

Prolog

- **Logic programming (declarative)**
 - Goals and subgoals
- **Prolog Syntax**
- **Database example**
 - rule order, subgoal order, argument invertibility, backtracking model of execution, negation by failure, variables
- **Data structures (lists, trees)**
 - Recursive Functions: append, member
 - Lazy evaluation, terms as trees, Prolog search trees
- **Models of execution**
 - Goal-oriented semantics
 - Procedural view

Intro to Logic Programming

- **Specifies relations between objects**
larger (2,1), father (tom, jane)
- **Separates control in PL from description of desired outcome**
father(X, jane) :- male(X), parent(X, jane).
- **Computation engine: theorem proving and recursion**
 - Higher-level PL than imperative languages
 - More accessible to non-technical people

Horn Clauses

- **Conjunct of 0 or more conditions which are atomic formulae in predicate logic (constants, predicates, functions)**

$$h_1 \wedge h_2 \wedge \dots \wedge h_n \rightarrow c$$

- **Means c if h_1, h_2, \dots, h_n are all true**
- **Can have variables in the h_i 's or c**

$$c(x_1, x_2, \dots, x_m) \text{ if } h(x_1, x_2, \dots, x_m, y_1, \dots, y_k)$$

means for all objects x_1, x_2, \dots, x_m , c holds if there are objects y_1, \dots, y_k such that h holds.

$$\text{father}(X, \text{jane}) \text{ :- } \text{male}(X), \text{parents}(X, Y, \text{jane})$$

Logic Programming

- **Goal-oriented semantics**
 - **goal is true for those values of variables which make each of the subgoals true**
 - *father(X, jane)* will be true if *male(X)* and *parents(X, Y, jane)* are true with specific values for X and Y
 - **recursively apply this reasoning until reach rules that are facts.**
 - **called *backwards chaining***

Logic Programming

- **Nondeterminism**

- Choice of rule to expand subgoal by
- Choice of subgoal to explore first

father(X, jane):- male(X), parents(X, Y, jane).

father (X,jane):- father (X,Y), brother(Y, jane).

which rule to use first? which subgoal to explore first?

- **Prolog tries rules in sequential order and proves subgoals from left to right. - Deterministic!**

Victoria Database Program

```
male(albert).
male(edward).
female(alice).
female(victoria).
parents(edward,victoria,albert).
parents(alice,victoria,albert).
?- male(albert).
```

yes

```
?- male(alice).
```

no

```
?-female(X).
```

X = alice ;

X = victoria ;

no

predicate

constants

variable

victoria.pl
from Clocksin
and Mellish

By responding
<cr> you quit the query
; <cr> you continue to
find another variable
binding that makes the
query true.

Victoria Example

- **Problem: facts alone do not make interesting programs possible. Need variables and deductive rules.**

```
?-female(X) .           a query or proposed fact
X = alice ;           ; asks for more answers
X = victoria ;       if user types <cr> then no more
                    answers given
no    when no more answers left, return no
```

- Variable X has been unified to all possible values that make female(X) true.
 - Performed by pattern match search
- Variables capitalized, predicates and constants are lower case

Victoria Example

```
?-sister_of(X,Y):-
    female(X),parents(X,M,F),parents(Y,M,F) .
?- sister_of(alice,Y) .           a rule ↗
Y = edward
?- sister_of(alice, victoria) .
no
```

- Prolog program consists of **facts, rules, and queries**
- A query is a proposed fact, needing to be proven
 - If query has no variables and is provable, answer is **yes**
 - If query has variables, the proof process causes some variables to be bound to values which are reported (called a **substitution**)

Victoria Example, cont.

```
sister_of(X,Y) :-  
    female(X),parents(X,M,F), parents(Y,M,F).
```

```
?- sister_of(alice, Y).
```

Y = edward

```
?- sister_of(X,Y).
```

X = alice

Y = edward ;

X = alice

Y = alice ;

no

```
3. female(alice).  
4. female(victoria).  
5. parents(edward,victoria,albert).  
6. parents(alice,victoria,albert).  
first answer from 3.+6.+5.  
second answer from 3.+6.+6.
```

Subgoal order, argument invertibility, backtracking, rule order

Victoria Example, cont.

```
sis(X,Y) :- female(X), parents(X,M,F),  
            parents(Y,M,F), \+(X==Y).
```

```
?- sis(X,Y).
```

X = alice

Y = edward ;

no

= means *unifies with*

== means *same in value*

\+ (P) succeeds when **P** fails;
called **negation by failure**

Negation by Failure

not(X) :- X, !, fail.

not(_) .

if X succeeds in first rule, then the goal fails because of the last term.

if we type “;” the cut (!) will prevent us from backtracking over it or trying the second rule so there is no way to undue the fail.

if X fails in the first rule, then the goal fails because subgoal X fails. the system tries the second rule which succeeds, since “_” unifies with anything.

