CS 4204 Computer Graphics

Introduction to Ray Tracing

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Reference: Ed Angle, Interactive Computer Graphics, University of New Mexico, class notes

Raytracing (Picture from Povray.org)

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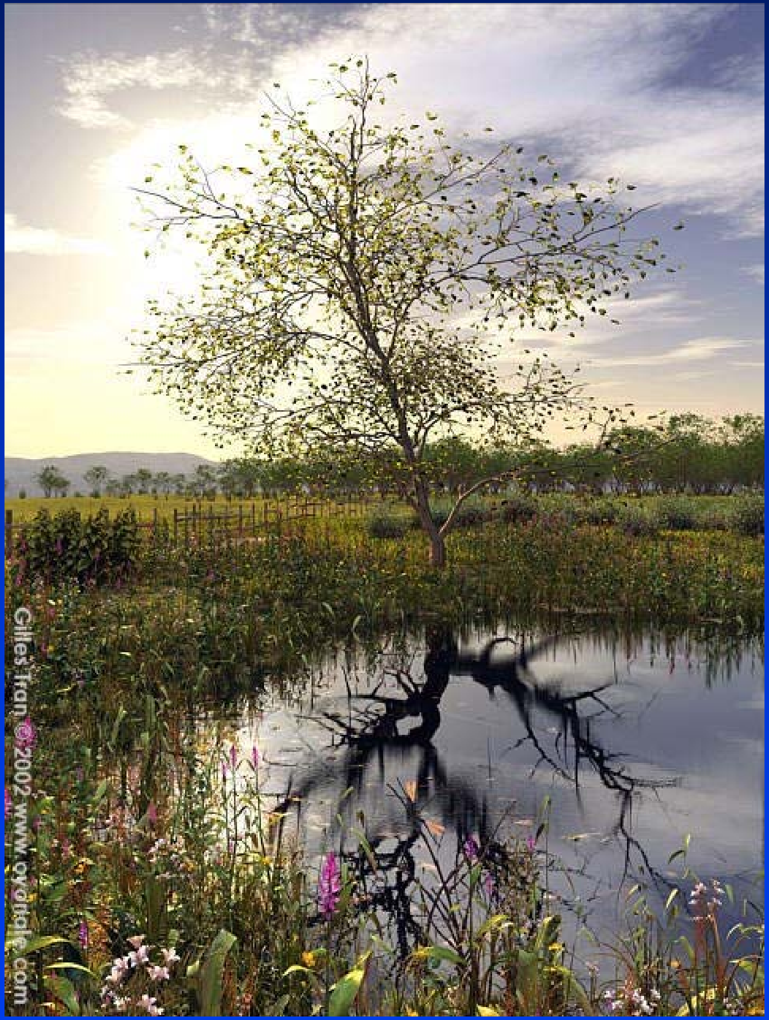

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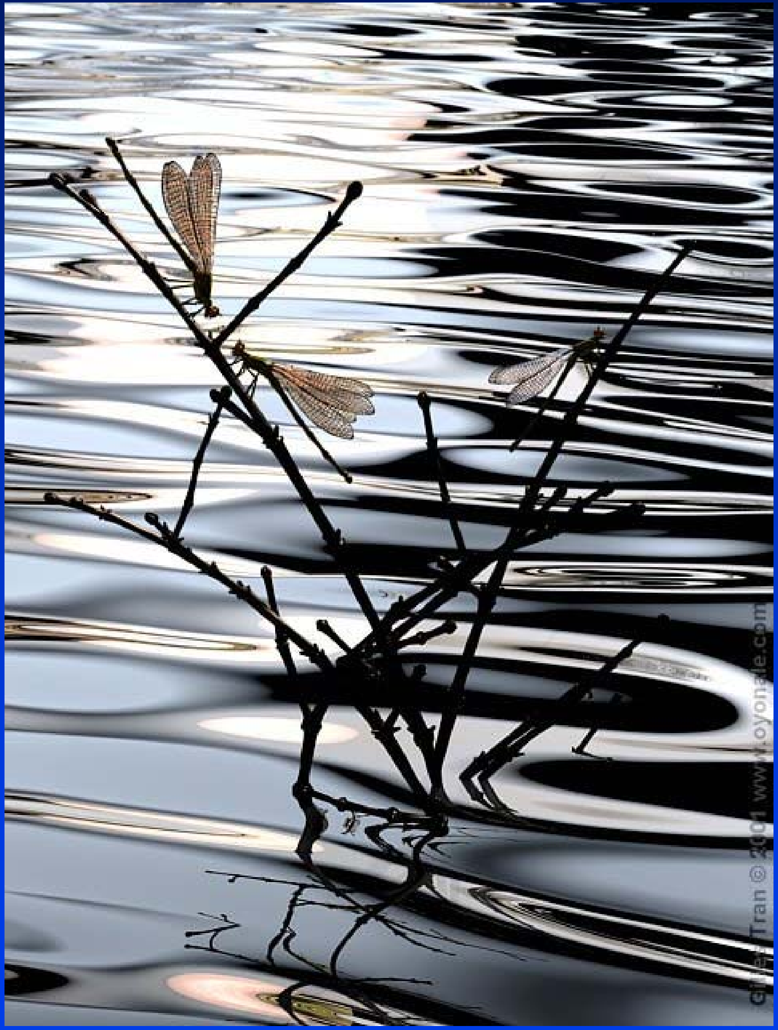

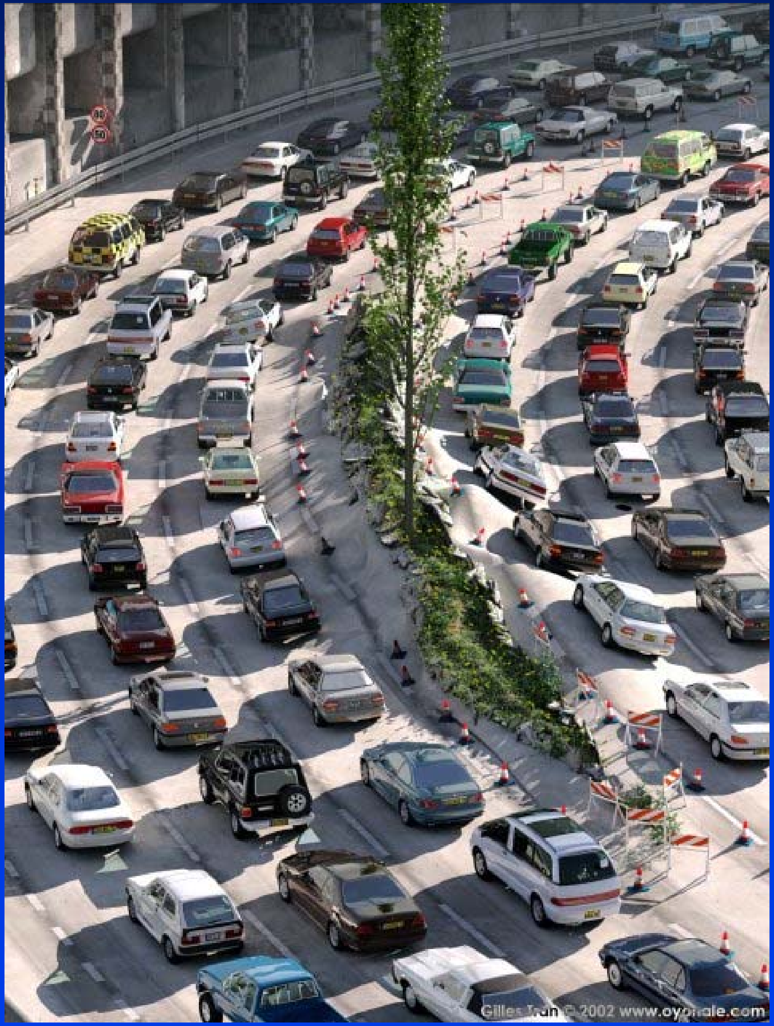

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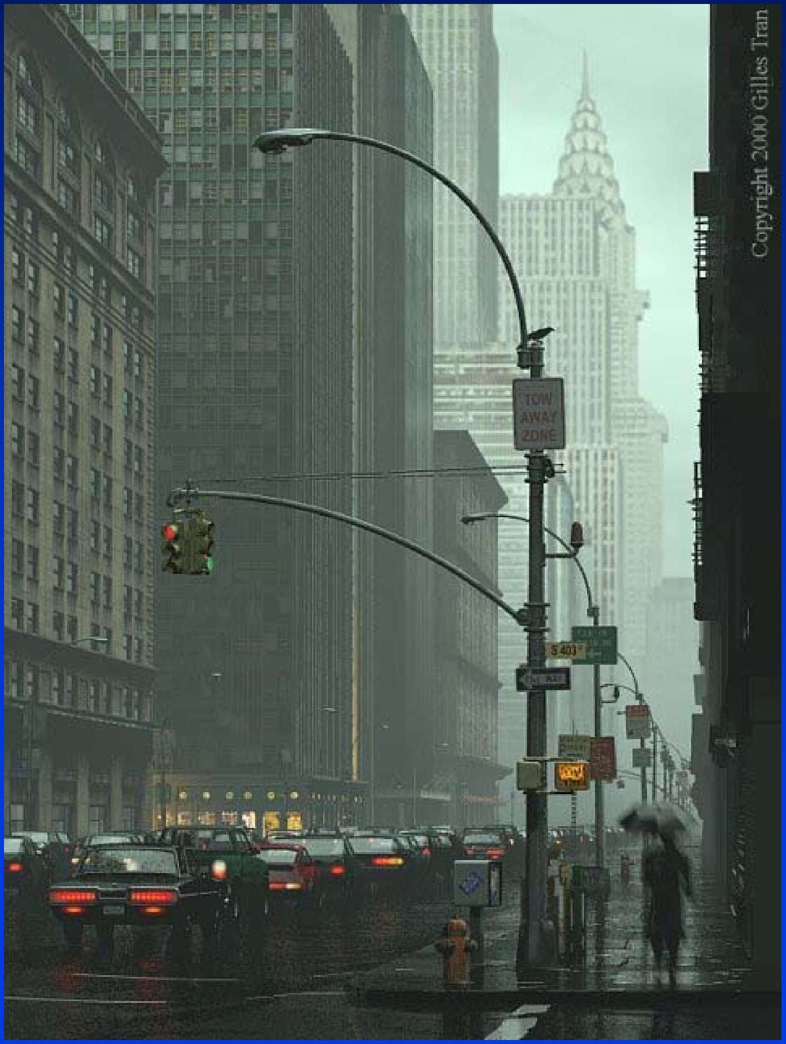
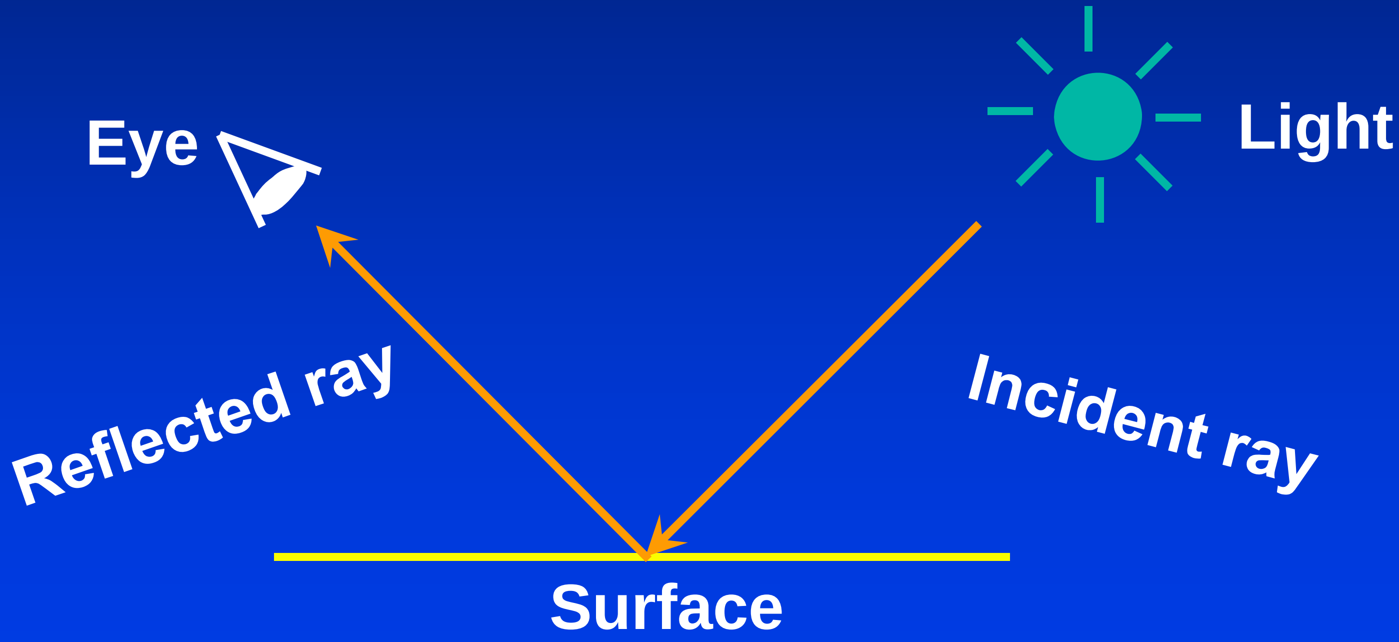


