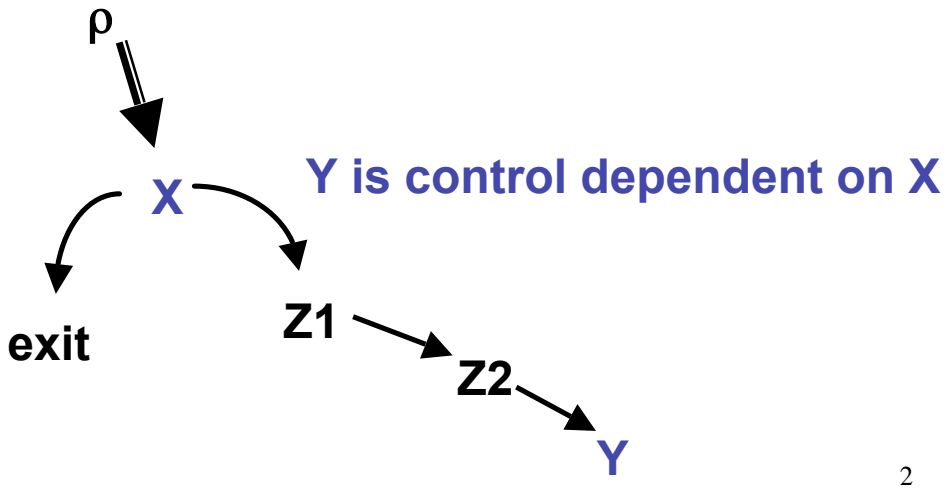
Static Single Assignment Form

- **What is *control dependence?***
  - **Dominators, postdominators**
- **SSA form - each use has one reaching defn**
  - **Dominance frontier**

R. Cytron, J. Ferrante, B. Rosen, M. Wegman, K. Zadeck, "Efficiently Computing Static Single Assignment Form and the Control Dependence Graph", ACM Toplas, vol 13, no 4, Oct 1991, pp 451-490.

Control Dependence

- **Node Y is *control dependent* on node X means there is a logical test at X whose outcome determines if Y is executed.**
- **Y *postdominates* Z iff every execution path from Z to program exit includes Y (analogous to domination on the reverse control flow graph)**



Control Dependence

X,Y ∈ N(CFG)

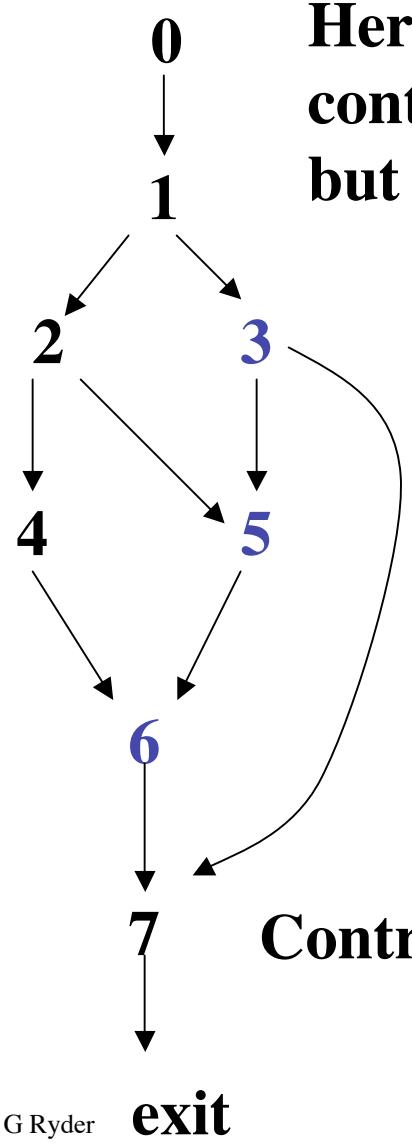
Y is control dependent on X iff

(i) ∃ path from X to Y (X, Z1, Z2,...,Zk, Y) such that ∀ Zi, Zi ≠ X, Zi is postdominated by Y, and

(ii) X is not postdominated by Y

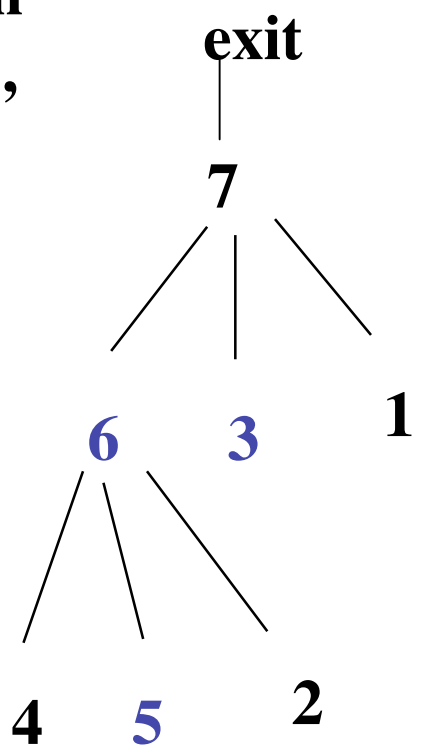
Idea: the predicate evaluated at X determines if Y executes, so once you know that X executes, you know if Y executes

Control Dependence - Example



Control flowgraph

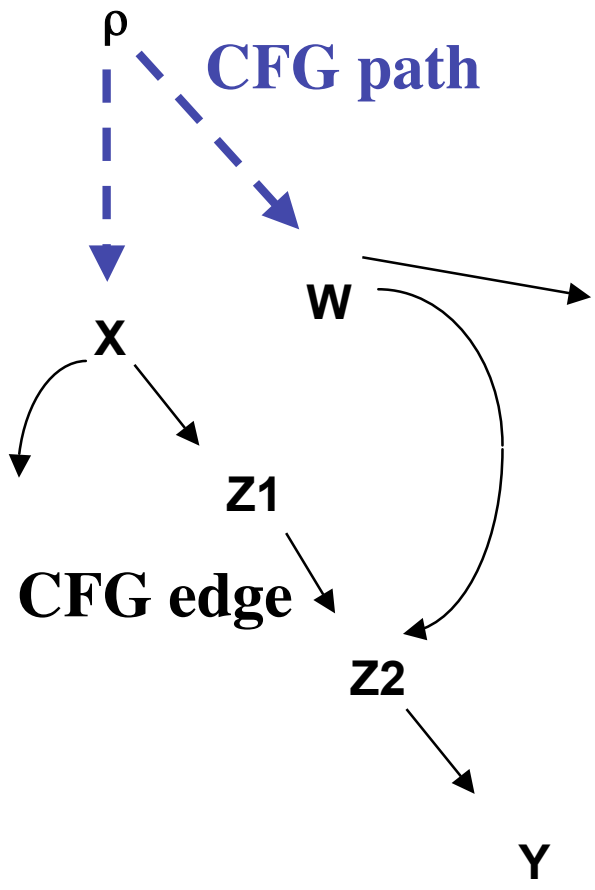
Here nodes 5,6 are both control dependent on 3, but 7 is not.



Postdominator Tree

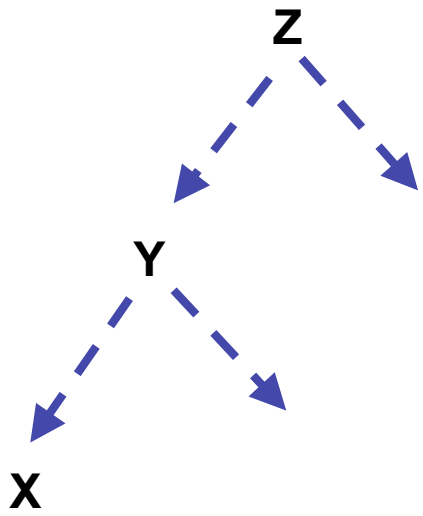
Properties

- **Relation is not unique**
  - **Y can be control dependent on more than one other CFG node**
  - **Y is control dependent on both W and X**



Properties

- **Relation is not transitive.**
  - **X is control dependent on Y, Y is control dependent on Z, but X is NOT control dependent on Z since X does not postdominate Y.**



Control Dependence Algorithm

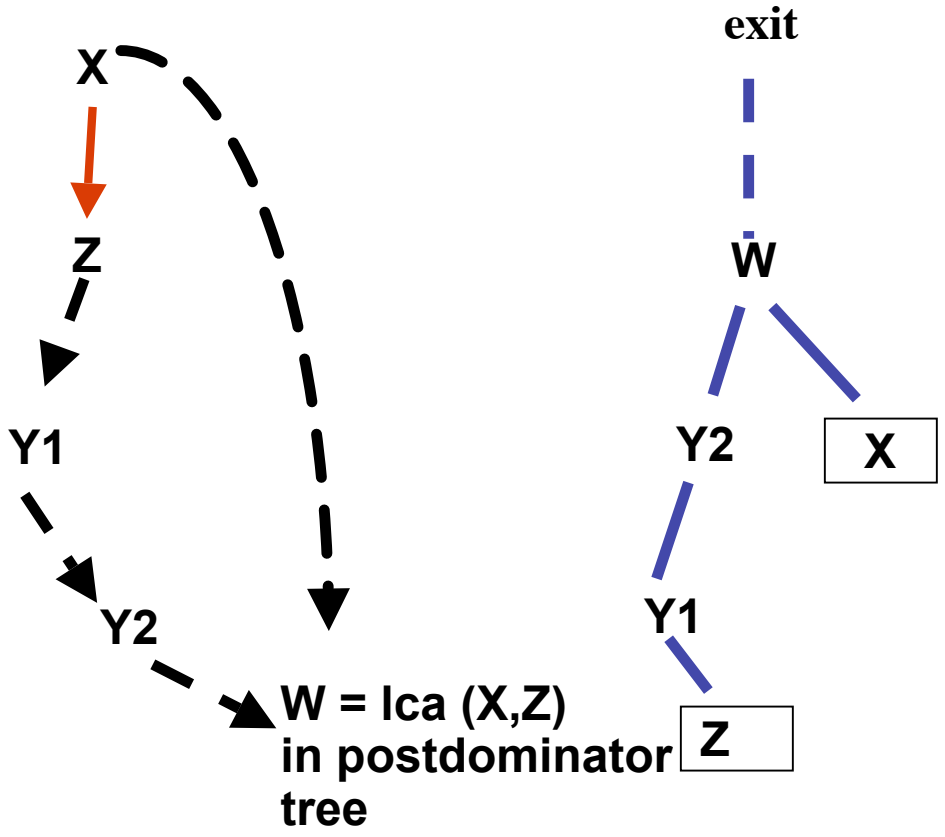
- **Intuition: look for CFG edges such that the target node does not postdominate the source node, then use the postdominator tree to find control dependences.**
- **Algorithm**
  1. **Find postdominators on CFG**
  2. **Form candidate edge set, S = { (X, Z) ∈ G | Z is not an ancestor of X in the postdominator tree}**
  3. **Find X and Z in postdominator tree (all ancestors of Z in tree postdominate Z)**

     Find all nodes that postdominate Z but not X, {Yi}.

     Z and {Yi} are all control dependent on X.

Illustration

Find X and Z in postdominator tree; (X,Z) is candidate edge; all ancestors of Z in tree postdominate Z. Find all nodes that postdominate Z but not X, {Yi}. Then Z and {Yi} are all control dependent on X.