Procedural Semantics

```
?-sister_of(X,Y):-  
    female(X),parents(X,M,F),parents(Y,M,F).
```

Semantics:

- First *find* an X to make **female(X)** true
- Second *find* an M and F to make **parents(X,M,F)** true for that X.
- Third *find* a Y to make **parents(Y,M,F)** true for those M, F
- This algorithm is recursive; each *find* works on a new “copy” of the facts+rules. eventually, each find must be resolved by appealing to facts.
- Process is called *backward chaining*.

Prolog Syntax in EBNF

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jane

X

$\langle \text{term} \rangle \rightarrow \langle \text{integer} \rangle \mid \langle \text{atom} \rangle \mid \langle \text{variable} \rangle \mid$
 $\langle \text{functor} \rangle (\langle \text{term} \rangle \{ , \langle \text{term} \rangle \})$

head :- body

$\langle \text{rule} \rangle \rightarrow \langle \text{predicate} \rangle (\langle \text{term} \rangle \{ , \langle \text{term} \rangle \}) :-$
 $\langle \text{term} \rangle \{ , \langle \text{term} \rangle \} . \mid \langle \text{fact} \rangle$

$\langle \text{fact} \rangle \rightarrow \langle \text{functor} \rangle (\langle \text{term} \rangle) \{ , \langle \text{term} \rangle \} .$

$\langle \text{query} \rangle \rightarrow ?- \langle \text{functor} \rangle (\langle \text{term} \rangle \{ , \langle \text{term} \rangle \}) .$

a proposed fact that must be proven

Syntax

- Names come from first order logic

$a(X,Y) :- b(c(Y)), integer(X).$

– **Predicates** are evaluated

– **Functors** with terms are unified

– Call this a clause or rule

cf. “Computing with Logic”, D. Warren and D. Meyers

Lists

list

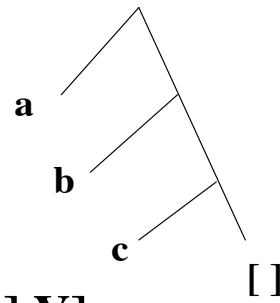
[a,b,c]

head

a

tail

[b,c]



[X, [cat], Y]

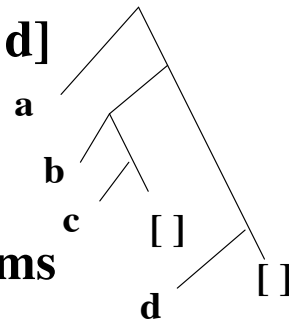
X

[[cat], Y]

[a, [b, c], d]

a

[[b,c], d]



[X | Y]

X

Y

a list consists of a sequence of terms

Unifying Lists

[X,Y,Z] = [john, likes, fish]

X = john, Y = likes, Z = fish

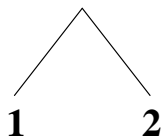
[cat] = [X | Y]

X = cat, Y = []

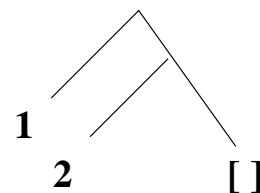
[[the, Y] | Z] = [[X, hare] | [is, here]]

X = the, Y = hare, Z = [is , here]

[1 | 2]

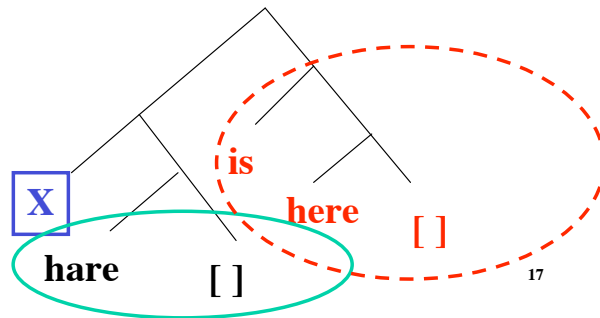
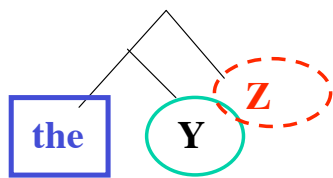


versus [1, 2]



Lists

- Sequence of elements separated by commas, or
- [first element | rest_of_list]
 - [car(list) | cdr(list)] notation
- [[the | Y] | Z] = [[X, hare] | [is, here]]



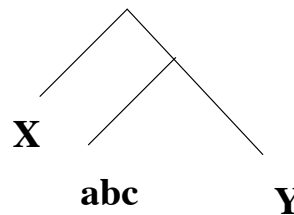
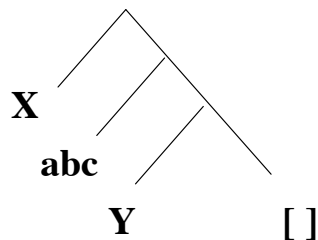
Prolog © BGR, Fall05

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Lists

[X, abc, Y] =? [X, abc | Y]

there is no value binding for Y, to make these two trees isomorphic.