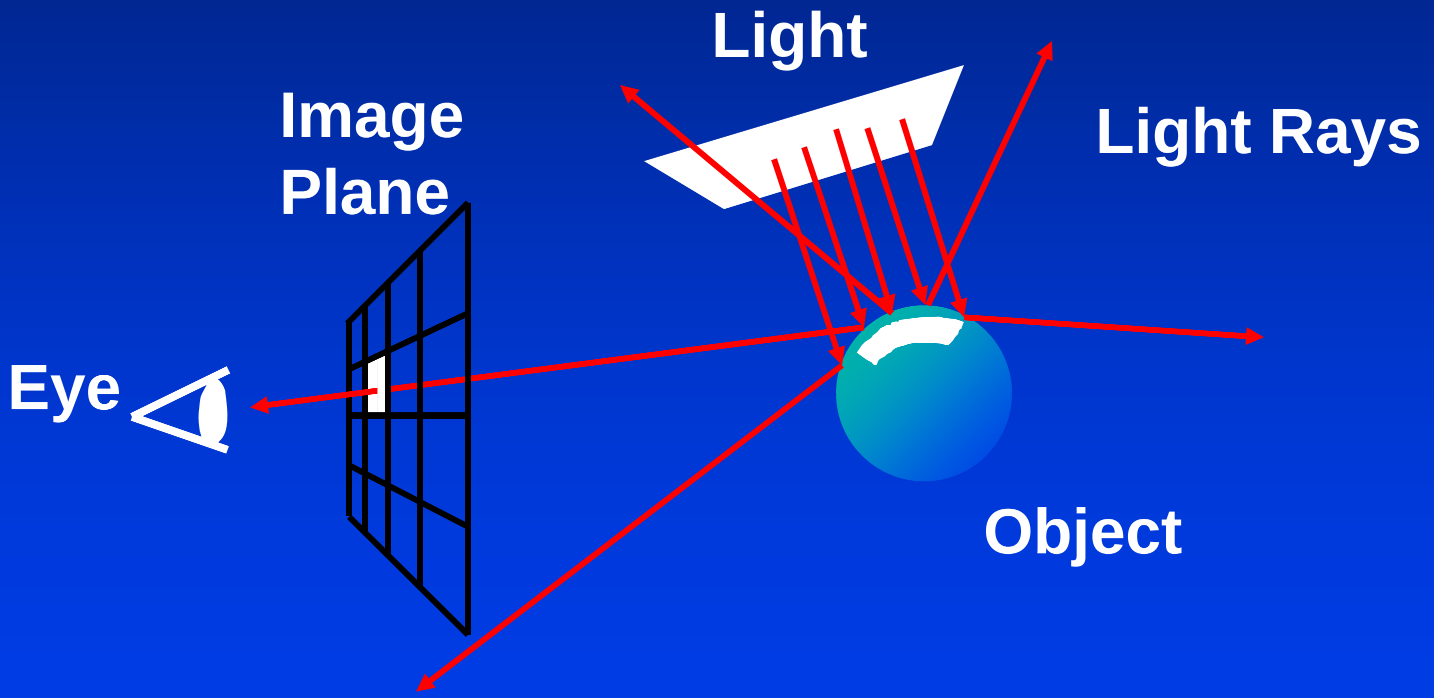
The Basic Idea

- ***Simulate light rays from light source to eye***



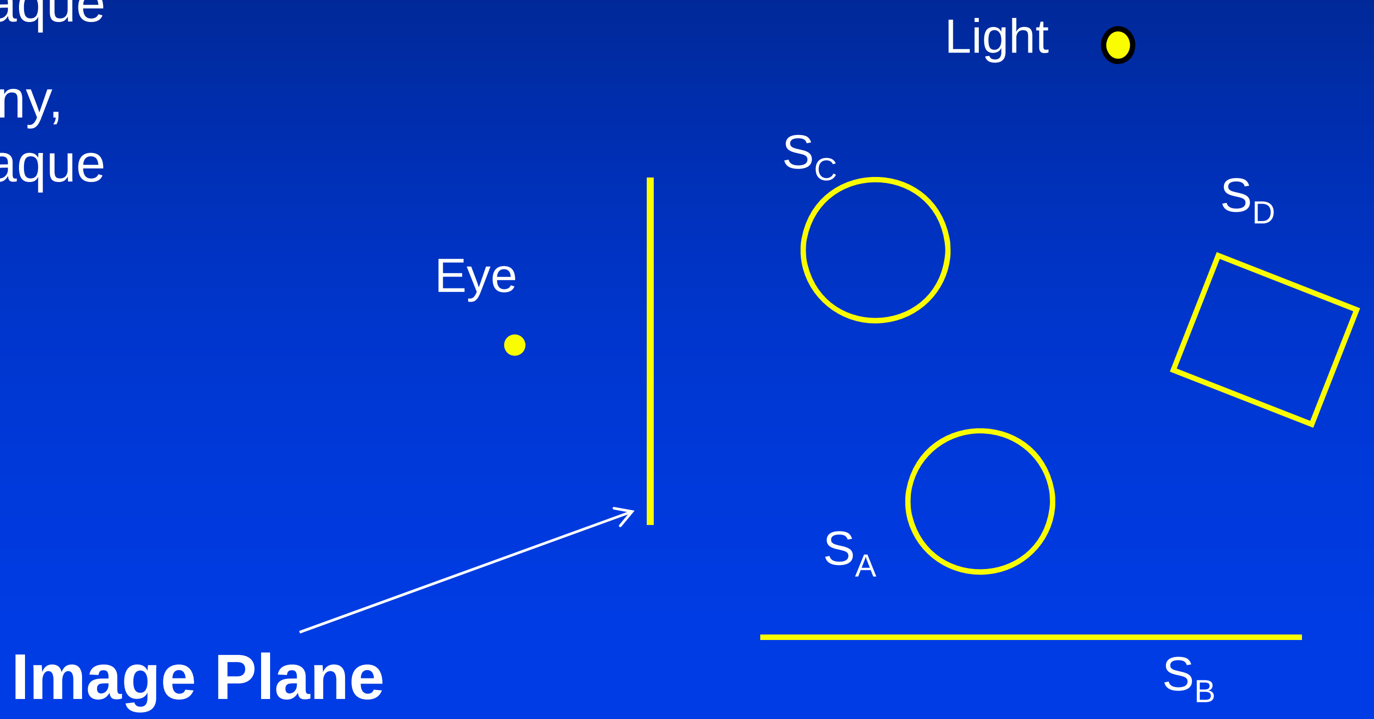
“Forward” Ray-Tracing

- *Trace rays from light*
- *Lots of work for little return*



Scene

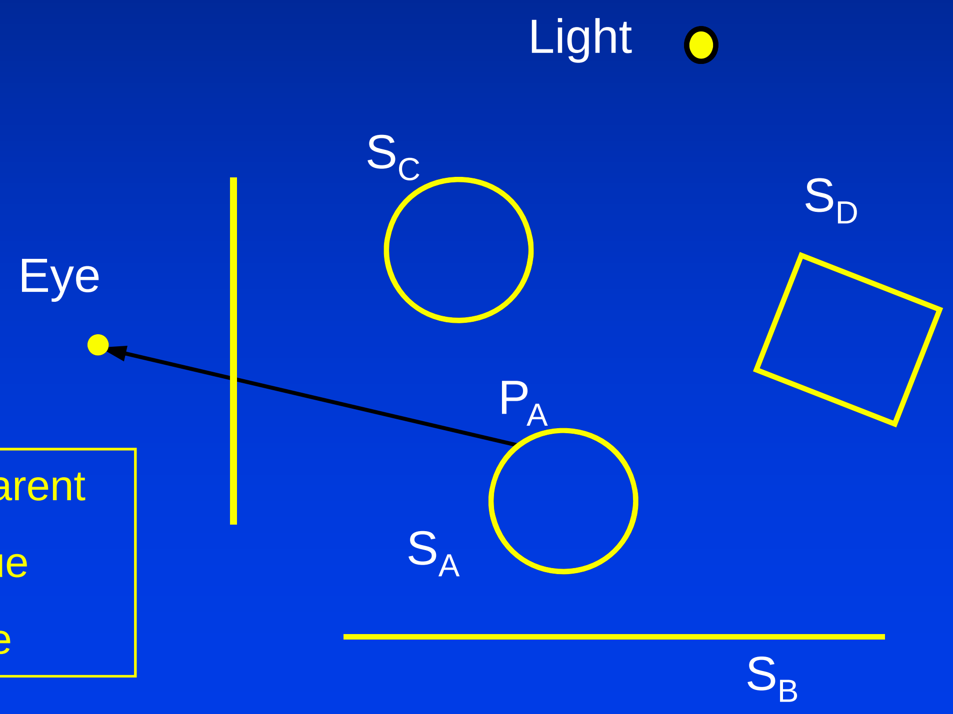
S_A	shiny, transparent
S_B,S_D	diffuse, opaque
S_C	shiny, opaque