Postdominators

- **Calculated on reverse CFG (same nodes, all edges reversed in direction) by fixed point iteration**

```
Pdom (exit) = {exit}   /* unique exit node */
for n ∈ N - {exit} do
    Pdom (n) = N    /* Max FP calculation */
while some Pdom(n) changes do
{   for n ∈ N - {exit} do
    Pdom(n) = {n} ∪ {∩ Pdom(j) }
                      j ∈ pred(n)
}
```

- **Forward dataflow problem on reversed CFG, meet semilattice**
- **Reflexive relation**

Validation

- **Claim: Given (X,Z) candidate edge in CFG, the least common ancestor(X,Z) in postdominator tree is X or parent(X).** **(Ferrante, et.al., "The Program Dependence Graph and Its Use in Optimization, TOPLAS, July 1987)**

  Proof: Let W= parent(X) in postdom tree. W ≠ Z because X not postdominated by Z. Assume W does not postdominate Z. Then ∃ path from Z to *exit* not containing W. But then adding (X,Z) to that path, creates a path from X to *exit* not containing W. CONTRADICTION.

  Therefore, W postdominates Z.

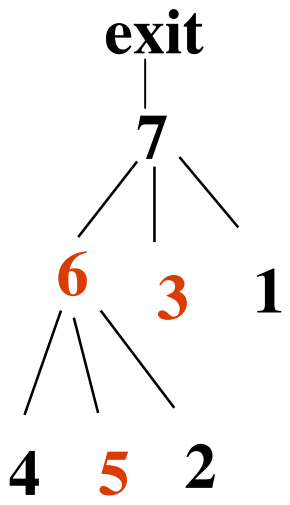
  Therefore, W is ancestor(Z) in postdom tree.

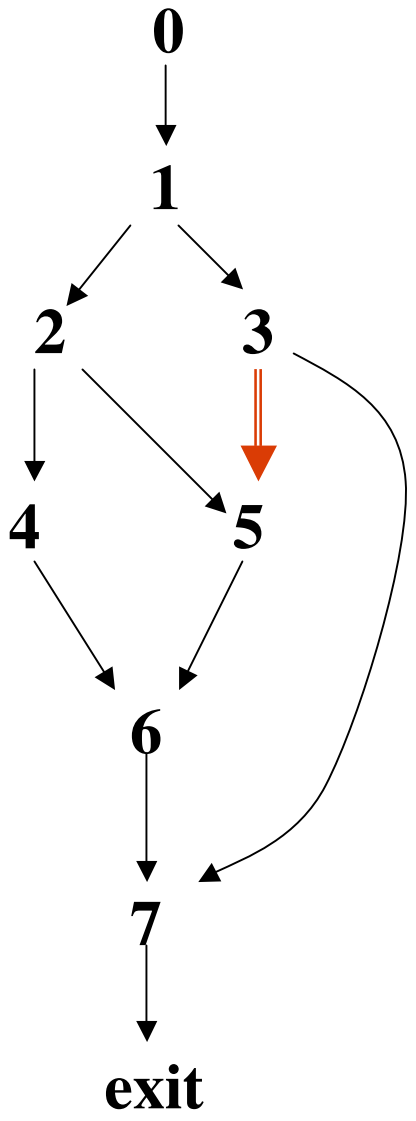
  Therefore, W or X is least common ancestor (X,Z) in postdom tree. qed.

Case 1

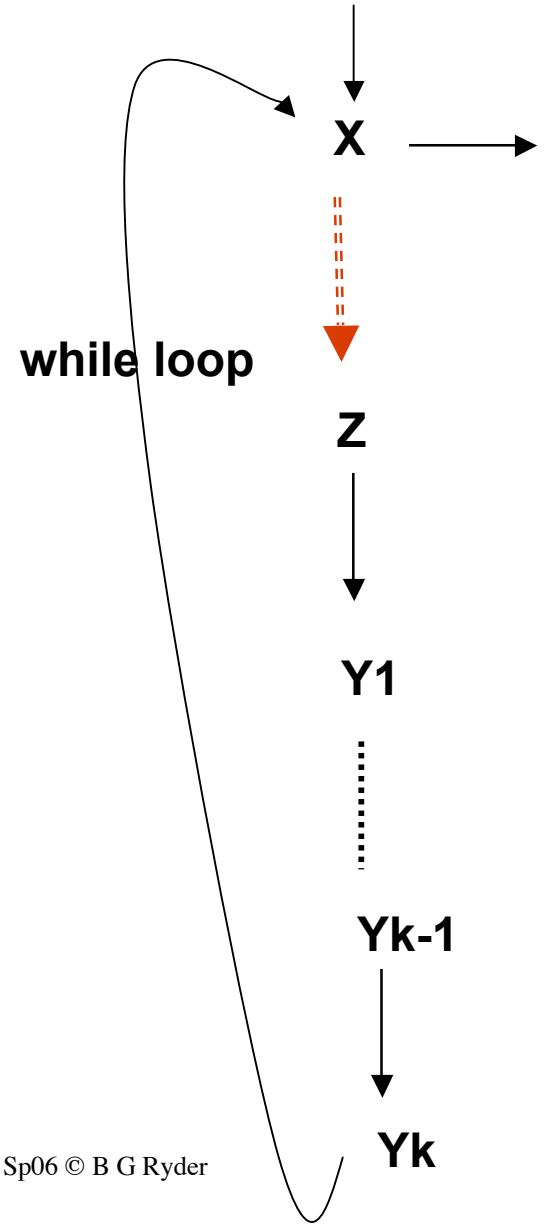
First case: if parent(X) = lca(X,Z), all nodes on postdom tree path (parent(X), Z] are control dependent on X.

5 and 6 control dependent on 3





Case 2: Loops

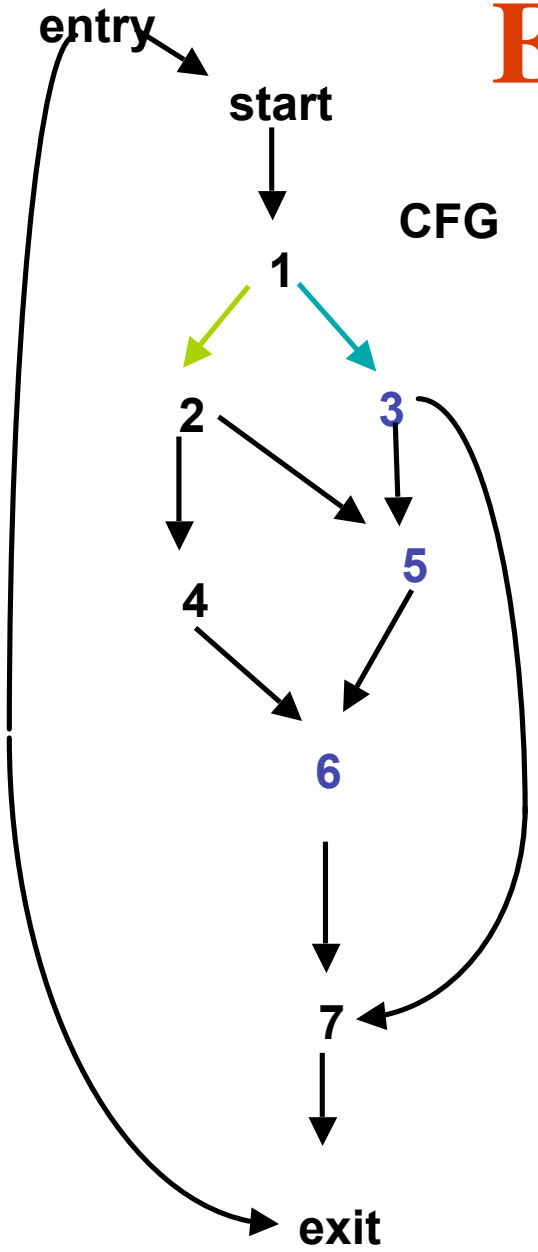


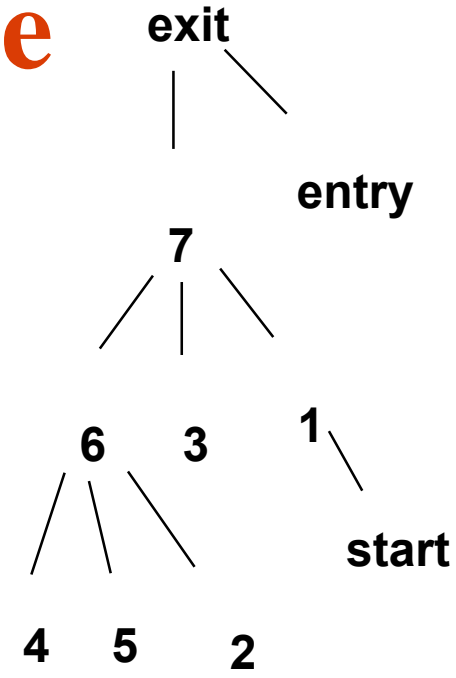
postdominator subtree

lca(X,Z) =X, and Z does not postdominate X.
Z,Y1,...,Yk are all control dependent on X.

Second case:
if X = lca(X,Z) all nodes on postdom tree path (X, Z] are control dependent on X.

Example



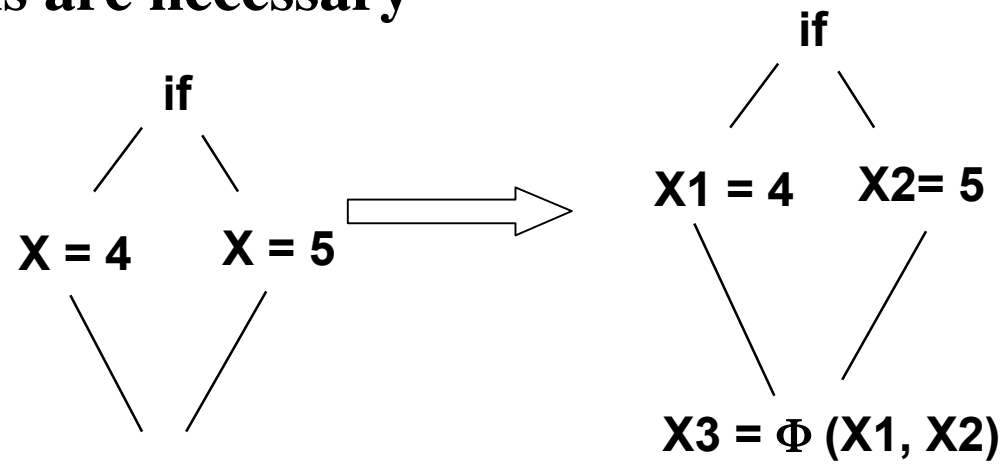


Find all edges (X,Z) st Z does not postdominate X.

(1,2) mark {2,6 } cd on 1.
(1,3) mark { 3 } cd on 1.
(2,4) mark { 4 } cd on 2.
(2,5) mark { 5 } cd on 2.
(3,5) mark { 5,6 } cd on 3.
(entry,start) mark {start,1,7} cd on entry.

