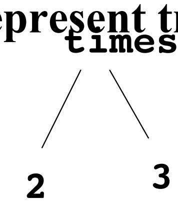
Prolog II

- **Unification**
  - Informally
  - Formal description
  - Problems in compilation
- **Factorial**
  - Example of generate and test
  - Cut (!)

Trees

- **Can use Prolog terms to represent trees**

  2 * 3 can be `times(2,3)`

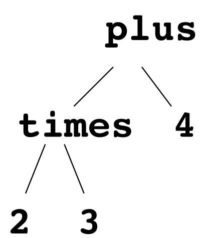


  - Then can design recursive Prolog clauses to "walk" the tree, gathering terms.
  - Example, generating code from an abstract syntax tree for an arithmetic expression

Example

```
treewalk(W,[W]) :- integer(W).
treewalk(times(X,Y),Walk) :- treewalk(X,W1),
   treewalk(Y,W2),append(W1,[*],A1),
   append(A1,W2,Walk).
treewalk(plus(X,Y), Walk):- treewalk(X,W1),
   treewalk(Y,W2), append(W1,[+],A1),
   append(A1,W2,Walk).
append([ ],A,A).
append([A|B],C,[A|D]) :- append(B,C,D).
```

Generating Code from AST



A Prolog data structure:
`plus(times(2,3),4))`
representation for
2*3+4

This Prolog query produces code from the tree represented as a Prolog data structure (a term):

```
?-treewalk(plus(times(2,3),4),X)).
```

*X = [2, *, 3, + , 4]*

Note code generated here is a correct inorder traversal but will not generate correct expressions from the input because it ignores operator precedence.

How `treewalk.pl` works?

- Second argument is always the code which corresponds to the AST which is the first argument.
- Base case finds leaf nodes which are integer constants with Prolog built-in

```
treewalk(W,[W]) :- integer(W).
```

- Tree exploration generates an inorder traversal of the nodes
- `Plus` and `times` clauses work the same

How `treewalk.pl` works?

- First, explore left subtree and get its code bound to W1 (left operand)

```
 treewalk(times(X,Y),Walk) :-
   treewalk(X,W1), …
```

- Second, explore right subtree and get its code bound to W2 (right operand)

```
 … treewalk(Y,W2),…
```

- Third, insert proper operator for this node …
  `append(W1,[*], A1), …`
- Fourth, append rest of expression

```
… append(A1,W2,Walk).
```

Unification Examples

```
unify(X,Y):- X = Y.
| ?- unify(a,X).
```

X = a ;

no

```
| ?- unify(a,X),unify(X,Y).
```

X = Y = a ;

no

```
| ?- unify(a,X),unify(b,Y),unify(X,Y).
```

no

```
| ?- unify(X,Y).
```

X = Y = _24 ;

no

Unification Examples

```
unify(X,Y):- X = Y.
| ?- unify(X,Y), unify(X,a).
```

X = Y = a

```
| ?- unify(X,dummy(a)).
```

X = dummy(a)

```
| ?- unify(X,dummy(a)),unify(X,Y).
```

X = Y = dummy(a)

```
| ?- unify(X,dummy(Y)).
```

X = dummy(Y),

Y = _45 ;

no

Unification, Informally

- Intuitively, unification between 2 Prolog terms tries to associate values with the variables so that the resulting trees, representing the terms, are isomorphic (including matching labels)
- To use a Prolog rule, we must unify the head of the rule with the subgoal to be proved, “matching” term by term

Unification, Informally

- Given a subgoal <functor>(<term>{, <term>}) how to unify it with a clause head?
  - Rule and subgoal have same name
  - Any uninstantiated variable matches any term
    - If term is also an uninstantiated variable, this means if either takes on a value, they both do
  - Integer and symbolic constants match themselves, only
  - A structured term matches another term iff
    - Has same relation name
    - Has same number of components (that is, terms within parentheses) and corresponding components match
  - Lists unify by matching element by element

Unification

- Unification looks for the most general (or least restrictive) value to assign
- A *substitution* (σ) is a finite map from variables to terms in the language

```
append([A|B],Y,[A|Z]):-
?- append([a,b],[c],W)
```

Rule head

query σ: A → a, B → [b], Y → [c], W → [a|Z]

- A term U is an *instance* of another term T, if there is a substitution σ such that U = T σ.

Unification

- Two terms S,T *unify* if they have a common instance U; that is,

$$S\,\sigma_1 = T\,\sigma_2 = U$$

  – Note: if variable X is contained in both S and T, then σ_1 and σ_2 both must have the same substitution for X.

  – If two terms unify, they can be made identical under some substitution

Unification

There may be more than one substitution to unify two terms

`times(Z,times(Y,7)) and times(4,W)`

σ_1: `Z = 4, Y = plus(3,5),`

`W = times(plus(3,5),7)`

σ_2: `Z = 4, W = times(Y,7)`

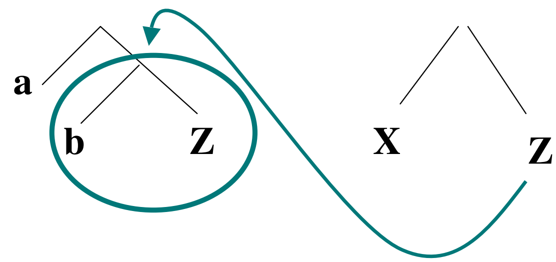
Which substitution is simpler or less restrictive on the values of the variables? σ_2