Three sources of light

The light that point P_A emits to the eye comes from:

light sources
other objects (reflection)
other objects (refraction)



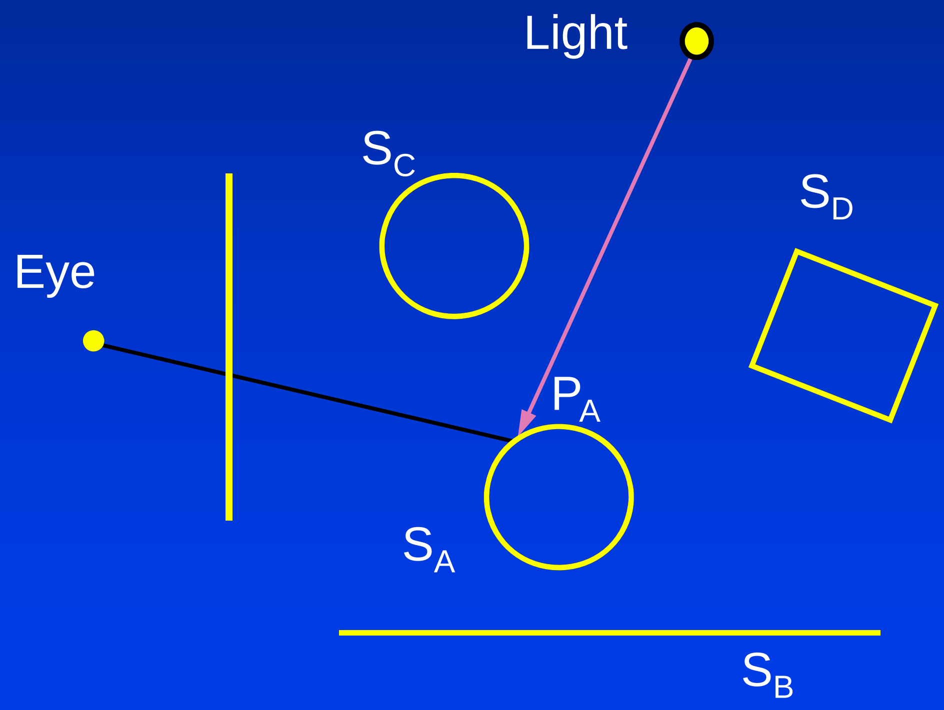
S_A	shiny, transparent
S_B,S_D	diffuse,opaque
S_C	shiny, opaque

Directly from light source

Local illumination model:

$$I = I_a + I_{diff} + I_{spec}$$

S_A	shiny, transparent
S_B,S_D	diffuse,opaque
S_C	shiny, opaque



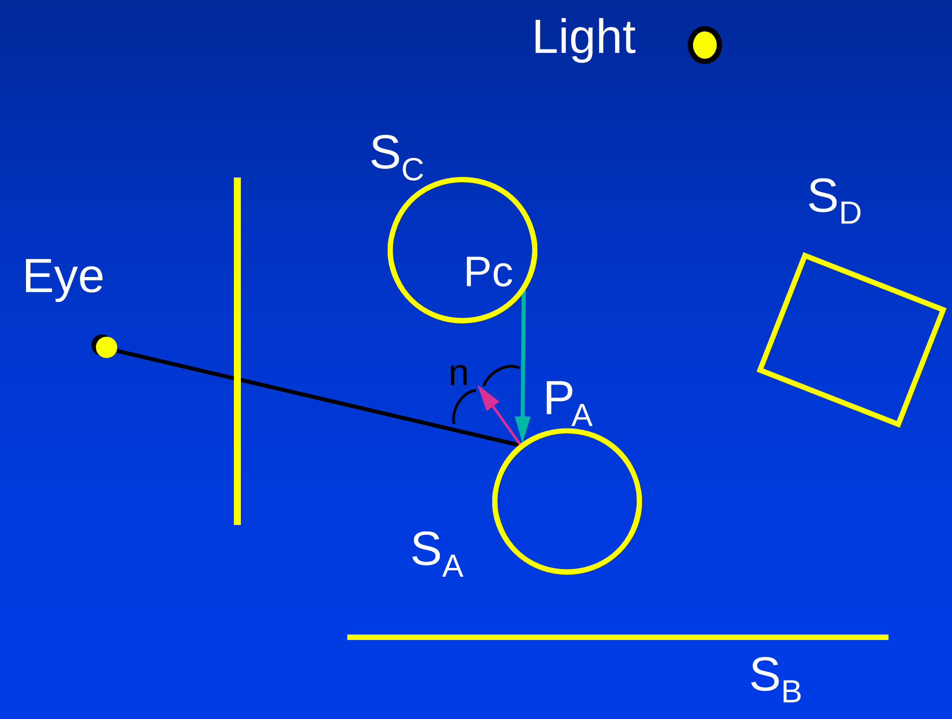
Reflection

What is the color that is reflected to P_A ?

The color of P_C.

What is the color of P_C ?

S_A	shiny, transparent
S_B,S_D	diffuse,opaque
S_C	shiny, opaque



Reflection

What is the light that is reflected to P_A ?

The color of P_C . as viewed by P_A

What is the color of P_C reflected towards P_A?

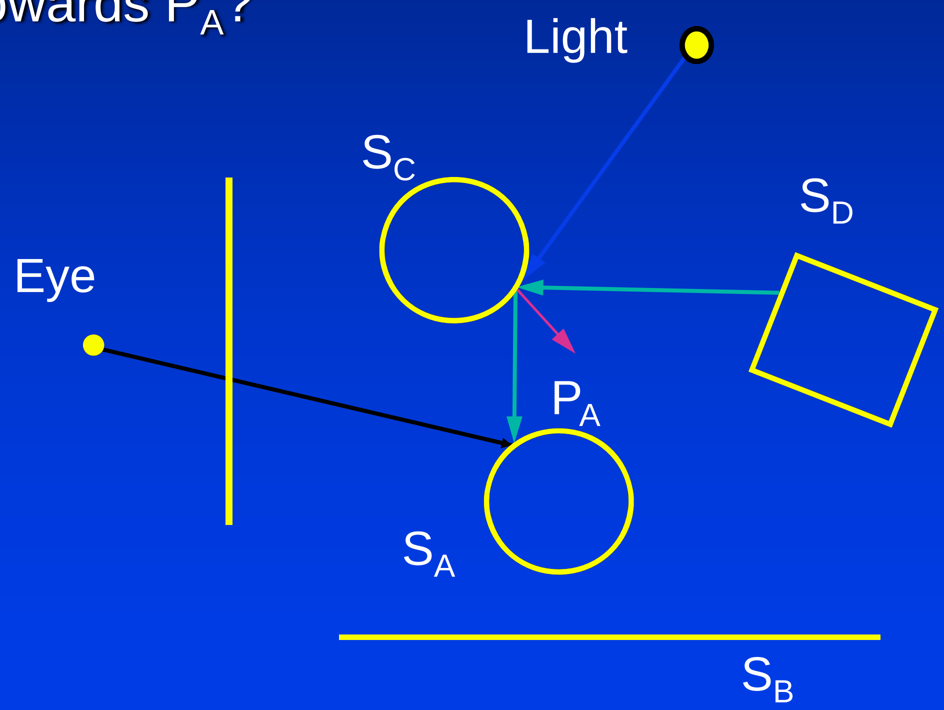
Just like P_A :

raytrace P_C i.e compute the

three contributions from

1. Light sources
2. Reflection
3. refraction

S_A	shiny, transparent
S_B,S_D	diffuse,opaque
S_C	shiny, opaque



Refraction

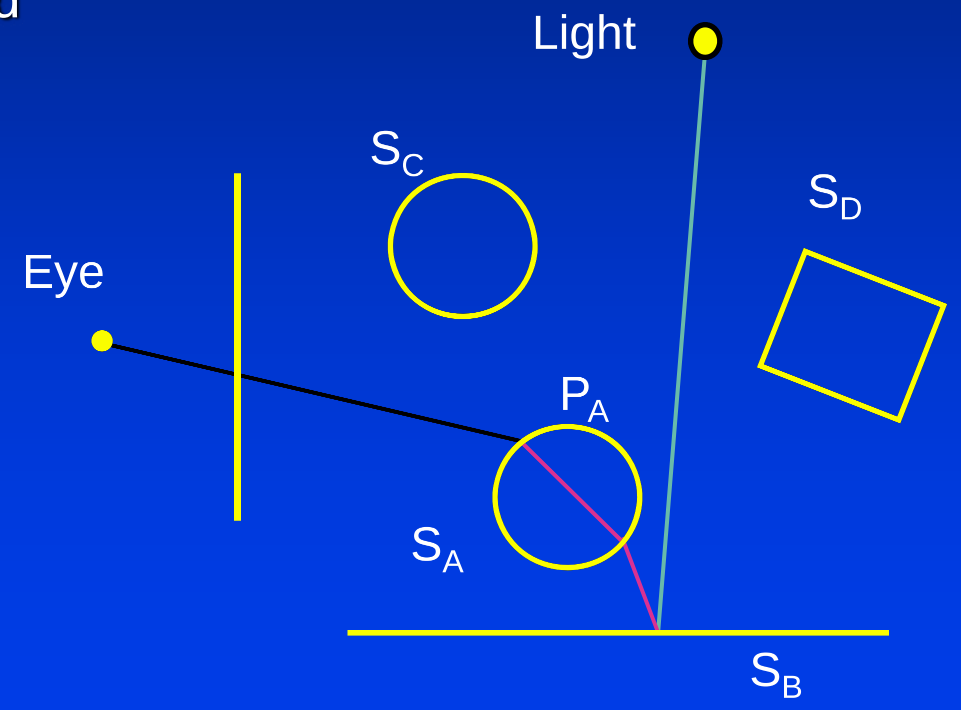
Transparent materials

S_A	shiny, transparent
S_B,S_D	diffuse,opaque
S_C	shiny, opaque

How do you compute the refracted contribution?

You raytrace the refracted ray.

1. *Lights*
2. *Reflection*
3. *Refraction*



What are we missing?