Static Single Assignment

- **Idea:** each assignment of variables to be given a unique name; each use of a variable to be reached by only one definition of that variable
- Need to create Φ functions at join nodes reached by more than one definition of same variable
  - **Dominance frontier** is a subset of these join nodes where Φ functions are necessary



Dominance Frontier

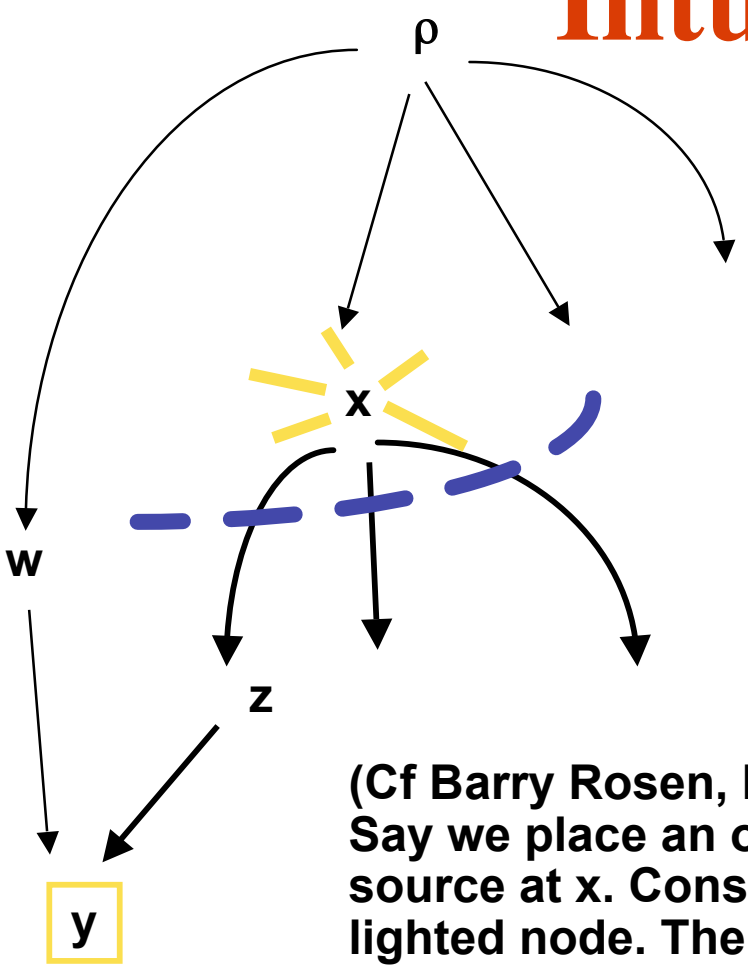
- **Idom(X) is the immediate dominator of X**
  - **Idom(X) ≥ X means Idom(X) dominates X**
  - **Idom(X) >> X means Idom(X) strictly dominates X (i.e., Idom(X) ≠ X)**
- ***Dominance frontier of X (DF(X))*** **is the set of all CFG nodes Y, such that X dominates predecessor of Y in CFG, but X does not strictly dominate Y.**

  DF(X) = { Y | ∃ P ∈ pred(Y), X ≥ P and ¬(X >> Y)}

Dominance Frontier

- **DF of a CFG node containing an assignment to variable v, contains those nodes which other assignments to v may reach**
  - **These are places in the CFG where Φ functions will be necessary**
- ***Iterated dominance frontier* is the dominance frontier of all definitions for a variable, including the added Φ functions**

Intuition



(Cf Barry Rosen, IBM TJ Watson Research Center) Say we place an opaque shade at node x with a light source at x. Consider all dark edges incident on a lighted node. Their targets form the dominance frontier of node x.

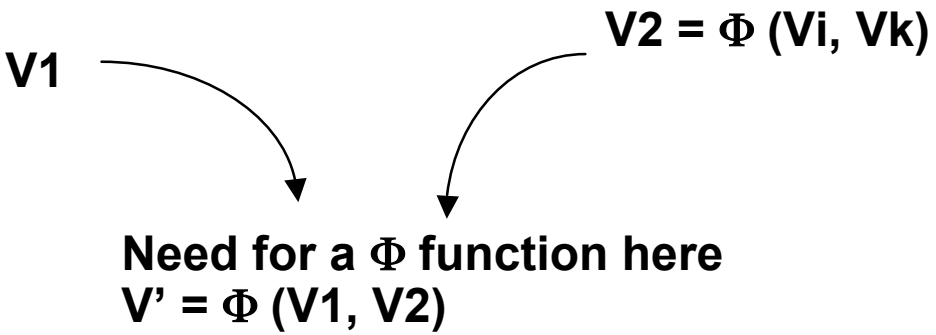
SSA Algorithm

- **Intuitive view:**
  - **Find dominance frontiers for all nodes in CFG**
  - **Locate where Φ functions are needed for all variables v**
    - **Find all definitions of variable v**
    - **Find iterated dominance frontier for v**
  - **Rename variables, propagating definition names to their uses (v --> vi)**

SSA Form

- **Two step translation**
  - **Put Φ functions at some control merge nodes**
  - **Rename variables in branch expressions or on either side of an assignment including Φ assignments**

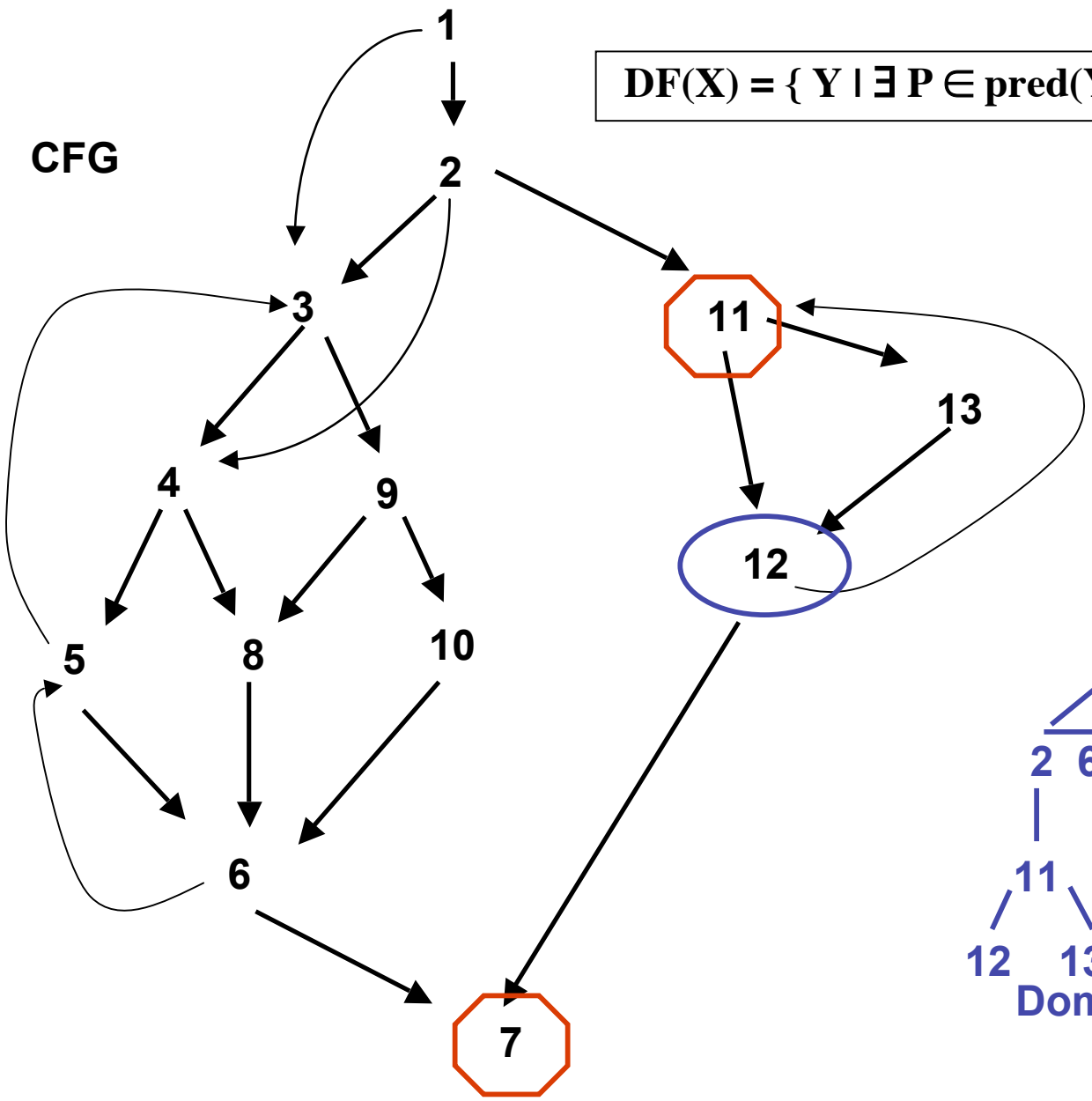
    (V --> Vi)



- ***Minimal* SSA form**
  - **Each use of a variable is target of only one assignment statement in the program text**
  - **Number of Φ functions inserted is as small as possible**

SSA

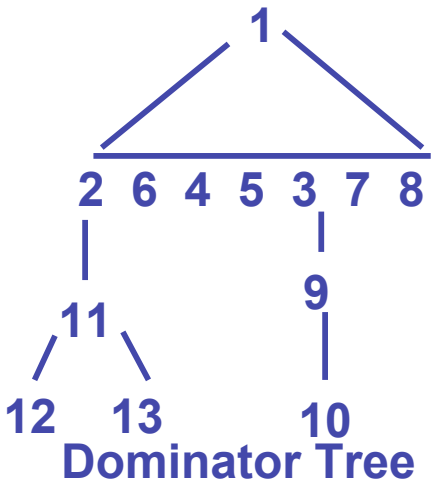
- ***Pruned*** **SSA form**
  - **Only creates Φ functions at program join points p for a variable that has live uses at p or after p.**
- **Special cases: arrays, structures, aliasing**
- **Main work in SSA algorithm: figure out where to put the Φ functions?**



$$DF(X) = \{ Y \mid \exists P \in pred(Y),\ X \geq P \text{ and } \neg(X >> Y)\}$$

