) + C(3) + C(3) = 7 + 10 +10 = 27
...	...
n	(n+1) + 2*C(n/2) for n assumed even

Note: this calculation is SAME as best case quickSort() !

How to obtain a closed form?

$$C(n) = (n+1) + 2*C(n/2)$$

$$C(n/2) = (n/2+1) + 2*C(n/4)$$

$$C(n/4) = (n/4+1) + 2*C(n/8)$$

Assumes length of the queue is a power of 2

$$\begin{aligned} C(n) &= (n+1) + 2*C(n/2) \\ &= (n+1) + 2*(\ (n/2+1) + 2*C(n/4)\ ) \\ &= (n+1) + (n+2) + 4*C(n/4) \\ &= (n+1) + (n+2) + 4*(\ (n/4+1) + 2*C(n/8)\ ) \\ &= (n+1) + (n+2) + (n+4) + 8*C(n/8) \end{aligned}$$

Closed form: worst case mergeSort()

$$C(n) = (n+1) + (n+2) + (n+4) + 8*C(n/8)$$

$$= (n+1) + (n+2) + (n+4) + \ \ldots \ + (2n)$$

$$< (2n)*\ lg_2(n)$$

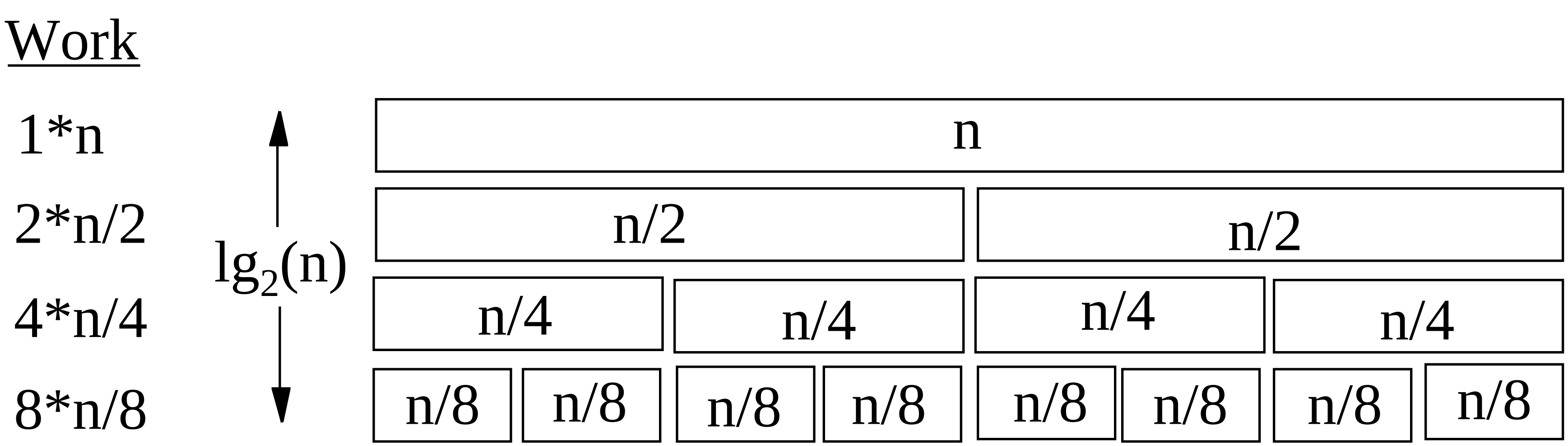
largest value ↗ (2n)