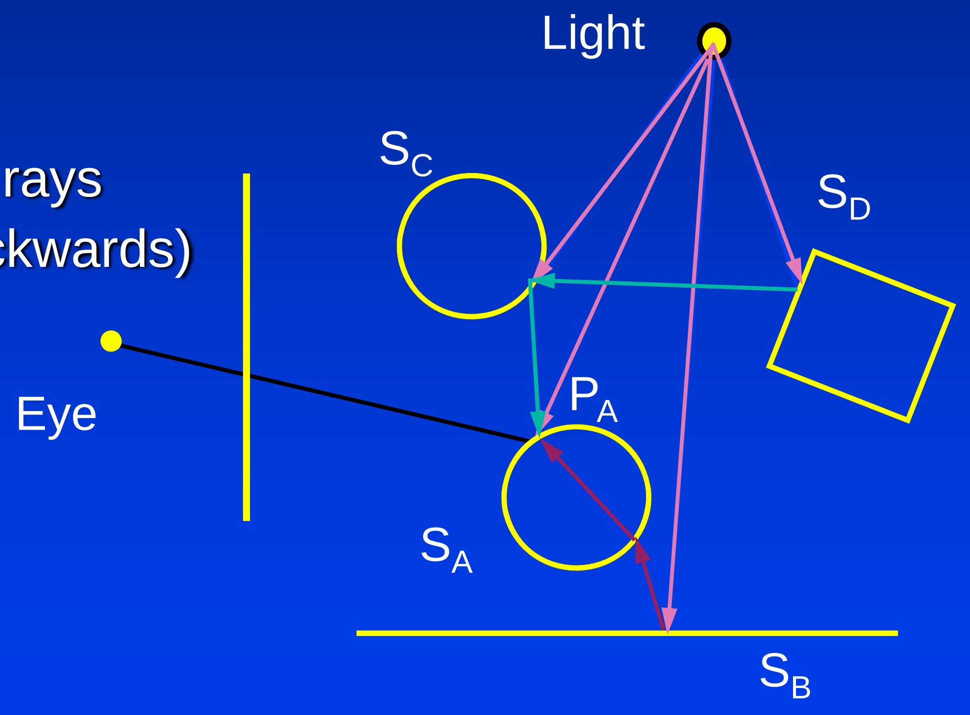
- ***Diffuse objects do not receive light from other objects.***

Three sources of light together

The color that the pixel is assigned comes from:
- light sources
- other objects (reflection)
- other objects (refraction)

It is more convenient to trace the rays from the eye to the scene (backwards)

S_A	shiny, transparent
S_B,S_D	diffuse,opaque
S_C	shiny, opaque



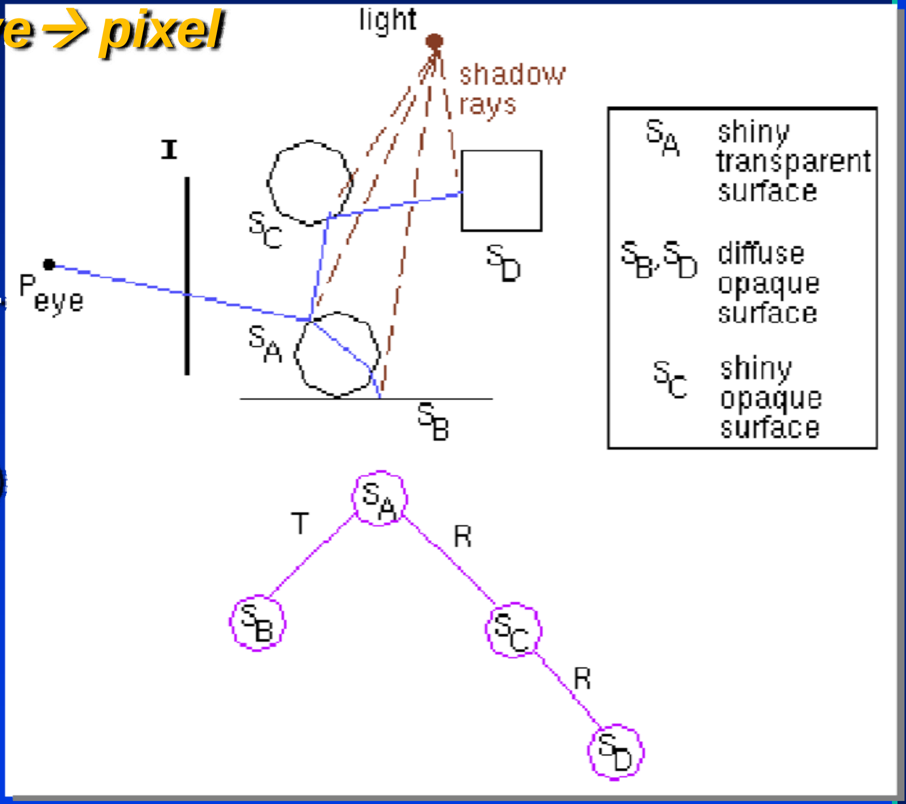
Backwards Raytracing Algoritm

- ***For each pixel construct a ray: eye→ pixel***

raytrace(ray)

P = closest intersection
color_local = ShadowRay(light1, P)+…
+ ShadowRay(lightN, P)
color_reflect = raytrace(reflected_ray)
color_refract = raytrace(refracted_ray)
color = color_local
+ k_{re}*color_reflect
+ k_{ra}*color_refract

return(color)



How many levels of recursion do we use?

- ***The more the better.***
- ***Infinite reflections at the limit.***

Stages of raytracing

- ***Setting the camera and the image plane***
- ***Computing a ray from the eye to every pixel and trace it in the scene***
- ***Object-ray intersections***
- ***Shadow, reflected and refracted ray at each intersection***

Setting up the camera

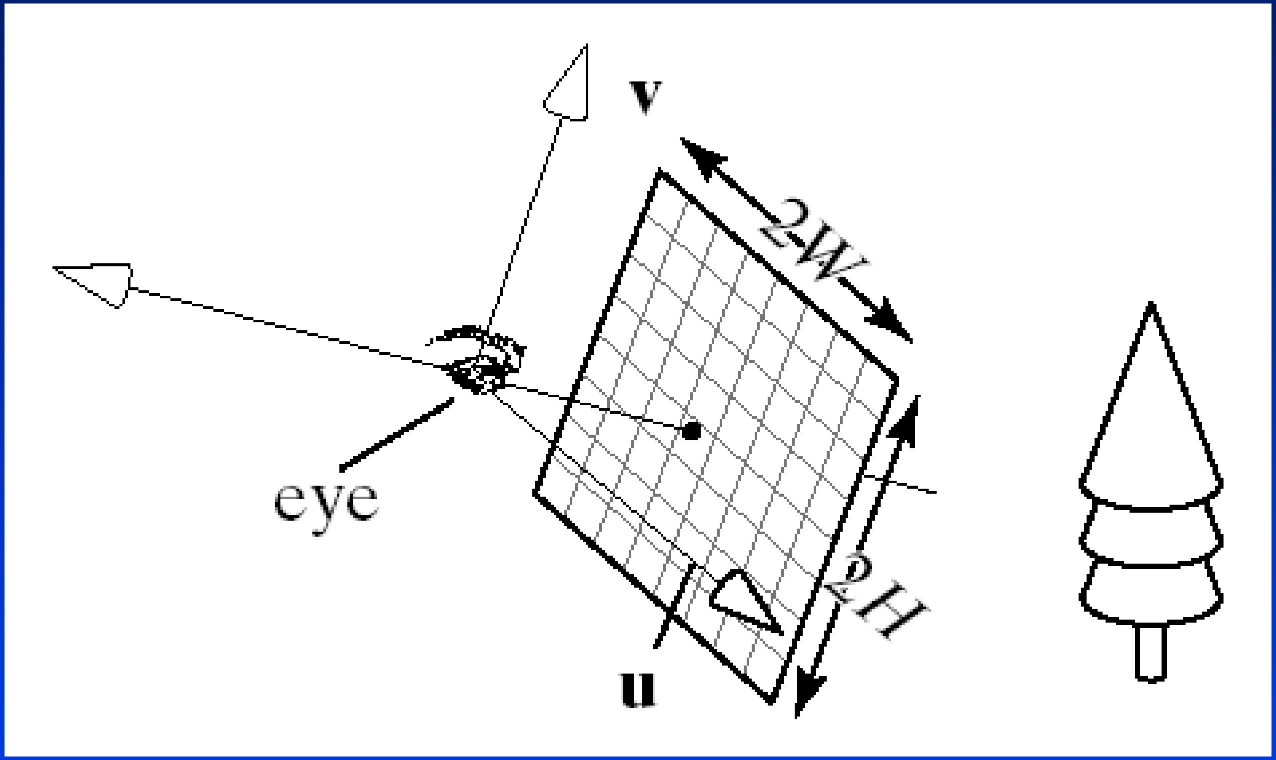
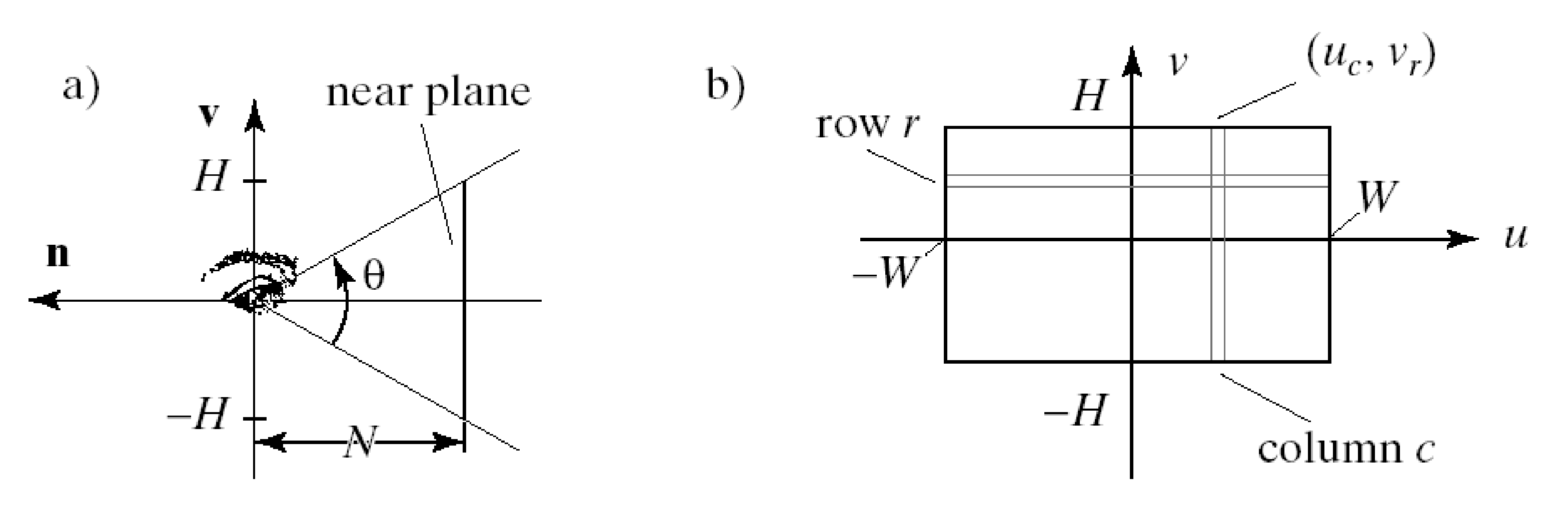