DF(12) = {7,11}

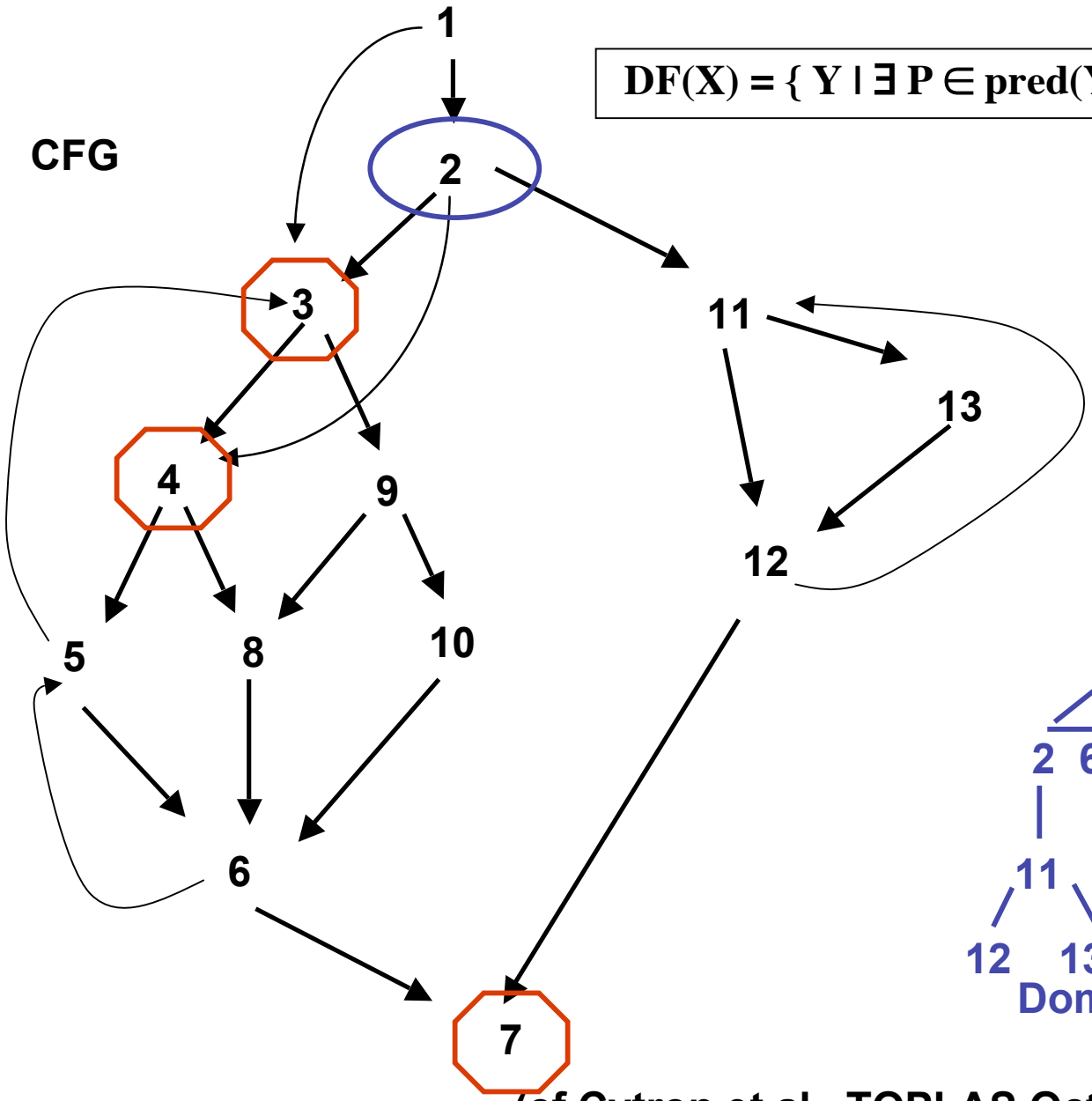
(i)12 ≥ 12 and ¬(12 >>7)
so 7 ∈ DF(12)
(ii)12 ≥ 12 and ¬ (12 >>11)
so 11 ∈ DF(12)



(cf Cytron et.al., TOPLAS Oct 1991)

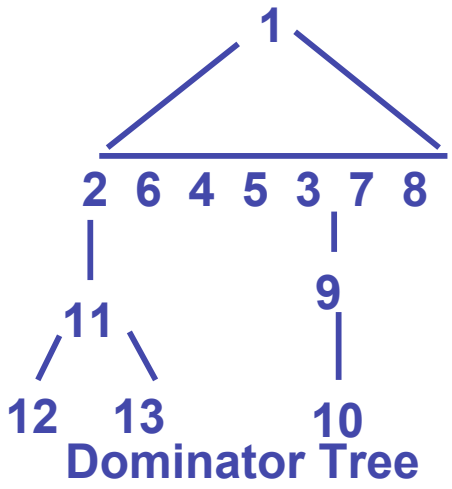
$$DF(X) = \{ Y \mid \exists P \in pred(Y),\ X \geq P \text{ and } \neg(X >> Y)\}$$

CFG



DF(2)= {3,4,7}

(i)2 ≥ 2 and ¬(2 >> 3)
(ii)2 ≥ 12 and ¬(2 >> 7)
(iii)2 ≥ 2 and ¬(2 >> 4)



Dominator Tree

(cf Cytron et.al., TOPLAS Oct 1991)

Initial Algorithm for DF

Give control flowgraph G and its dominator tree.

1. DomBy(X) = {Z | X is ancestor of Z in dominator tree}

2. foreach (X ∈ N) do

3. foreach Y ∈ DomBy(X) do

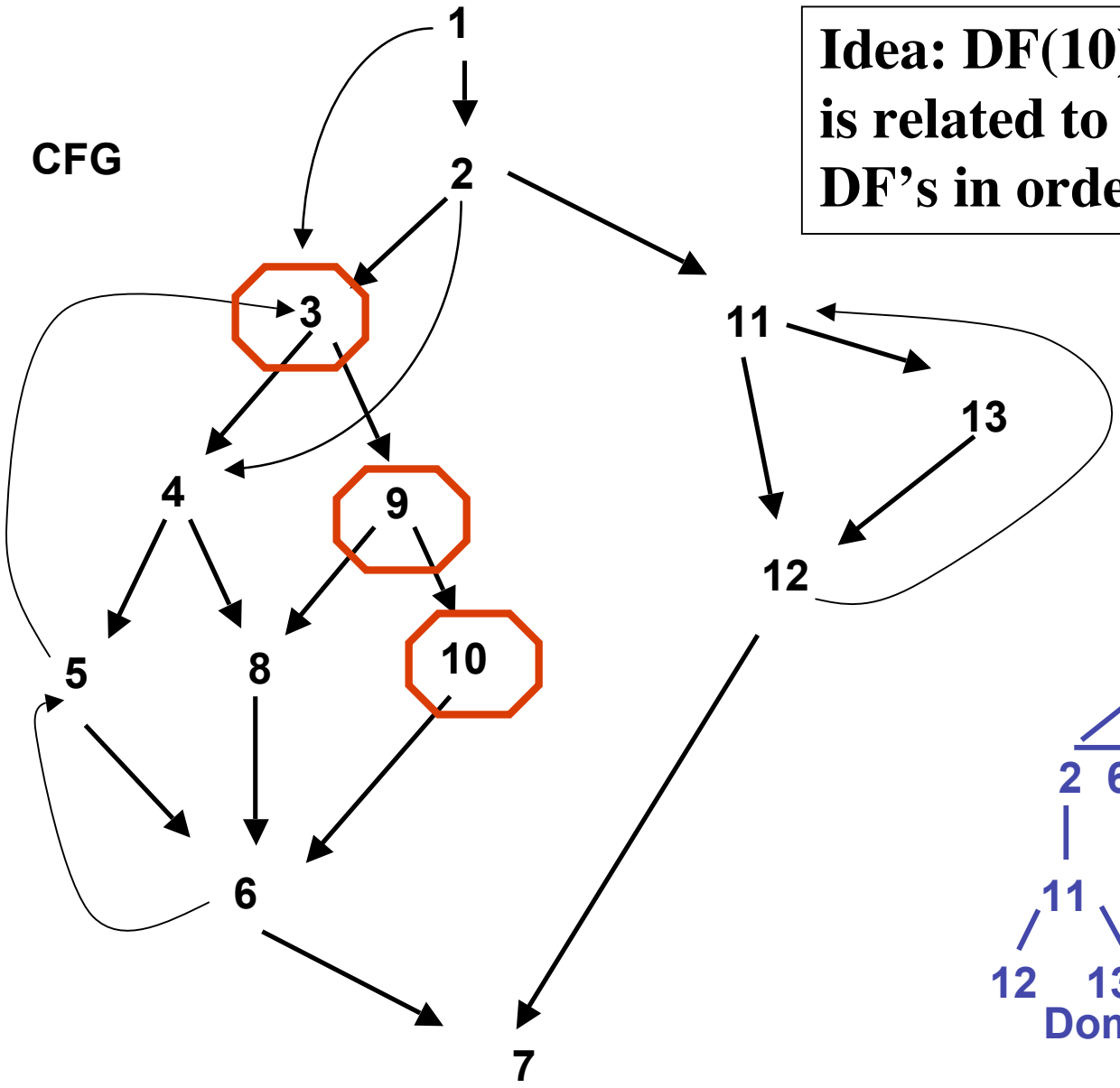
{ foreach Z ∈ succ(Y) do

if ¬ (Z ∈ (DomBy(X) - {X})) then

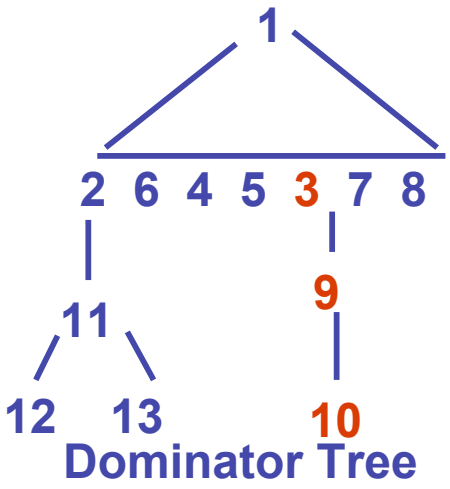
DF(X) <-- DF(X) ∪ {Z}

}

- **This is inefficient because we are reusing the dominance relation for every node in steps 1 and 3. we could accomplish the same work just knowing the immediate dominance relation, if we processed the nodes in a 'good' order in step 2.**



Idea: DF(10) is related to DF(9) is related to DF(3); so calculate DF's in order for 10, 9, 3.



Improved DF Calculation

- Define $DF(X) = DF_{local}(X) \cup DF_{up}(Z)$ where

  Z is child(X) in dominator tree

  $DF_{local}(X) = \{Y \in succ(X) \mid \neg(X >> Y)\}$

  $DF_{up}(Z) = \{ Y \in DF(Z) \mid \neg(idom(Z) >> Y\}$
- Then can recursively calculate DF for all nodes in a bottom up traversal of the dominator tree.
- Easier, equivalent conditions to use

  $DF_{local}(X) = \{Y \in succ(X) \mid idom(Y) \neq X \}$

  $DF_{up}(Z) = \{ Y \in DF(Z) \mid idom(Y) \neq parent(Z)\}$

Improved DF Algorithm

Traverse dominator tree in bottom up order; at node X do:

DF(X) <-- ∅

foreach Y ∈ succ(X) do

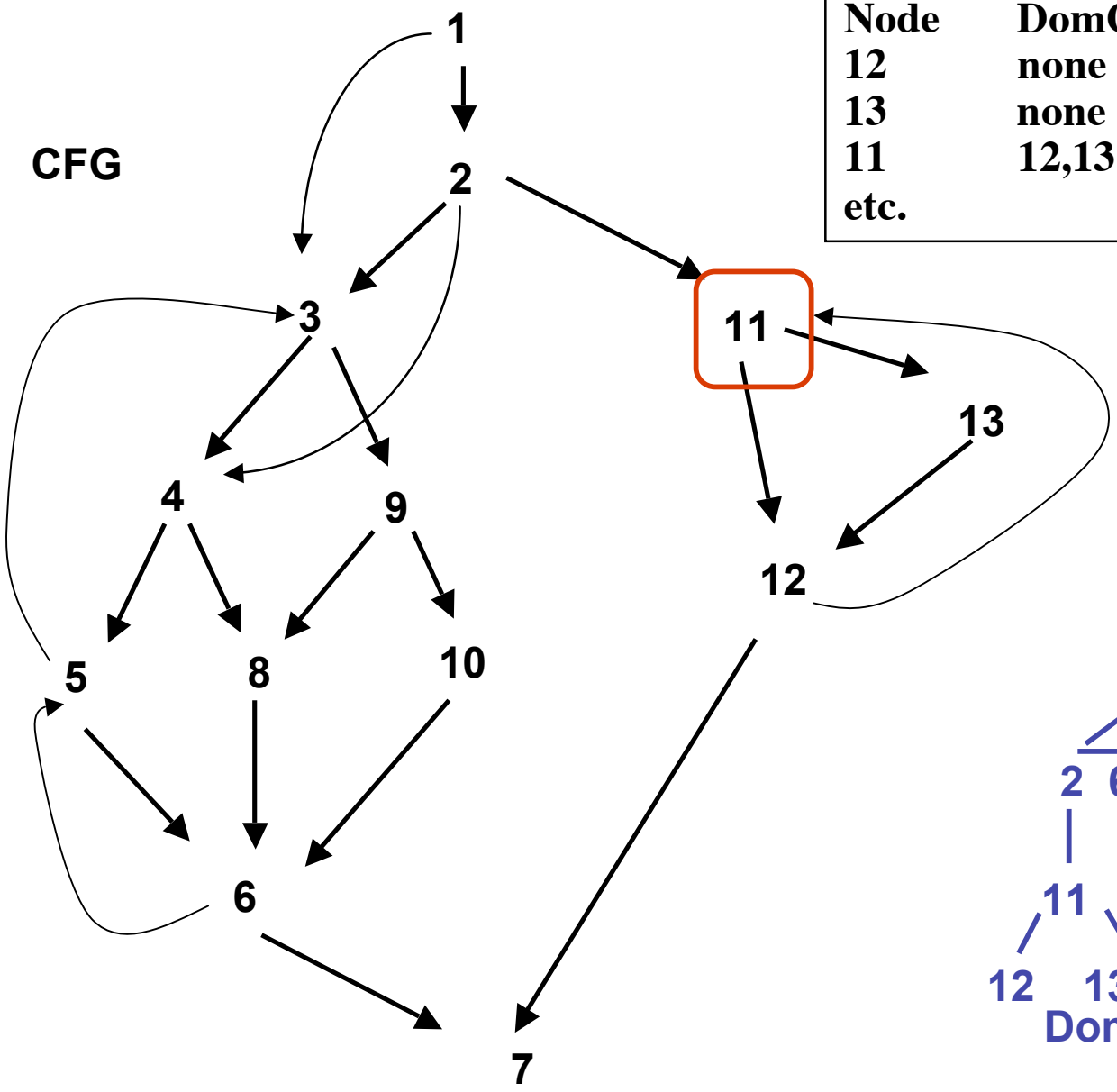
if (idom(Y) ≠ X) then DF(X) <-- DF(X) ∪ {Y}

foreach Z ∈ Children (X) do

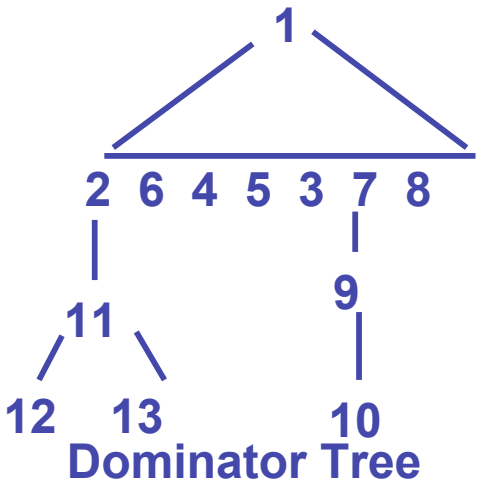
foreach Y ∈ DF(Z) do

if (idom(Y) ≠ X) then DF(X) <-- DF(X) ∪ {Y}

Note: no storage needed for DF $_{local}$ or DF $_{up}$



Node	DomChi	DF(Node)	DF_{local}	DF_{up}
12	none	7,11	7,11	∅
13	none	12	12	∅
11	12,13	7,11	∅	7,11
etc.				

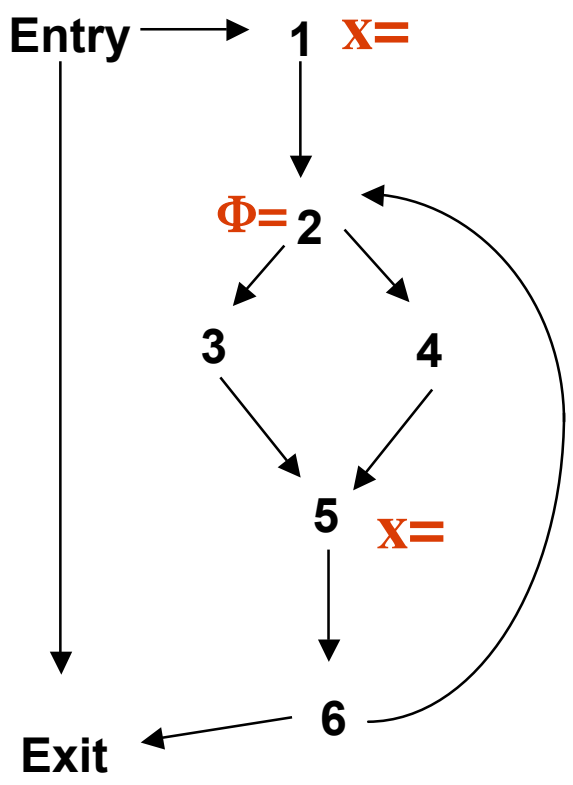


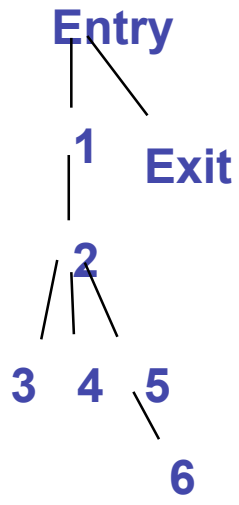
Where to place the Φ's?

- **Want dominance frontiers of all nodes containing Φ functions or source level definitions for a particular variable v**
- ***Iterated Dominance Frontier***
  - **Let S = all nodes containing a definition of variable v in program; then *DF(S)* will be set of nodes where Φ functions for variable v should be place initially.**
  - $DF_1 = DF(S)$
  - $DF_{i+1} = DF(S \cup DF_i)$
- **We need to calculate the iterated dominance frontier with respect to every variable in the program to convert it into SSA form**

Example

```
x = 1;
y = 1;   /*1*/
Repeat /*2*/
   if( )then y = x /*3*/
   else y = x + 1; /*4*/
   print (x,y); /*5*/
   x = 2 * x;
until (  );  /*6*/
```





Find defns of x.

Calculate DF(1), DF(5):

DF(1) = {Exit}; DF(5) = { 2,Exit }

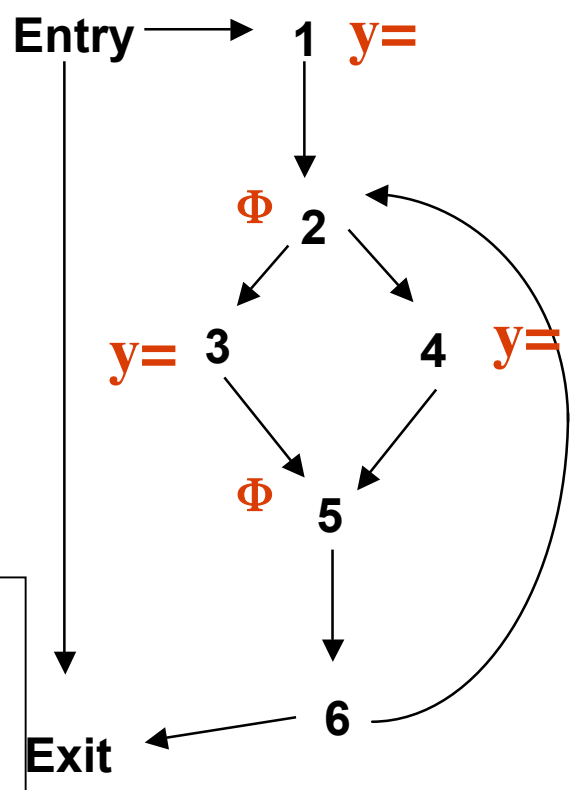
Calculate the iterated DF for x: $DF_1 = DF(1,5) = \{ 2 \}$

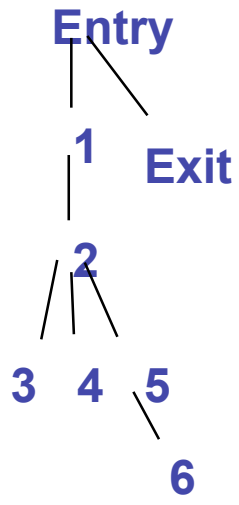
$DF_2 = DF(1,2,5) = \{ 2 \}$

so $DF_2 = DF_1$

Example

```
x = 1;
y = 1;   /*1*/
Repeat /*2*/
   if( )then y = x /*3*/
   else y = x + 1; /*4*/
   print (x,y); /*5*/
   x = 2 * x;
until (  );  /*6*/
```





Find defns of y.

Calculate DF(1), DF(3), DF(4):

DF(1) = {Exit}; DF(3) ={5}; DF(4) = {5}

Calculate the iterated DF for y:

DF_1 = DF(1,3,4) = { 5 }

DF_2 = DF(1,3,4,5) = { 5,2 }

$DF_3 = DF_2$

Variable Renaming

new code:

```
X1 = 1;
Y1 = 1;
repeat
   X2 = Φ(X1, X3);
   Y2 = Φ(Y1, Y5);
    if ( ) then Y3 = X2
          else Y4 = X2 + 1;
   Y5 = Φ(Y3, Y4);
   print (X2, Y5);
   X3 = 2*X2;
until (  );
```

old code:

```
x = 1;
y = 1;  /*1*/
Repeat /*2*/
   if( )then y = x /*3*/
   else y = x + 1; /*4*/
   print (x,y); /*5*/
   x = 2 * x;
until (  );  /*6*/
```

Renaming done through top down traversal of the dominator tree; process user definitions and Φ functions in same traversal;

For each variable Z need stack for current index i for Zi and count of total # defs for Z seen so far

More on control dependence

- **Dominance frontiers in reverse CFG yield the control dependence relation**
  - **Y is control dependent on X in CFG *iff* X ∈ DF(Y) on reverse CFG**
    - **For structured programs (i.e., *whiles*, *ifs*, straight-line code), DF(n) contains ≤ 2 nodes; therefore, every node is control dependent on at most 2 other nodes in structured programs**
  - **If there is a non-null path from X to Y such that Y postdominates every node after X on that path in the CFG, then Y will dominate those nodes, but not X in the reverse CFG**
  - **If Y does not strictly postdominate X in the CFG, then Y will not strictly dominate X in the reverse CFG**

Experimental Observations

- **Size of dominance frontier varies linearly with size of program**
- **Size of control dependence graph is linear in size of program**
- **Found no relation between program size and**
  - **total #assignments in final program/ total # assignments in original program**
  - **total # defns and uses in final program/total # defns and uses in original program**
- **Claimed number of Φ functions varies linearly with program size (Cytron et.al., Fig 21)**

Complexity

- Each assignment statement is mapped to a tuple of identifiers <u,v,z..> = <...>, same length tuple of expressions
- Finding DFs for CFG takes $O(E + \Sigma_n |DF(n)|)$ *0*
- Let $A_{total}(n) = A_{original}(n) + A_{\Phi}(n)$

Then work of Φ function placement (i.e., finding iterated dominance frontiers) is $O(\Sigma_n (A_{total}(n) * |DF(n)|))$ *1*

where $|DF(n)|$ is observably small in practice, so this is effectively $O(A_{total}(n) * averDF)$, for averDF being weighted average of |DF(X)| sizes

- Renaming takes $O(M_{tot})$ *2* where M_{tot} is total number of variable occurrances in resulting program
- So worst case complexity is the sum of these three terms.