number of values ↖ $lg_2(n)$

$C(n) = O(n*lg_2(n))$ in worst case

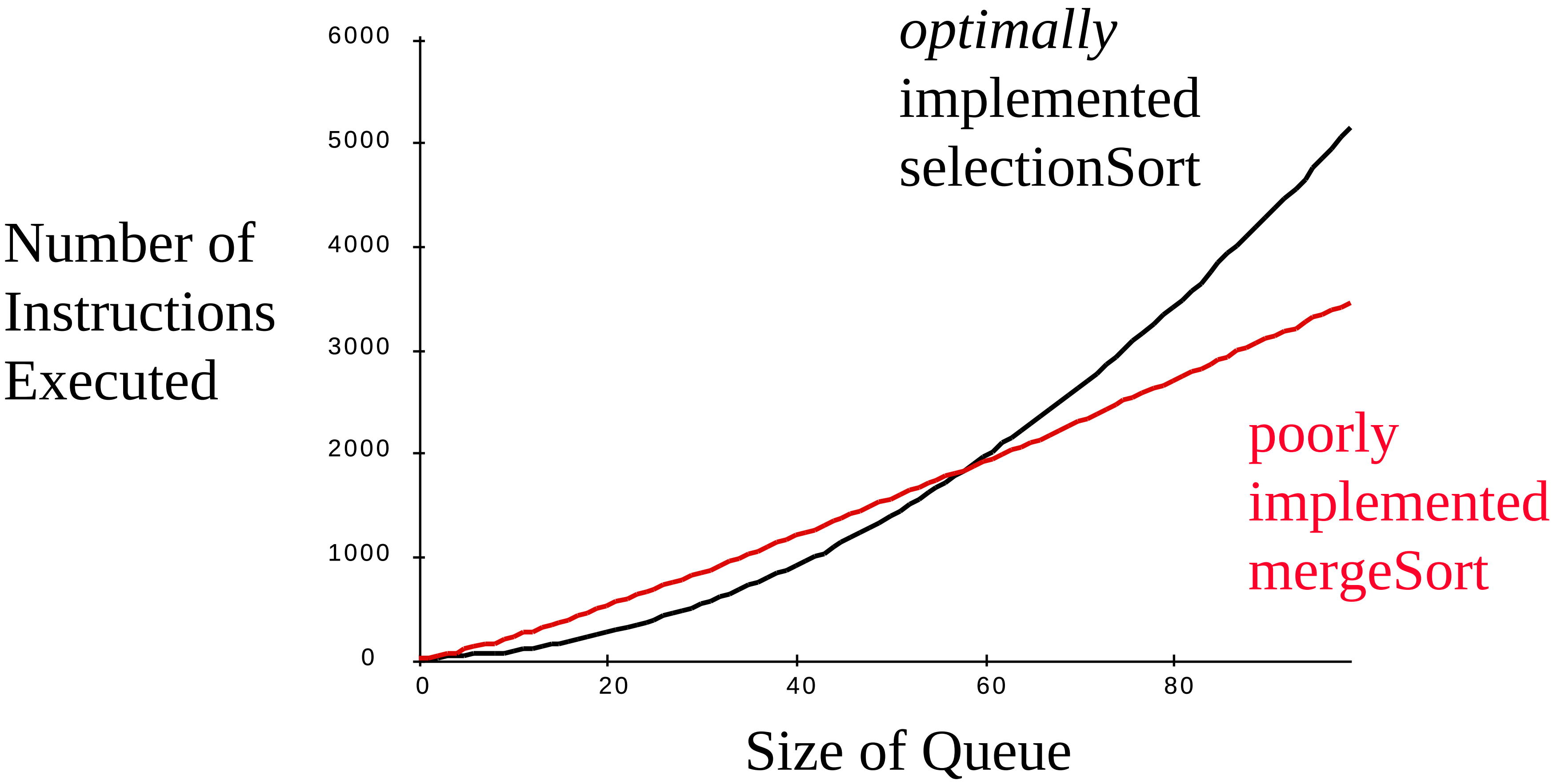
Work at each recursive step

- How much work at each level   n
- How many levels $lg_2(n)$
- Total work is $n * lg_2(n)$ $= O(n*lg_2(n))$



Experiments

Randomly selected values; varied number of values and type of sort applied; counted instructions executed