Image parameters

- *Width 2W, Height 2H*
  Number of pixels nCols x nRows
- *Camera coordinate system (eye, u,v,n)*
- *Image plane at -N*

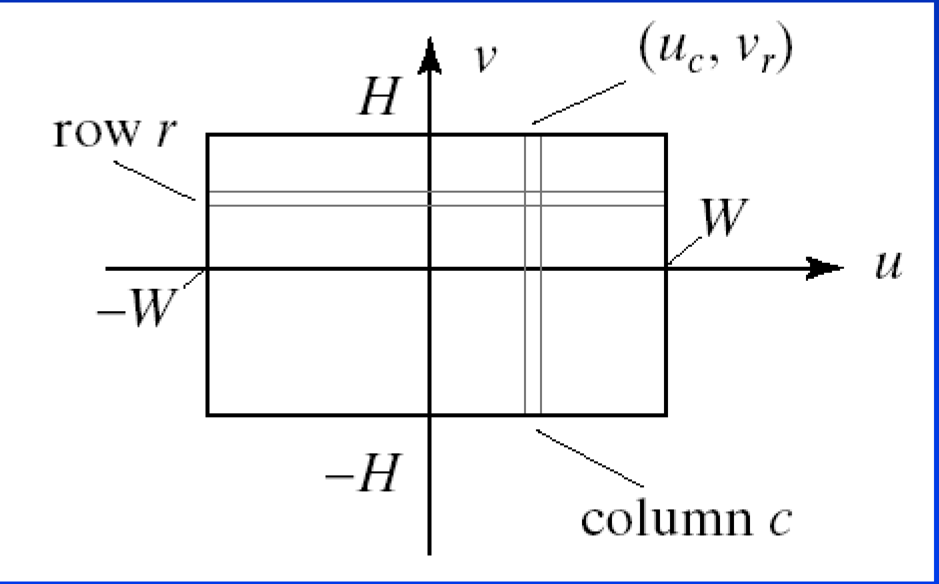


Pixel coordinates in camera coordinate system

- ***Pixel P(r,c) has coordinates in camera space:***

$$u_c = -W + W\frac{2c}{nCols}, \quad c = 0, 1, \ldots, nCols - 1,$$

$$v_r = -H + H\frac{2r}{nRows}, \quad r = 0, 1, \ldots, nRows - 1,$$



Ray through pixel

- ***Pixel location***

$$Camera\ \ coordinates:\quad P(r,c) = (u_c, v_r, -N)$$

$$Wolrd\ \ coordinates:\quad P(r,c) = eye - N\mathbf{n} + u_c\mathbf{u} + v_r\mathbf{v}$$

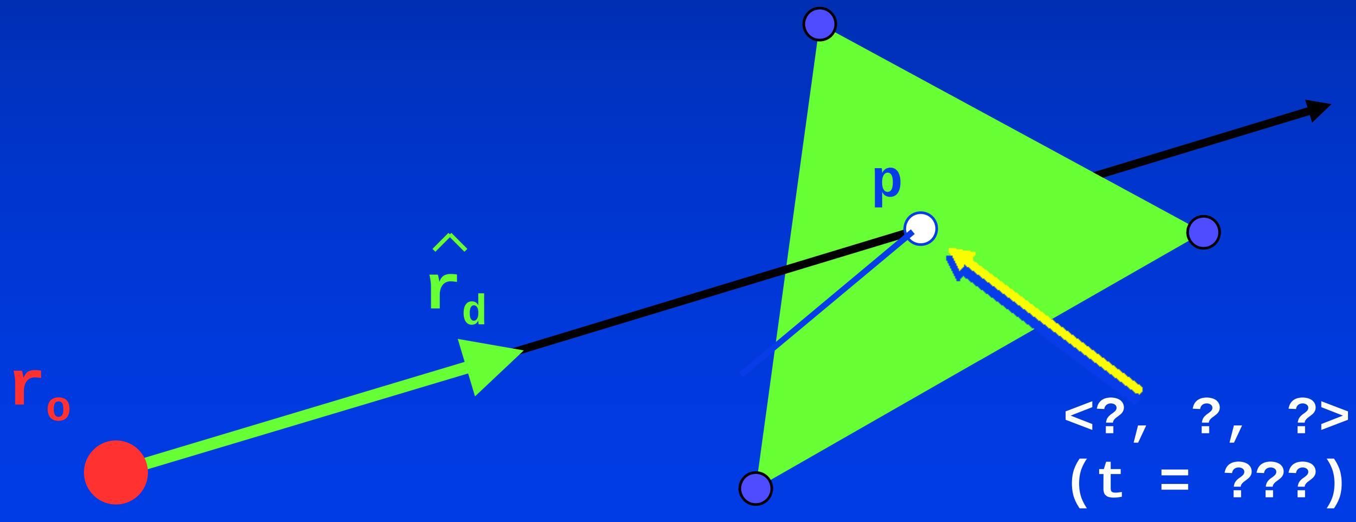
- ***Ray through pixel:***

$$ray(r,c,t) = eye + t(P(r,c) - eye)$$

$$ray(r,c,t) = eye + t(-N\mathbf{n} + w(\frac{2c}{nCols} - 1)\mathbf{u} + H(\frac{2r}{nRows} - 1)\mathbf{v})$$

Triangle Intersection

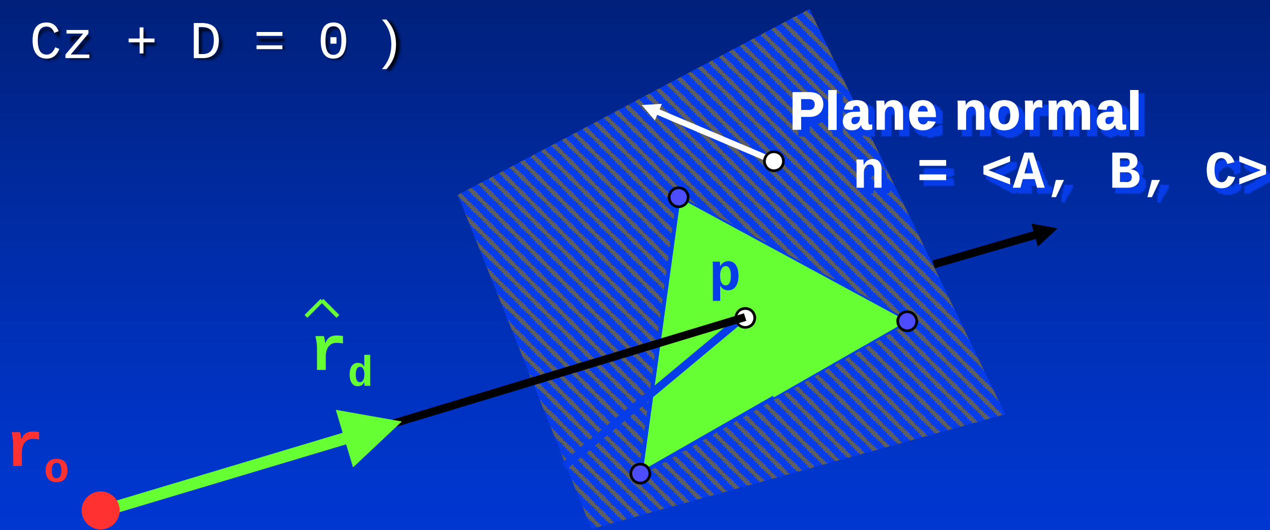
- *Want to know: at what point (p) does ray intersect triangle?*
- *Compute lighting, reflected rays, shadowing from that point*



Triangle Intersection

- ***Step 1 : Intersect with plane***

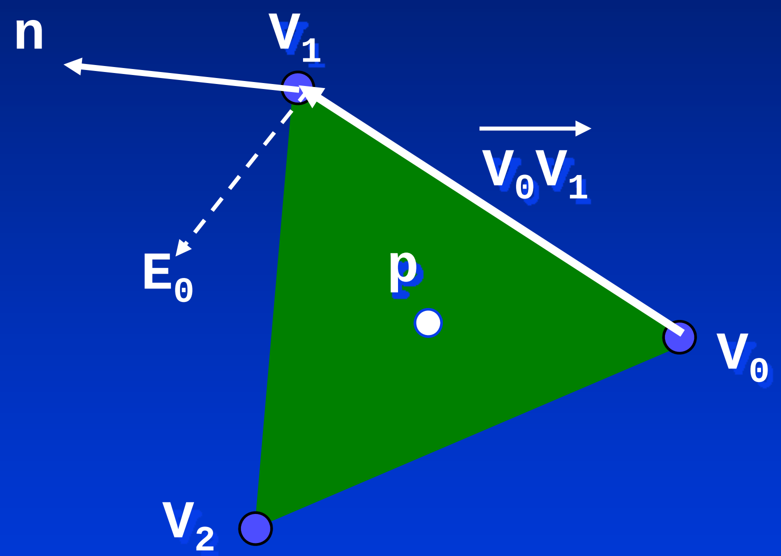
$(Ax + By + Cz + D = 0)$



$$p = -(\hat{n} \cdot r_o + D) / (\hat{n} \cdot \hat{r}_d)$$

Triangle Intersection

- ***Step 2 : Check against triangle edges***



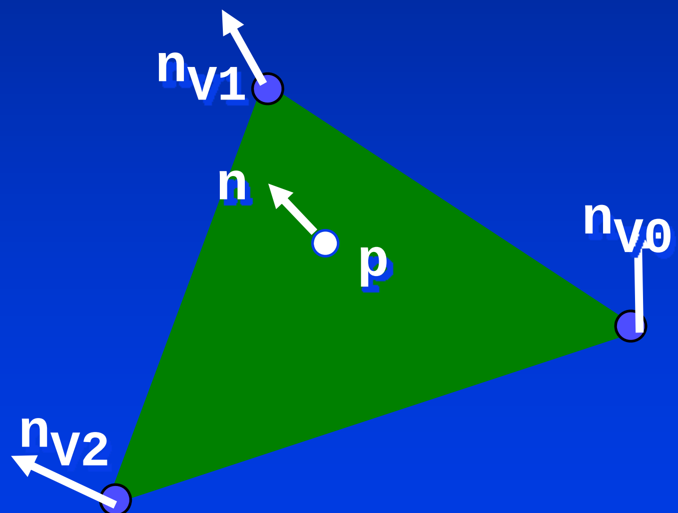
$$E_i = \overrightarrow{V_iV_{i+1}} \times n \quad \text{(plane A, B, C)}$$
$$d_i = -A \cdot N \quad \text{(plane D)}$$

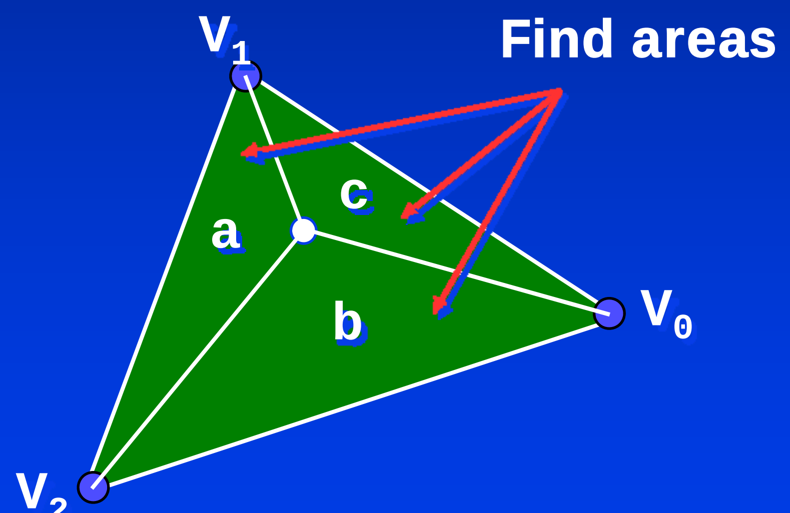
Plug p into ($p \cdot E_i + d_i$) for each edge

if signs are all positive or negative,
point is inside triangle!