Optimizing the DF algorithm

"Computing Φ nodes in Linear Time Using DJ graphs",
V. Shreedhar and G. Gao, JI Programming Languages 1995

- **Idea: if Y is an ancestor of X in the dominator tree, then if DF(X) has been computed, you can use it to compute DF(Y).**
  - **Yields a linear time algorithm for Φ placement**
- **Use new graphical representation, *DJ graph*, related to dominator tree**
  - **Nodes are same as CFG**
  - **Edges are either dominator tree edges or join edges**
    - **CFG edge (X,Y) is a *join edge*, if X does not strictly dominate Y (Y called *join node*)**

DJ Graph

"Computing Φ nodes in Linear Time Using DJ graphs",
V. Shreedhar and G. Gao, JI Programming Languages 1995

- **DJ graph built from dominator tree and CFG**
  - **D-edges: from dominator tree (entire tree included)**
  - **J-edges: all (X,Y) from CFG st ¬(X>>Y)**
    - **join edges, Y is join node**
    - **Find by marking all edges (X,Y) where X¬=idom(Y)**
  - **Essentially, DJ graph is dominator tree augmented with join edges**

DJ Graph

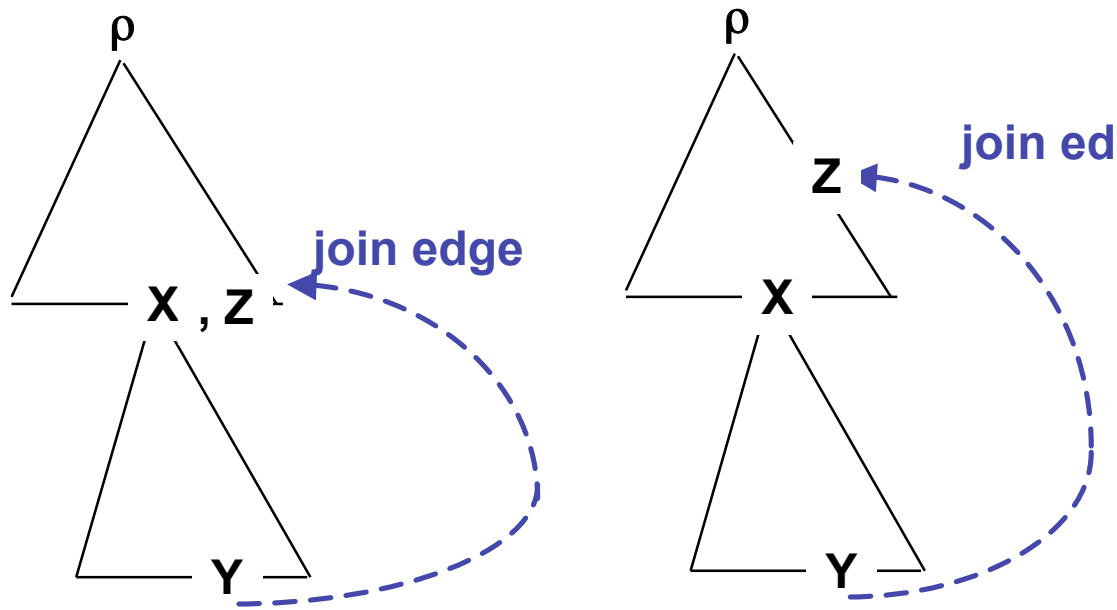
- **DJ graph is linear in size of CFG (Thm 3.1) because $|E_{DJ}| < |N| + |E|$**
- **Assign *level number* to each dominator tree node, equal to its depth in dominator tree from root**
  - **Use these to order the DF computations**
- **if X ∈ DJ graph, then X.level ≤ Y.level,**

∀ Y ∈ DF(X) and ∀ Y ∈ IDF(X) (Thm 3.2)

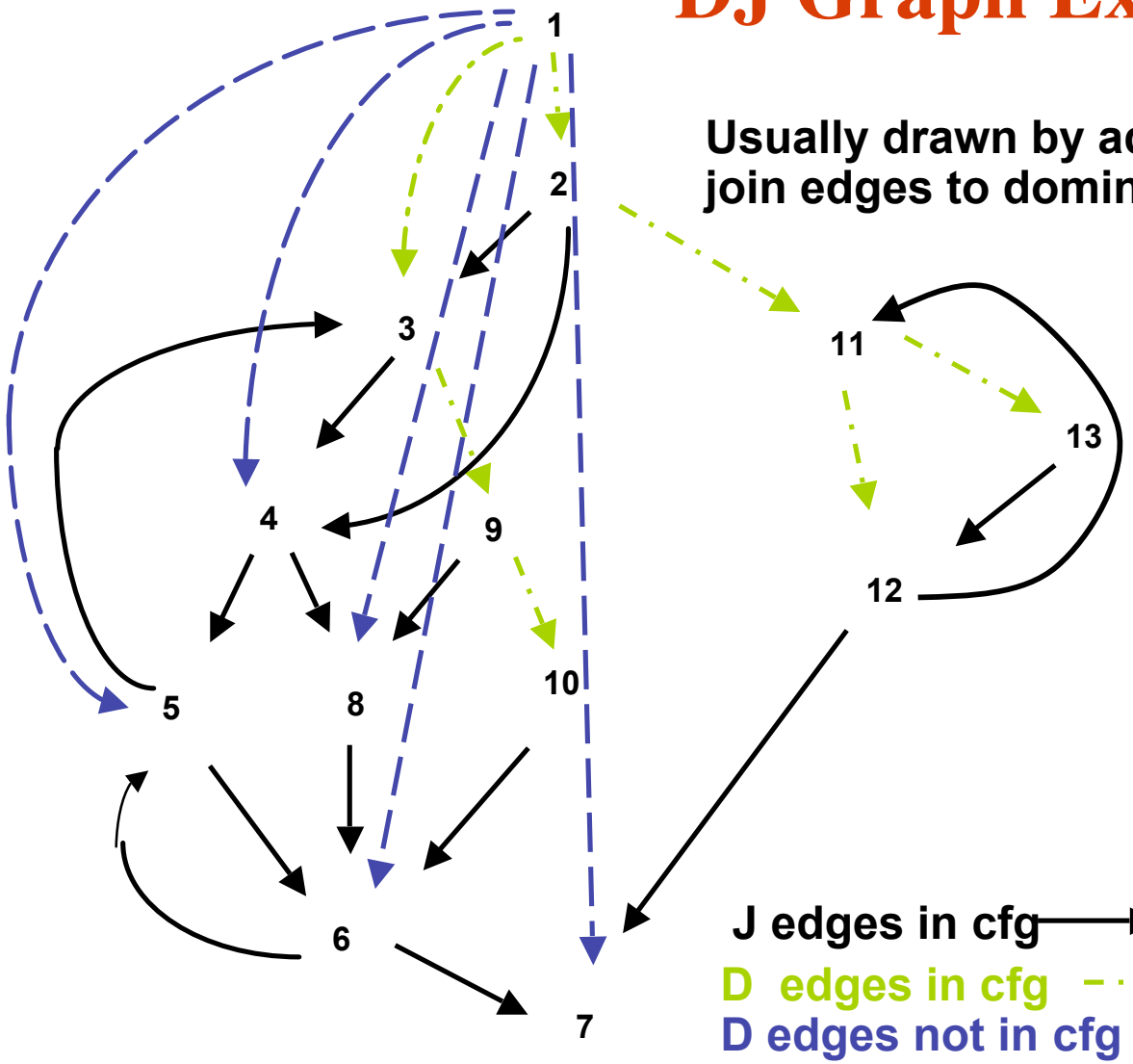
- **Idea: calculate DF(X) in bottom up order on dom tree; Use D-edges to order the DF calculation; use J-edges to identify the Φ nodes**

DJ Graph

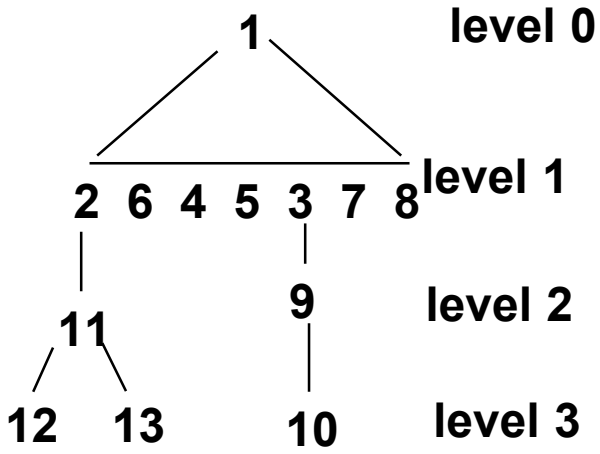
- **A node Z ∈ DF(X) *iff* ∃ Y ∈ subtree of dom tree rooted at X with (Y,Z) as a J-edge and Z.level ≤ X.level (lemma 3.1)**



DJ Graph Example



Usually drawn by adding join edges to dominator tree.