Most General Unifier

- We say γ is the *most general unifier (mgu)* of two terms, T and W, iff for all other unifiers σ of T and W, $T\sigma$ is an instance of $T\gamma$; therefore, σ can be obtained by a substitution δ applied to γ, $\sigma = \gamma \bullet \delta$
  - ?- `member(A,B)` returns A=_123, B=[A| _] when it could return A= _123, B=[A,b] or A=_123, B= [A, c, d] etc. Note, the 2nd and 3rd B values are obtainable from the mgu by additional substitutions

Occurs Check

- **There are problems with the unification done in some Prolog compilers, which result in an unbounded unification being attempted. Called an *occurs check***
  - **[a,b |Z] = [X | Z] X → a, Z → [b, Z]**



Occurs Check

- **If try to evaluate value of Z, compiler will return `Z=[b,b,b,…` a value that results in an infinite loop in the Prolog interpreter**
- **Unification should check that it doesn't unify a variable with a term containing that same variable**
- **Occurs check was left out of Prolog by Colmerauer because of efficiency (to avoid the run-time cost**
  - **Current Prolog compilers have it**
  - **Example of safety yielding to efficiency (O(n) instead of O(n^2) on list concatenation)**

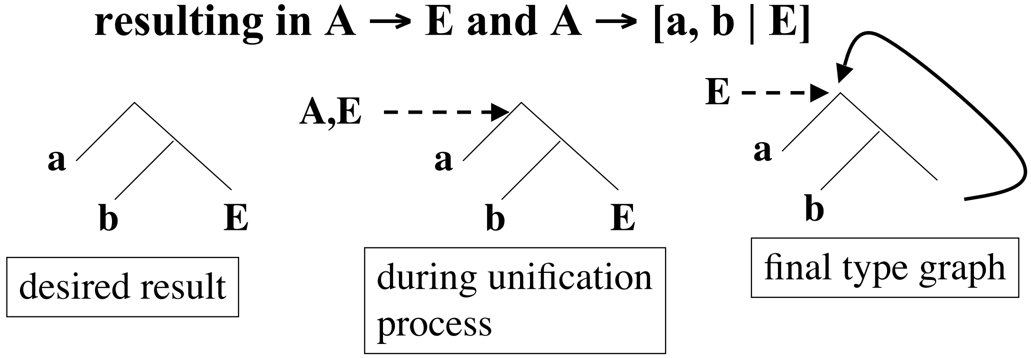
Occurs Check

occursCheck.pl

Useful recursive type to build, a not-fully-evaluated list

```
?-append([ ],E,[a,b|E] )
```

need to unify with `append([ ],A,A)`
resulting in A → E and A → [a, b | E]



Can't be built without occurs check

Generate and Test Paradigm

- **Use of cut (!) to change evaluation order of Prolog clauses.**
- **Already saw cut in definition of \\+**
- **A typical programming style in Prolog is *generate and test***
  - **Can write clauses to generate values and test if they satisfy the desired condition**
  - **Factorial example** fact.pl
  - **N Queens example** queens.pl

Factorial

- **Function to calculate X factorial if X is bound to an integer value**

```
factorial (0,1).
factorial(X,Y) :- W is X-1,
  factorial(W,Z),
   Y is Z*X.
```

If X is not bound to an integer value, then first subgoal (is clause) is undefined.

- **A top-down calculation: n! is (n-1)!*n**

Factorial

- **Add a guard to 2nd rule:**

```
factorial (0,1).
factorial(X,Y) :- integer(X), W is X-1,
  factorial(W,Z), Y is Z * X.
```

This builds f(n) from f(n-1), stepping down to f(0). If we query this new 2nd clause with `factorial(Y,6)`, it will not match, but it will not abort, either.

Factorial

- How about a bottom-up definition?

  ```
  f(0,1).
  f(X,Y):-f(W,Z), X is W+1, Y is Z*(W+1).
  ```

  Here we calculate f(3,Y) by building it up from f(0,1), f(1,1), f(2,2), f(3,6).
- This new definition works for f(3,Y) and f(X,6) but what about f(X,5)? It will infinitely loop on this query. We need a way to control the backtracking mechanism, so it stops computation once a factorial value greater than 5 is returned.

Cut

- Cut (!)
  - Commits system to all choices made since the parent goal was invoked
  - If the parent predicate is re-entered by a backtracking computation, it cannot be re-satisfied. Instead a previous predicate must be re-satisfied.

```
eat_lunch(joe,X):-available(X),cheap(X),!,
                  sick(joe, X).
```

use eat_lunch predicate in another computation:

```
…eat_lunch(joe,Y),…
```

If backtrack into eat_lunch, can't retry available(X) or cheap(X), and can't try another rule for eat_lunch(joe,Y).

Factorial, finally

```
fact(0,1).
fact(X,Y):-fact(W,Z),X is W+1,Y is Z*(W+1).
f2(X,Y):-integer(Y),fact(W,Z),Z>=Y,!,Z=Y,
  W=X.
f2(X,Y):-integer(X),var(Y),fact(X,Y),!.
f2(X,Y):-fact(X,Y).
```

Look at cases:

f2(int,var) - uses 2nd f2 rule for generation

f2(var or int, int) - uses 1st f2 rule to check (int,int) or generate (var, int)

f2(var,var) - uses 3rd f2 rule to generate factorial pairs