Triangle Normals

- ***Could use plane normals (flat shading)***
- ***Better to interpolate from vertices***

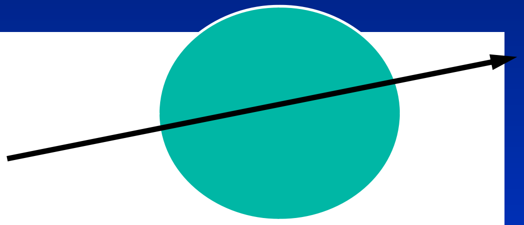




$$n = \frac{a\hat{n}_{V0} + b\hat{n}_{V1} + c\hat{n}_{V2}}{\mathrm{area}(v_0 v_1 v_2)}$$

Ray-object intersections

- ***Unit sphere at origin - ray intersection:***



$$ray(t) = S + \mathbf{c}t$$

$$Sphere(P) = |P| - 1 = 0$$

$$Sphere(ray(t)) = 0 \Rightarrow$$

$$|S + \mathbf{c}t| - 1 = 0 \Rightarrow (S + \mathbf{c}t)(S + \mathbf{c}t) - 1 = 0 \Rightarrow$$

$$\boxed{|\mathbf{c}|^2 t^2 + 2(S \cdot \mathbf{c})t + |S|^2 - 1 = 0}$$

- ***That's a quadratic equation***

Solving a quadratic equation

$$|\mathbf{c}|^2 t^2 + 2(S \cdot \mathbf{c})t + |S|^2 - 1 = 0$$

$$At^2 + 2Bt + C = 0$$

$$t_h = -\frac{B}{A} \pm \frac{\sqrt{B^2 - AC}}{A}$$

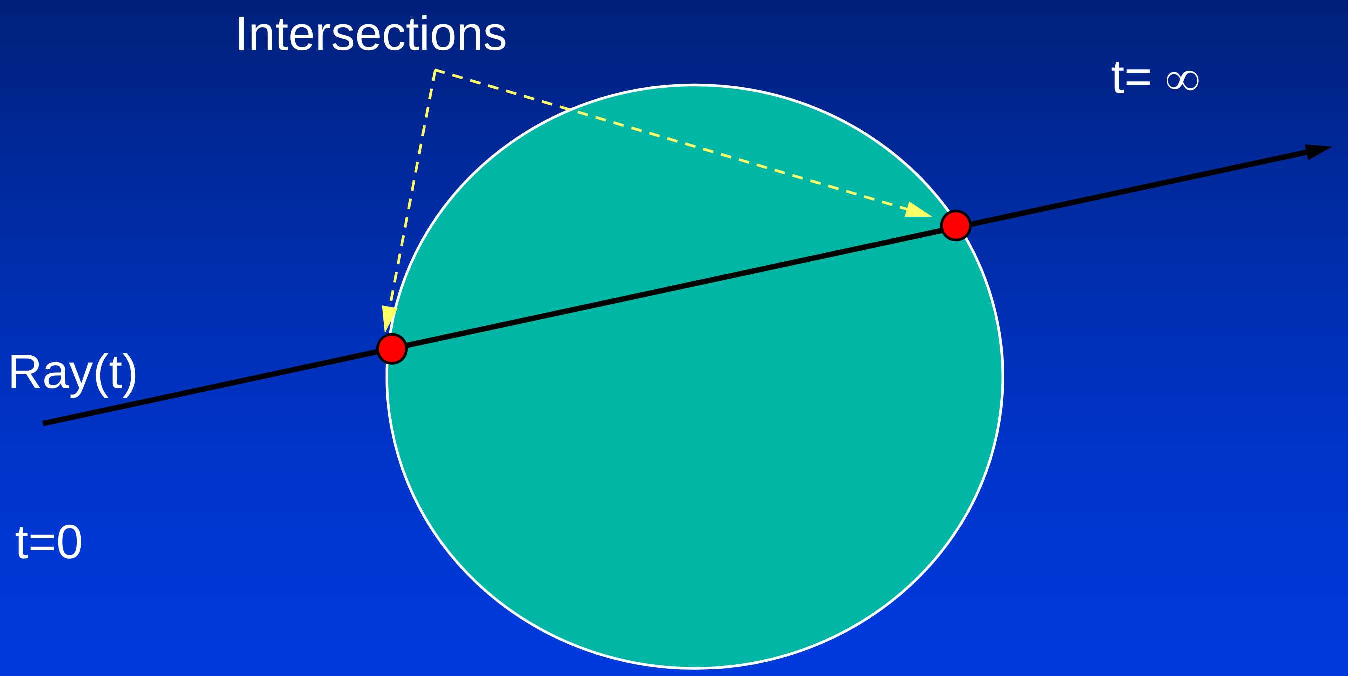
$$t_h = -\frac{S \cdot \mathbf{c}}{|\mathbf{c}|^2} \pm \frac{\sqrt{(S \cdot \mathbf{c})^2 - |\mathbf{c}|^2 (|S|^2 - 1)}}{|\mathbf{c}|^2}$$

If $(B^2 - AC) = 0$ one solution

If $(B^2 - AC) < 0$ no solution

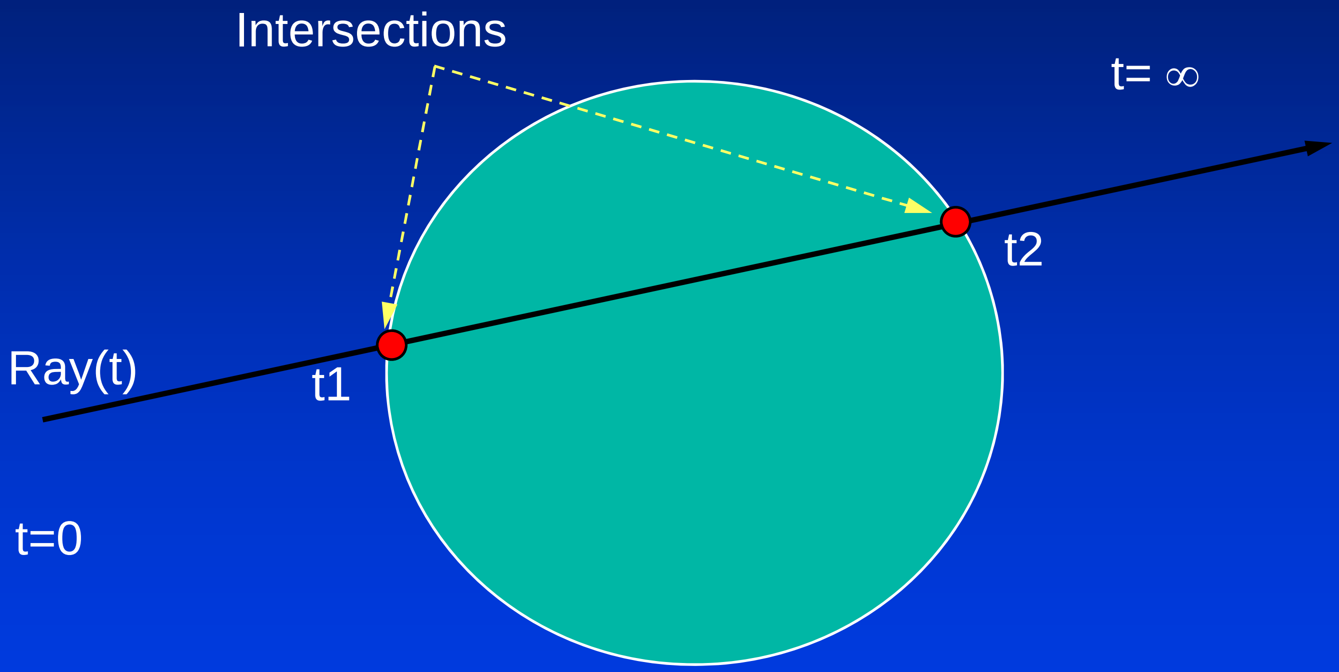
If $(B^2 - AC) > 0$ two solutions

First intersection?

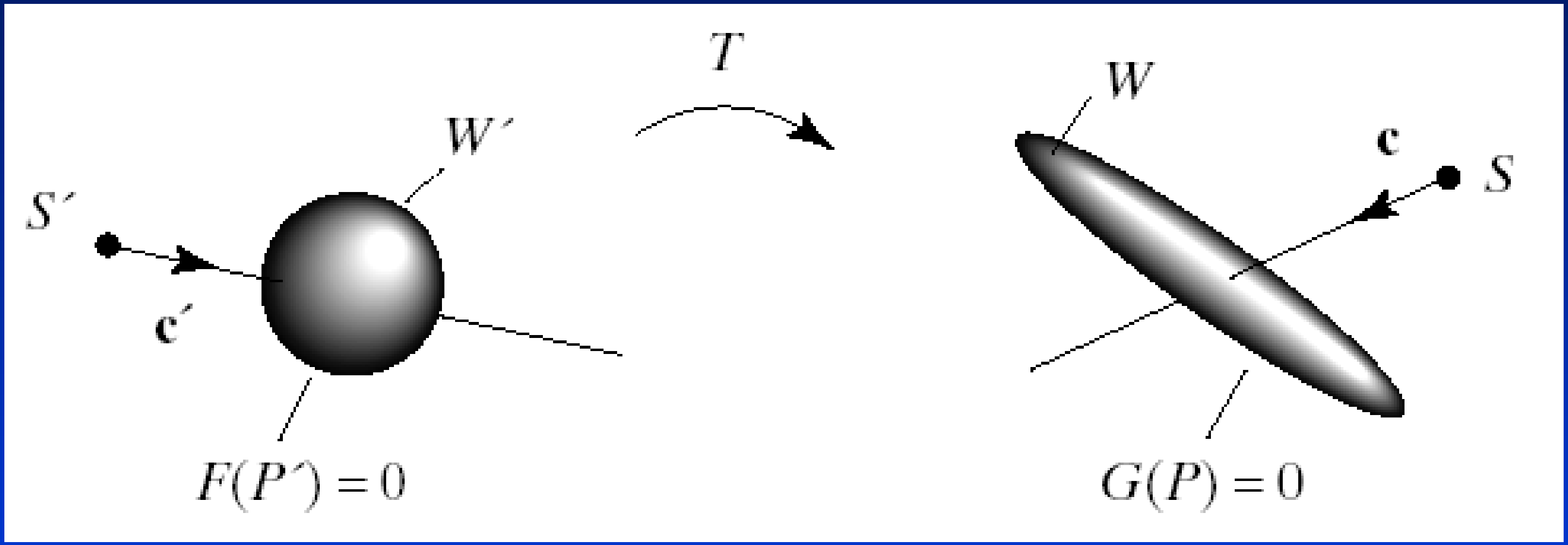


First intersection?