Dominator Tree

J edges in cfg

D edges in cfg

D edges not in cfg

Naive DF Calculation

```
DF(X) = ∅
∀Y ∈ Subtree(X)
{   If ((Y,Z) is a J edge)
        if (Z.level ≤ X.level)  DF(X) = DF(X) ∪ {Z}
}
```

- If use algorithm to compute DF's for all nodes and then apply definition of iterated DF (IDF), have a quadratic method
- But we can alter this approach to compute IDF in linear time ∀ CFG nodes
- Key observations
  - Order of DF computation wrt dominator tree is crucial
  - Can limit J edges to check to compute DF

Algorithm Defns

- **Each node can be marked**
  - **(visited, not visited), (in N_α , not in N_α), (in N_ϕ , not in N_ϕ), with its level number**
- **Each CFG edge is a D or J edge**
- ***Ordered buckets* - a restricted priority queue, implemented as an array of lists**
  - **Node W with level = i is saved to be processed in OrderedBuckets.list[i]**

Algorithm Overview

Input: set of initial variable definition nodes(N_α); Output: IDF(N_α)= N_Φ

/* Initialize all data structures; Insert N_α into OrderedBuckets */

while (OrderedBuckets ≠∅) do

CurrentRoot = GetNode(); //*removes node with max level from OrderedBuckets*

Mark CurrentRoot as visited;//*CurrentRoot is global variable*

Visit(CurrentRoot);

endwhile;

procedure visit(X)

{ ∀ Y ∈ succ(X) in DJ graph do

{ if ((X,Y) is J edge) then

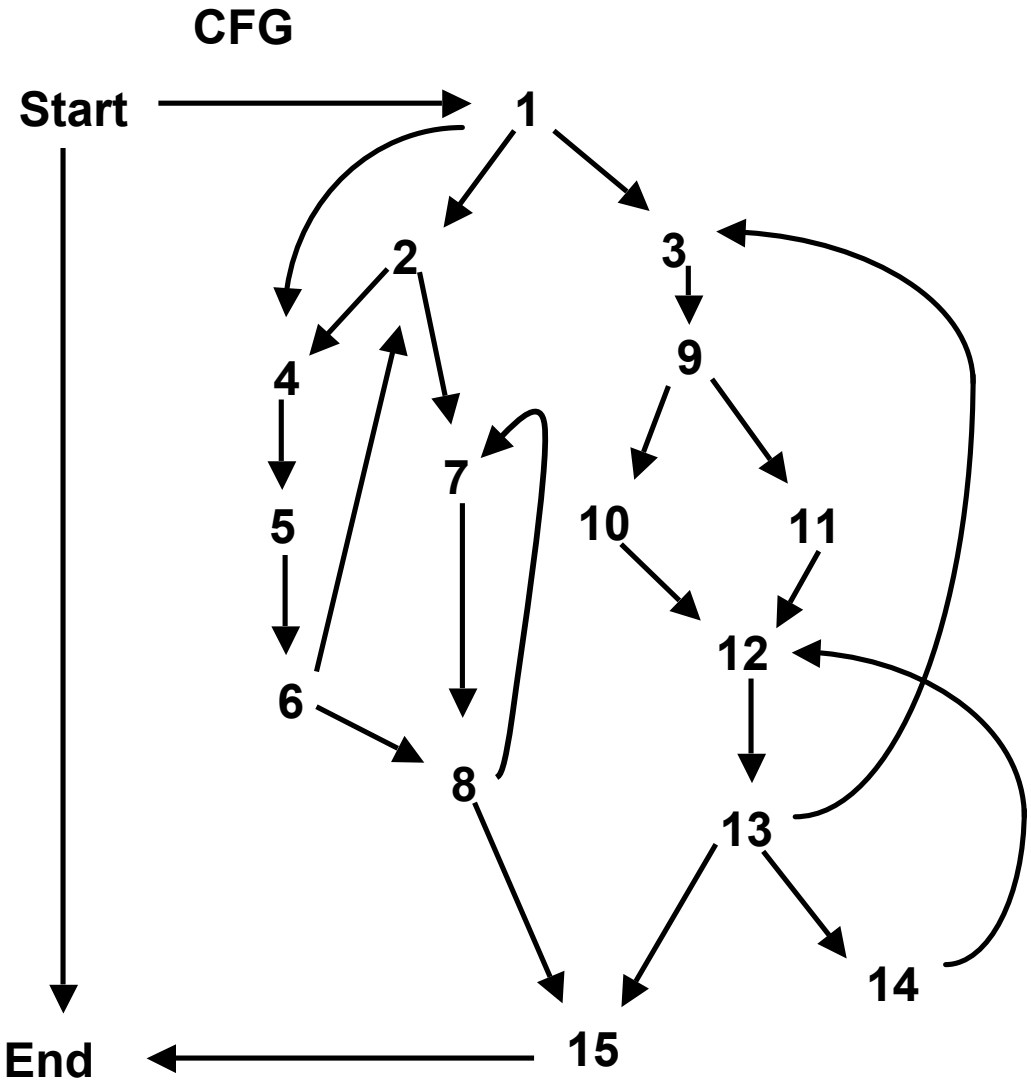
{ if (Y.level ≤ CurrentRoot.level) then

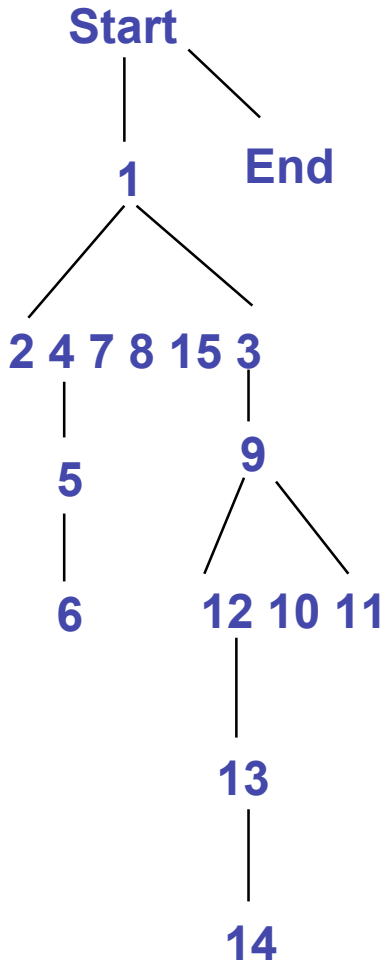
// Add Y to IDF(CurrentRoot) if not already there and mark it added (in N_ϕ); add Y to OrderedBuckets unless Y there already}

else // (X,Y) is D edge; if Y is not yet visited, then mark it visited and execute visit(Y);

}

Example

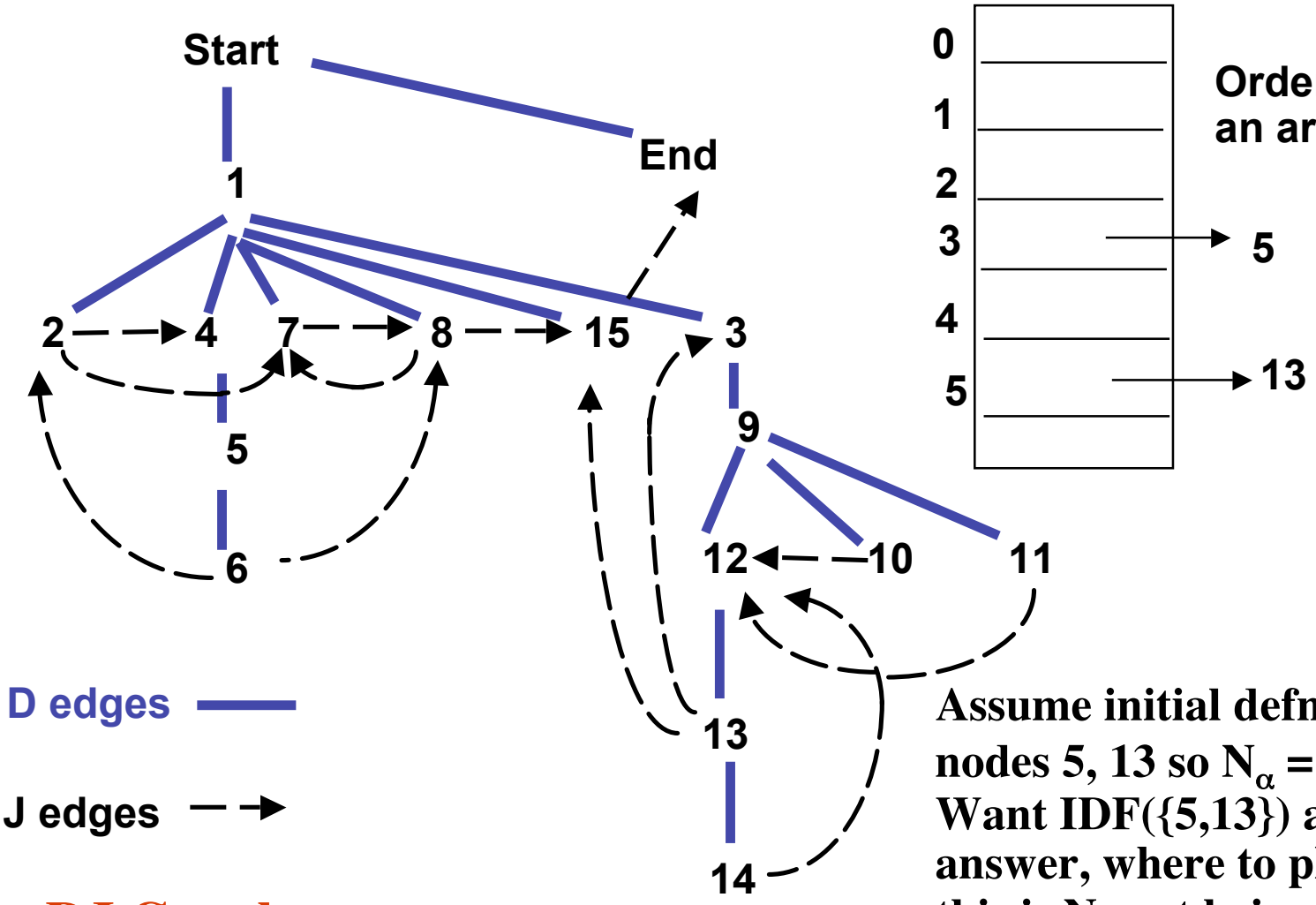




Dom Tree

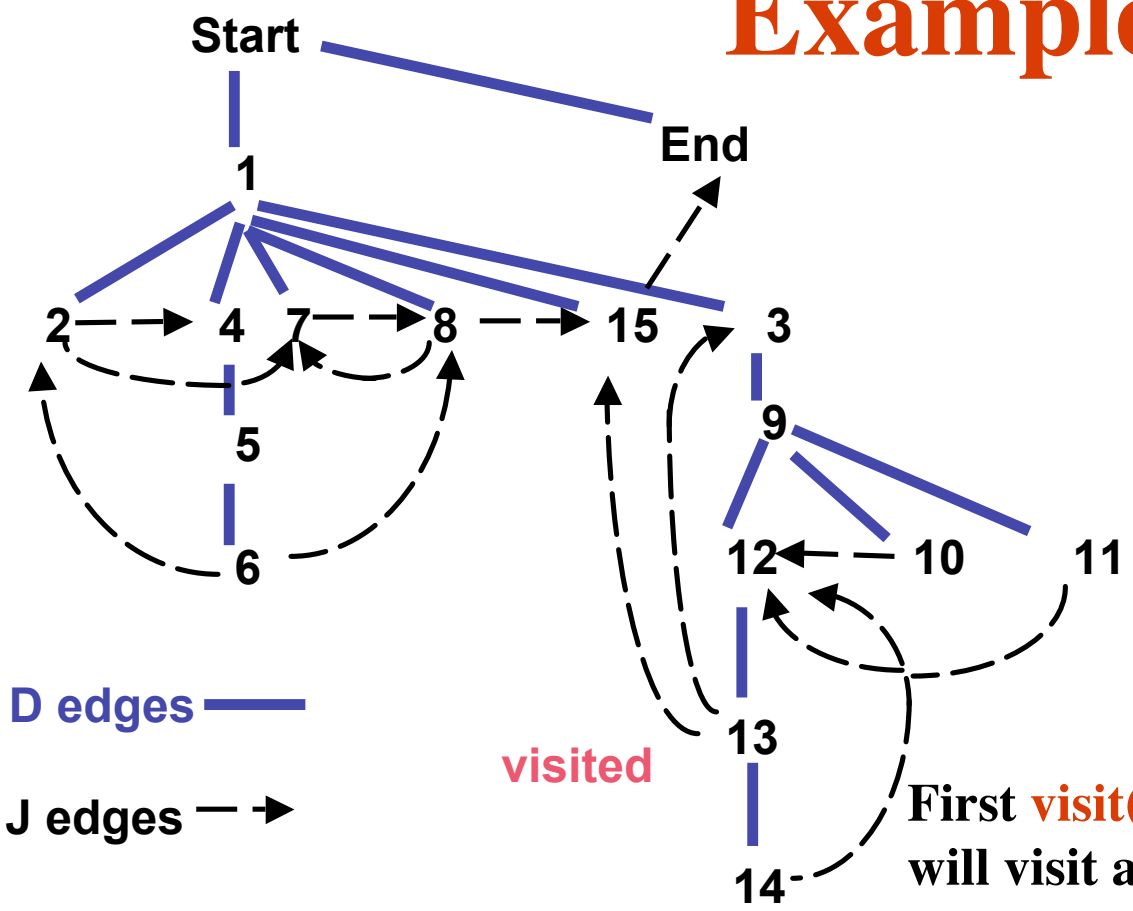
"Computing Φ nodes in Linear Time Using DJ graphs",
V. Shreedhar and G. Gao, JI Programming Languages 1995, p196