Prolog © BGR, Fall05

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Lists

$[a,b \mid Z] =? [X \mid Y]$

don't care variable
unifies with anything

$X = a, Y = [b \mid Z], Z = _$

look at the trees to see why this works!

$[a, b, c] = [X \mid Y]$

$X = a, Y = [b,c] ;$

no

Member_of Function

$\text{member}(A, [A \mid B])$.

$\text{member}(A, [B \mid C]) :- \text{member}(A, C)$.

goal-oriented semantics: can get value assignment for goal $\text{member}(A,[B|C])$ by showing truth of subgoal $\text{member}(A,C)$ and retaining value bindings of the variables

procedural semantics: think of head of clause as procedure entry, terms as parameters. then body consists of calls within this procedure to do the calculation. variables bindings are like “returned values”.

Example

?- member(a, [a, b]).

yes

?- member(a, [b, c]).

no

?- member(X, [a, b, c]).

X = a ;

X = b ;

X = c ;

no

Invertibility of Prolog arguments

1. member(A, [A | B]).

2. member(A, [B | C]) :- member(A, C).

Example

?- member(a, [b, c, X]).

X = a ;

no

?- member(X, Y).

X = _123

Y = [X | _124] ;

X = _123

Y = [_125, X | _126] ;

X = _123

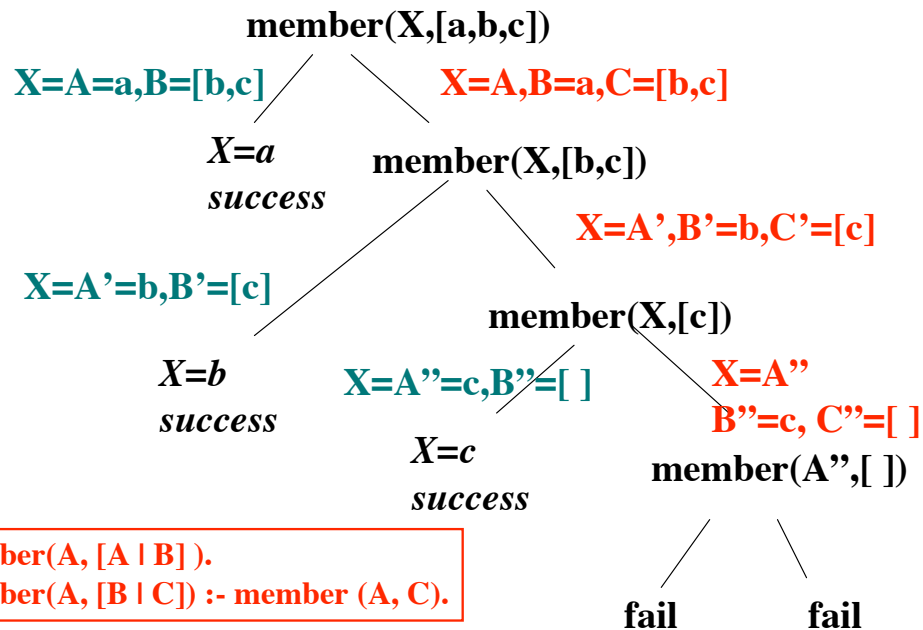
Y = [_127, _128, X | _129]

1. member(A, [A | B]).

2. member(A, [B | C]) :- member(A, C).

Lazy evaluation of a *a priori* unbounded list structure. Unbound X variable is first element, then second element, then third element, in a sequence of generated lists of increasing length.

Prolog Search Tree



1. member(A, [A | B]).
 2. member(A, [B | C]) :- member(A, C).

?- member(X, [a,b,c]).

match rule 1. member(A, [A | B]) so $X = A = a, B = [b,c]$

$X = a$;

match rule 2. member(A, [B | C]) so $X = A, B = a, C = [b,c]$

then evaluate subgoal member(X, [b,c])

match rule 1. member(A', [A' | B']) so $X = b, B' = [c]$

$X = b$;