t1 < t2

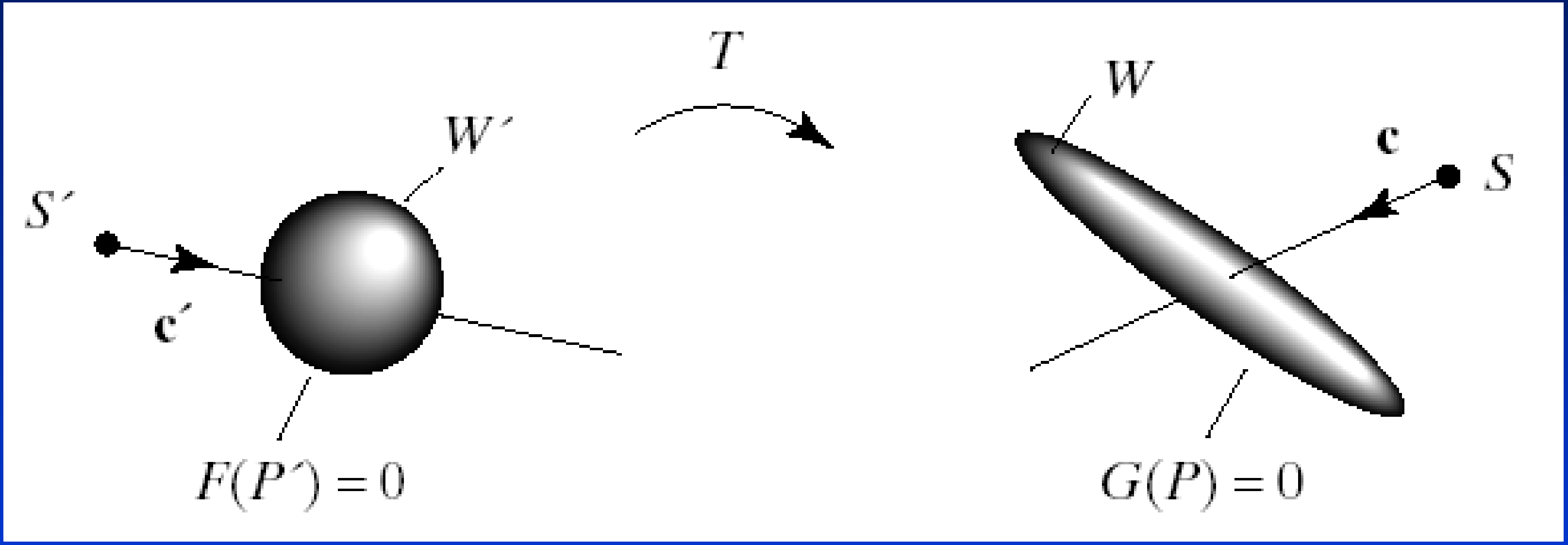


Transformed primitives?



- ***Where does S+ct hit the transformed sphere G ?***

Linear transformation

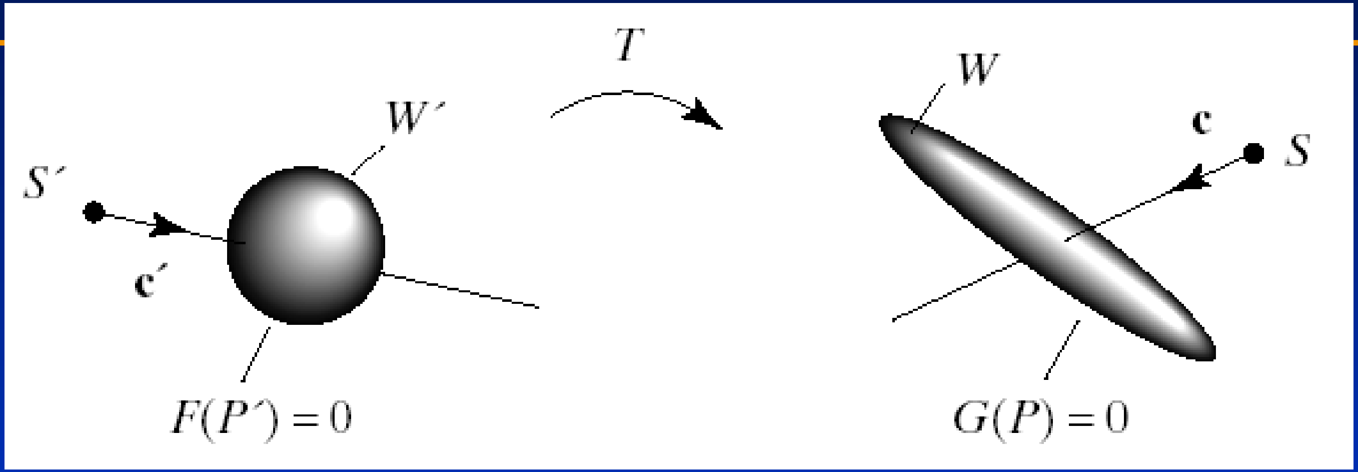


Implicit equation $G(P) = 0$.

Untransformed implicit equation F(P') = 0.

$$P = MP' \Rightarrow P' = M^{-1}P$$

Linear transformation



$$P = MP' \Rightarrow P' = M^{-1}P$$

$$F(P') = F(T^{-1}(P)) = 0 \Rightarrow F(T^{-1}(P)) = 0$$
$$F(T^{-1}(S + \mathbf{c}t)) = 0 \Rightarrow$$
$$F(T^{-1}(S) + T^{-1}(\mathbf{c}t)) = 0$$

Which means that we can intersect the inverse transformed ray with the untransformed primitive.

Final Intersection

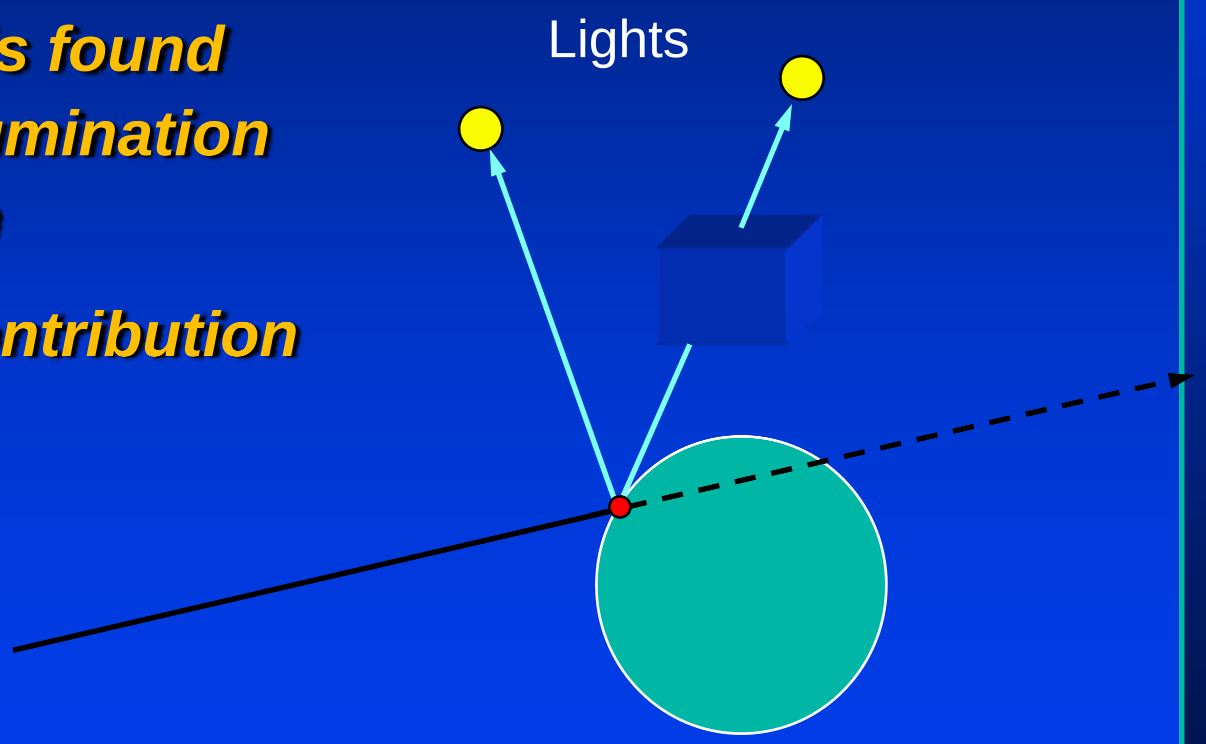
- ***Inverse transformed ray***

$$\tilde{r}(t) = M^{-1}\begin{pmatrix} S_x \\ S_y \\ S_z \\ 1 \end{pmatrix} + M^{-1}\begin{pmatrix} c_x \\ c_y \\ c_z \\ 0 \end{pmatrix} = \tilde{S}' + \tilde{\mathbf{c}}'t$$

  - Drop 1 and O to get *S*'+c'*t*
- ***For each object***
  - Inverse transform ray getting *S*'+c'*t*
  - Find intersection t_h
  - Use t_h in the untransformed ray *S*+c*t* to find the intersection

Shadow ray

- *For each light intersect shadow ray with all objects.*
- *If no intersection is found apply local illumination at intersection*
- *If in shadow no contribution*