Experiments

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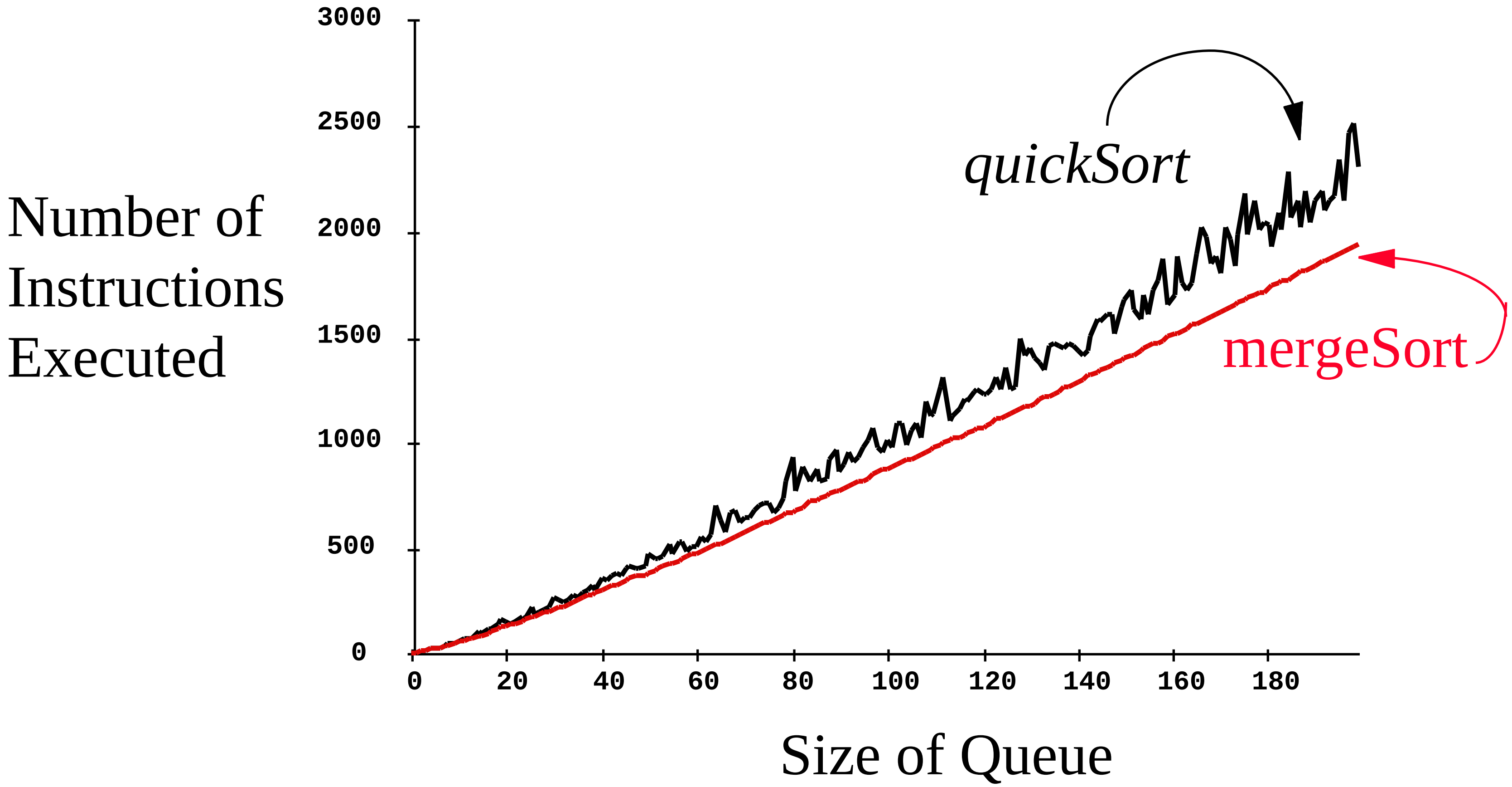
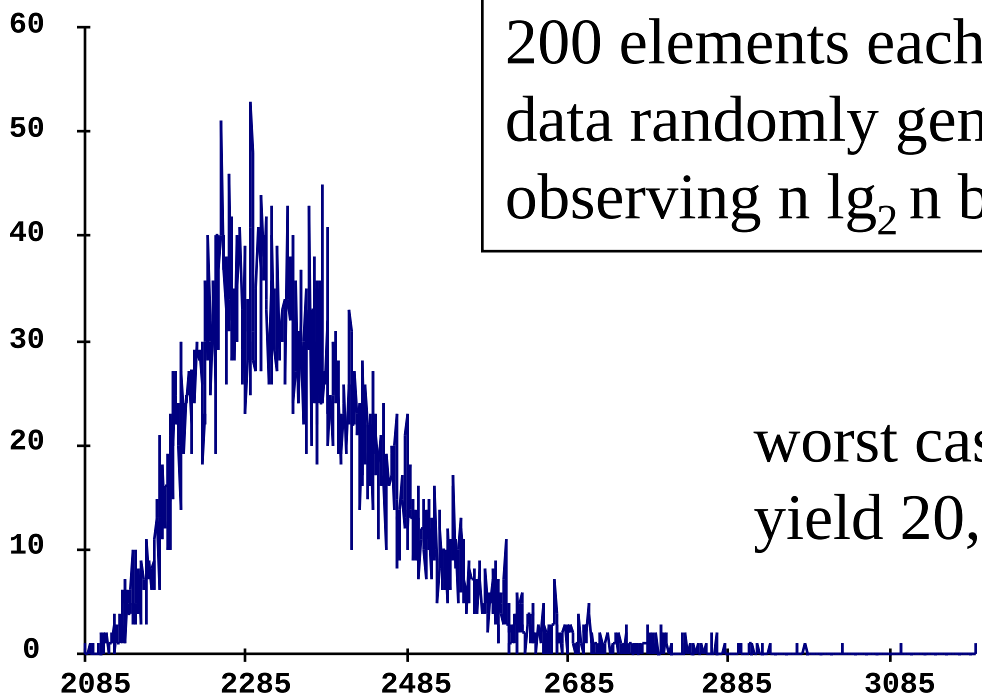