match rule 2. member(A', [B' | C']) so $X = A', B' = b, C' = [c]$

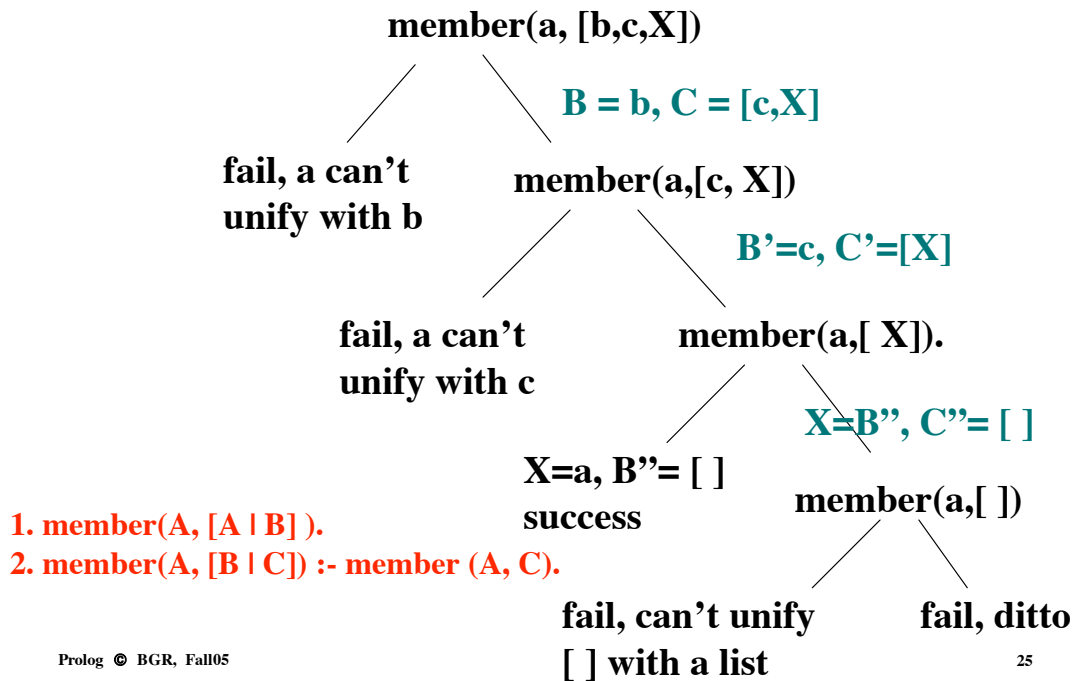
then evaluate subgoal member(X, [c])

match rule 1. member(A'', [A'' | B'']) so $X=A''=c, B''=[]$

$X = c$;

match rule 2. member(A'', [B'' | C'']) so $X=A'', B''=c, C''=[]$, but member(X, []) is unsatisfiable, *no*

Another Search Tree



Prolog Search Trees

- Really have built an evaluation tree for the query $\text{member}(X, [a, b, c])$.
- Search trees provide a formalism to consider all possible computation paths
- Leaves are **success** nodes or **failures** where computation can proceed no further
 - Can have more than 1 of each type of node
- By convention, to model Prolog, leftmost subgoal is tried first

Prolog Search Trees, cont.

- Label edges with variable bindings that occur by *unification*
- There can be infinite branches in the tree, representing non-terminating computations (performed *lazily* by Prolog);
 - *Lazy evaluation* implies only generate a node when needed.

Another Member_of Function

Equivalent set of rules:

```
mem(A, [A|_] ) .  
mem(A, [_| C] ) :- mem(A, C) .
```

Can examine search tree and see the variables which have been excised were auxiliary variables in the clauses.

Append Function

append ([], A, A) .

append ([A|B], C, [A|D]) :- **append**(B, C, D) .

- **Build a list**

?- **append**([a], [b], Y) .

Y = [a,b]

- **Break a list into constituent parts**

?- **append**(X, [b], [a,b]) .

X = [a]

?- **append**([a], Y, [a,b]) .

Y = [b]

More Append

?- **append**(X, Y, [a,b]) .

X = []

Y = [a,b] ;

X = [a]

Y = [b] ;

X = [a,b]

Y = [] ;

no

append ([], A, A) .

append ([A|B], C, [A|D]) :- **append**(B, C, D) .

Still More Append

- **Generating an unbounded number of lists**

?- `append(X, [b], Y).`

`X = []`

`Y = [b] ;`

`X = [_169]`

`Y = [_169, b] ;`

`X = [_169, _170]`

`Y = [_169, _170, b] ;`

etc.

```
append ([ ],A,A).
```

```
append([A|B],C,[A|D]):-append(B,C,D).
```

Common Beginner's Errors

- **Compile-time**

- Forget ending “.”
- Misspelled functors
- Need to override precedences with (..)

- **Runtime**

- Infinite loops - check your recursion
- Variables instantiated with unexpected values
- Circular definitions
- Giving wrong numbers of arguments to a clause

Examples Online on Paul

- **/grad/users/ryder/prolog/programs/**
contains examples of Prolog programs
- **/grad/users/ryder/prolog/newtraces/**
contains traces of runs of these programs
- **Remember, to run these you need to copy them to your own working directory so you can write on them**