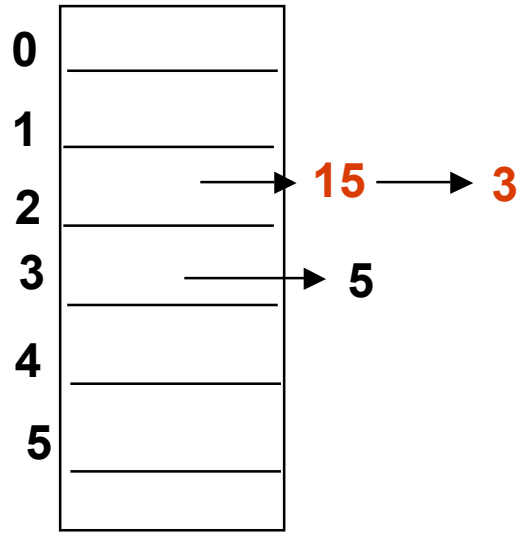
Example



Assume initial defns at nodes 5, 13 so N_α = {13,5}. Want IDF({5,13}) as final answer, where to place Φ nodes; this is N_Φ set being formed. Initialize OB with N_α level(13)=5, level(5)=3.

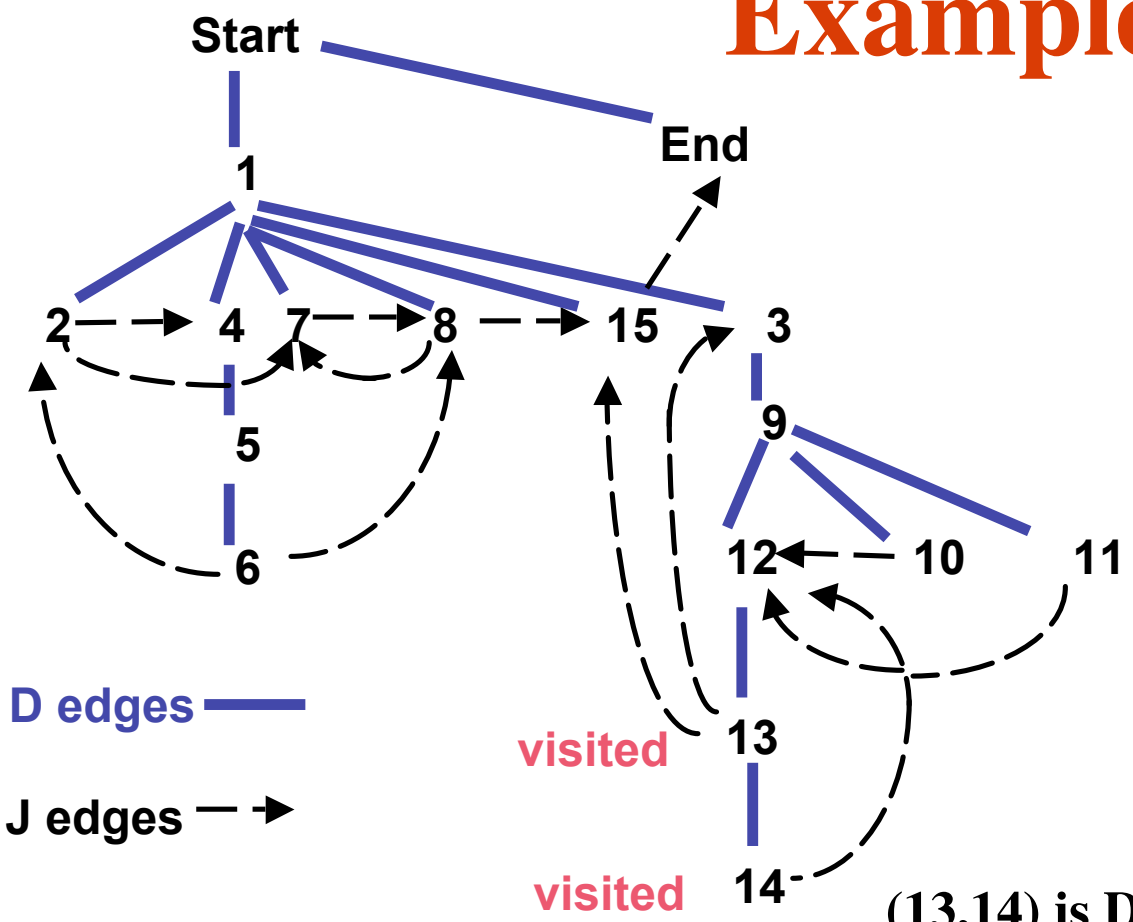
Example

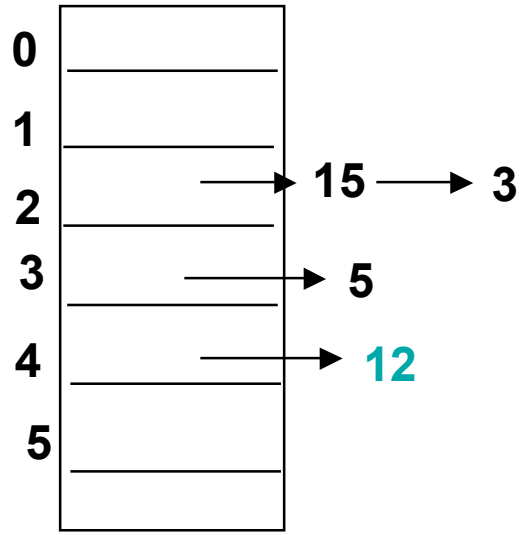




First visit(13), removing it from OB. will visit all succ(13) in DJ graph. CurrentRoot = 13. (13,15), (13,3) are J edges. test levels and find must add 15, 3 to N_Φ and OrderedBuckets;

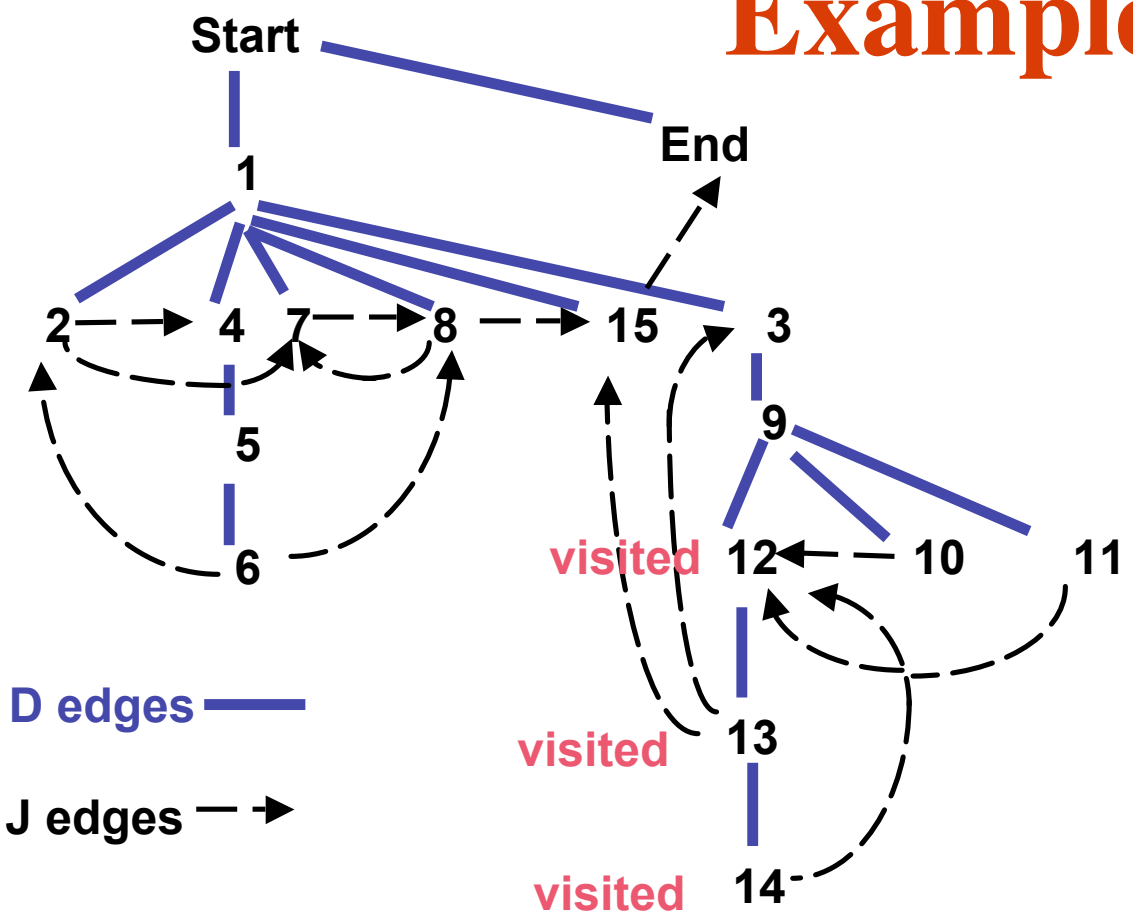
Example

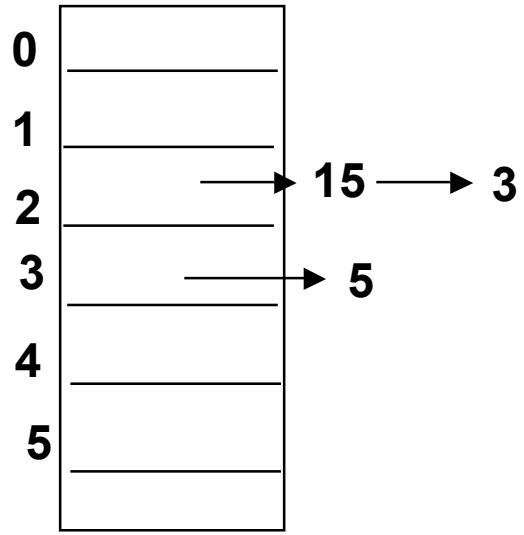




(13,14) is D edge so visit(14).
(14,12) is J edge so add 12 to N_Φ and OrderedBuckets;
then visit(13) terminates.

Example





Now do visit(12); since (12,13) is D edge and 13 marked visited, DF(12) = empty and it adds nothing to N_{Φ}. Next do visit(5), etc.