Histogram of quickSort()

Number of problems that required the given instruction count



10,000 quickSorts of 200 elements each; data randomly generated; observing $n \lg_2 n$ behavior

worst case would yield 20,000

Number of instructions executed

Asymptotic Complexity

$g(n) = O(f(n))$

pronounced: $g(n)$ is order $f(n)$

meaning: for large n, $g(n)$ grows no faster than $f(n)$

definition: $\exists\ n_0, C\ \ \forall\ n > n_0 \quad g(n) \leq C * f(n)$

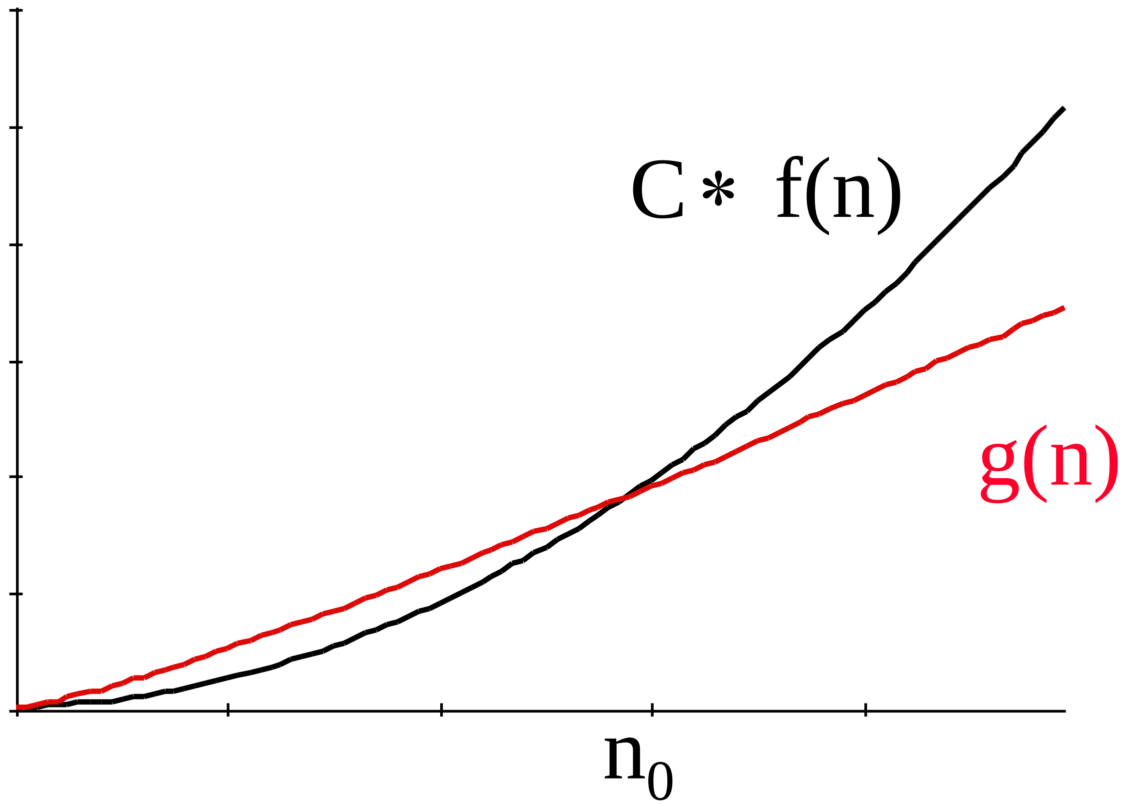
$f(n)$ is usually expressed in *reduced* form as a function of n

$1 \quad lg(n) \quad n^{1/2} \quad n \quad n * lg(n) \quad n^2$

Graphical Interpretation

$$\exists\ n_0, C \quad \forall\ n > n_0 \quad g(n) \leq C * f(n)$$

g(n) is order f(n)



Examples

- **Is $3n^2$ order n^2 ?**
  - **Can a constant be found to make it so?**
  - **Yes, for any n, $3 n^2 < 4 n^2$**
- **Is 17n order n^2 ?**
  - **Yes, for n >= 18, $17 n < n^2$**
- **Big O notation bounds from above**
  - **Means algorithm behaves no worse than this**
- **Other notations exist to bound from below and to talk about *on average* performance**