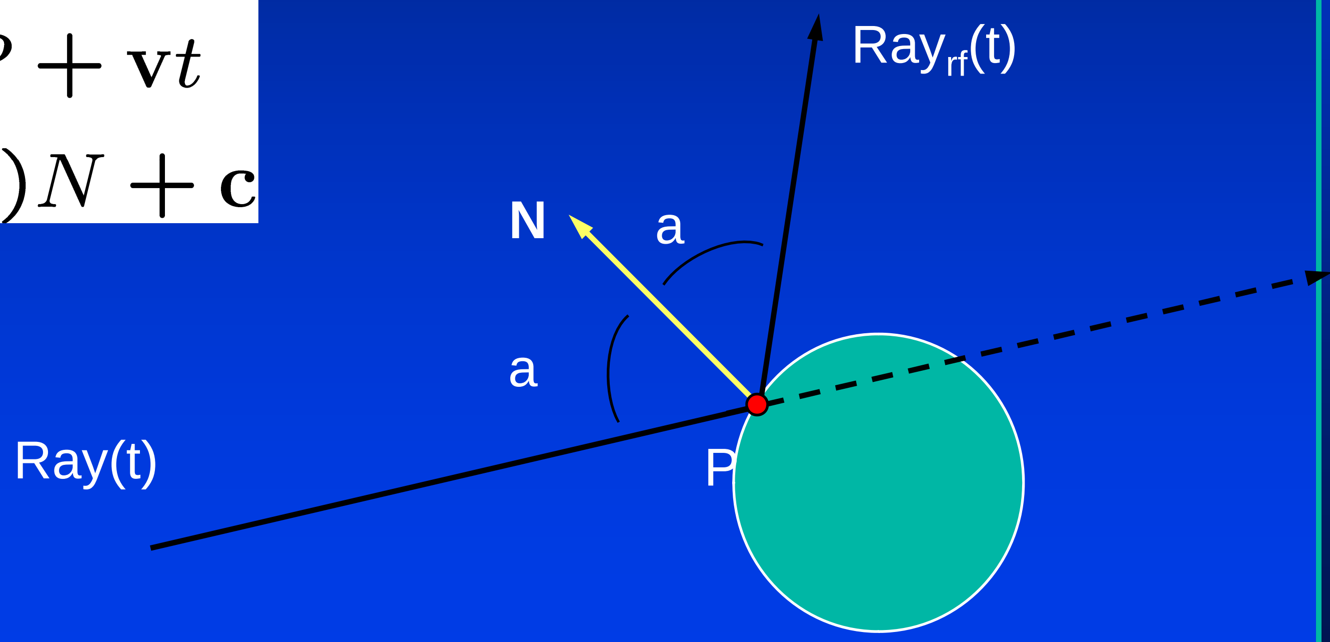
Reflected ray

- ***Raytrace the reflected ray***

$$Ray(t) = A + \mathbf{c}t$$

$$Ray_{rf}(t) = P + \mathbf{v}t$$

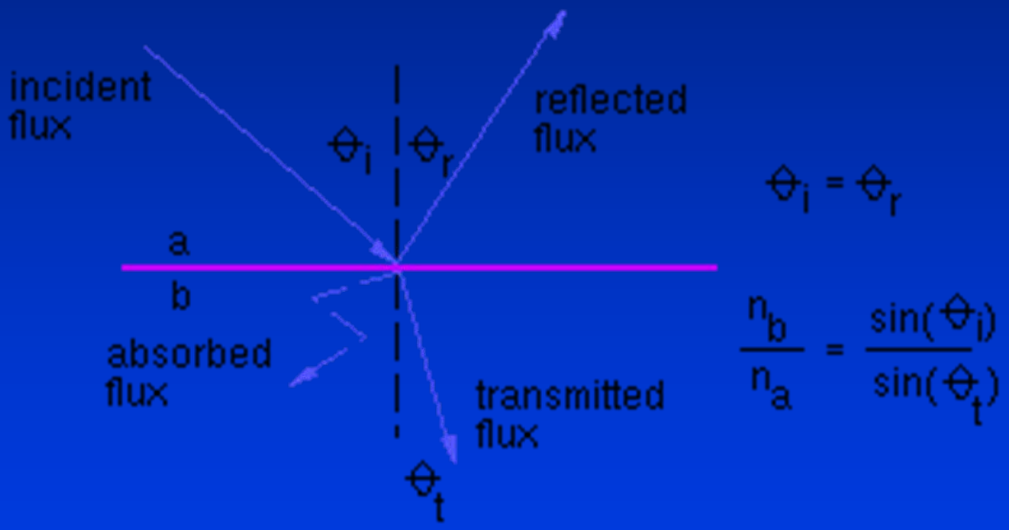
$$\mathbf{v} = -2(N \cdot \mathbf{c})N + \mathbf{c}$$

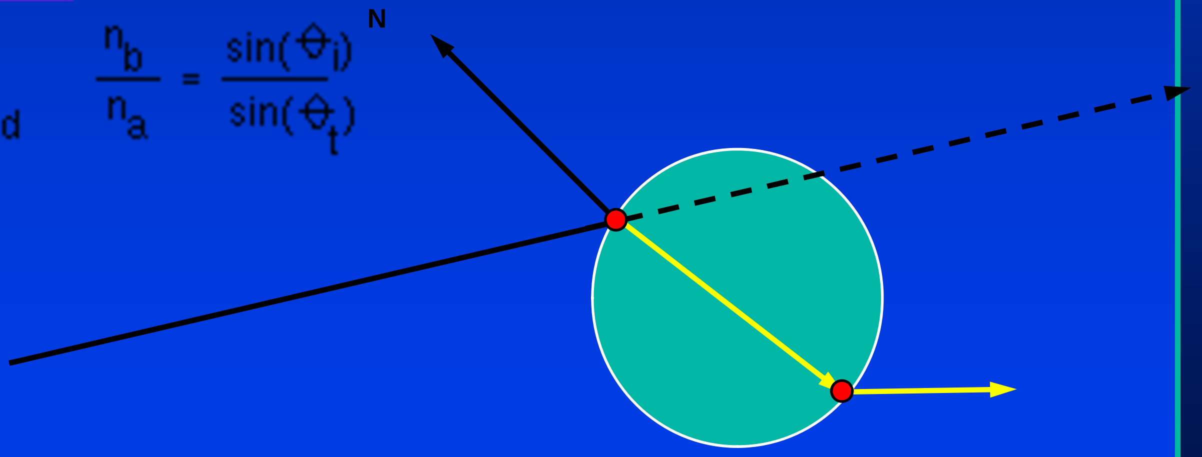


Refracted ray

- ***Raytrace the refracted ray***

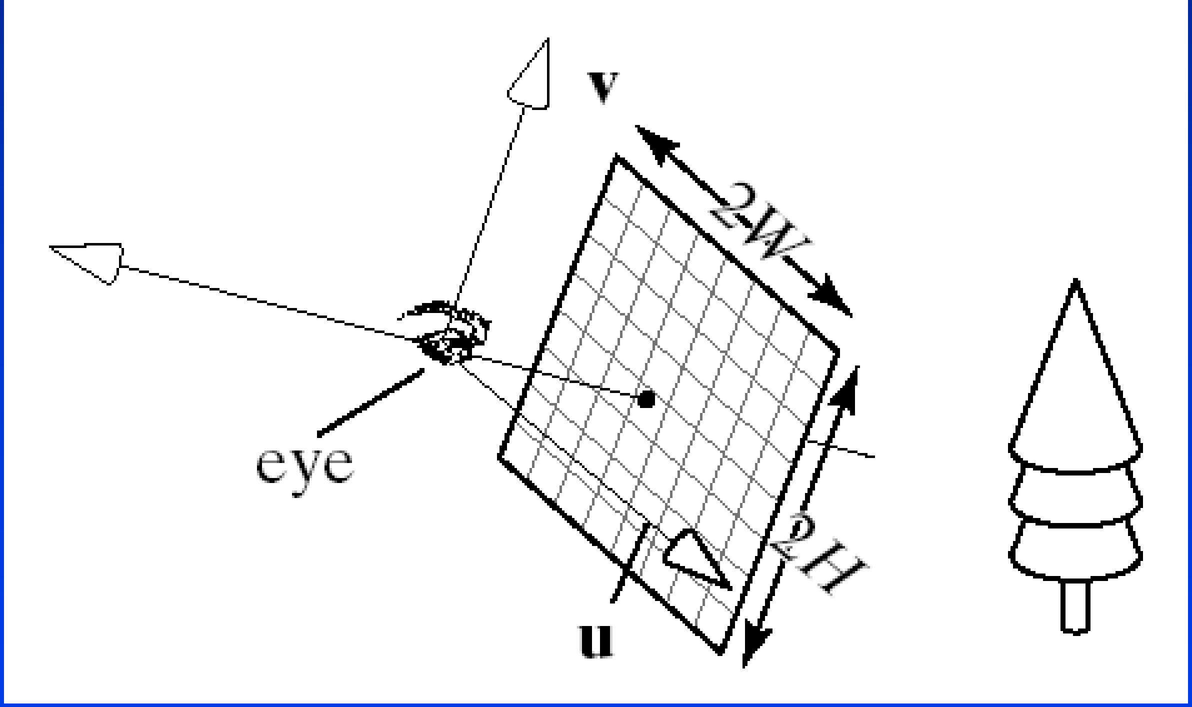
Snell's law





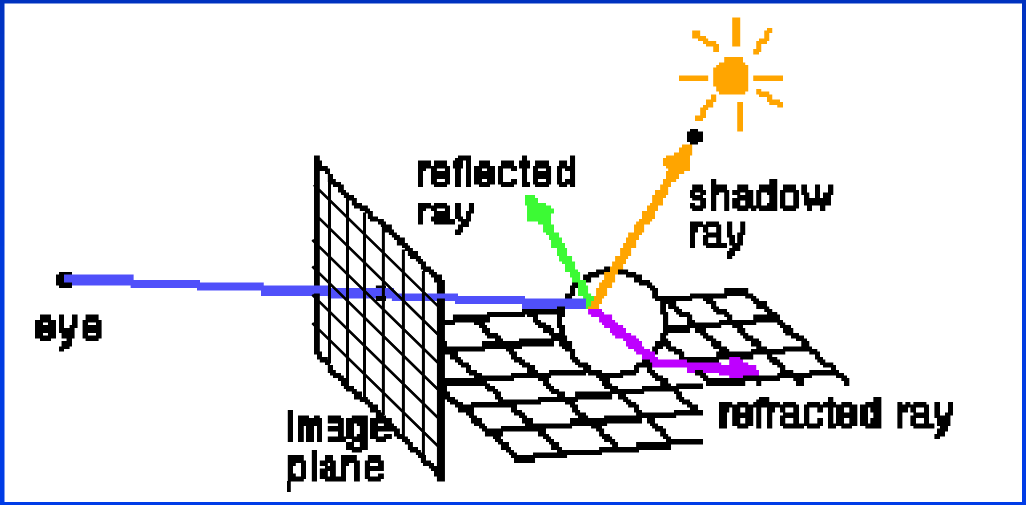
Add all together

- *color(r,c) = color_shadow_ray + Kf*color_rf + Kr*color_rfa*



Raytracing

```
for each pixel on screen
  determine ray from eye through pixel
  find closest intersection of ray with an object
  cast off reflected and refracted ray, recursively
  calculate pixel colour, draw pixel
end
```



Acceleration

- *1280x1024 image with 10 rays/pixel*
- *1000 objects (triangle, CSG, NURBS)*
- *3 levels recursion*

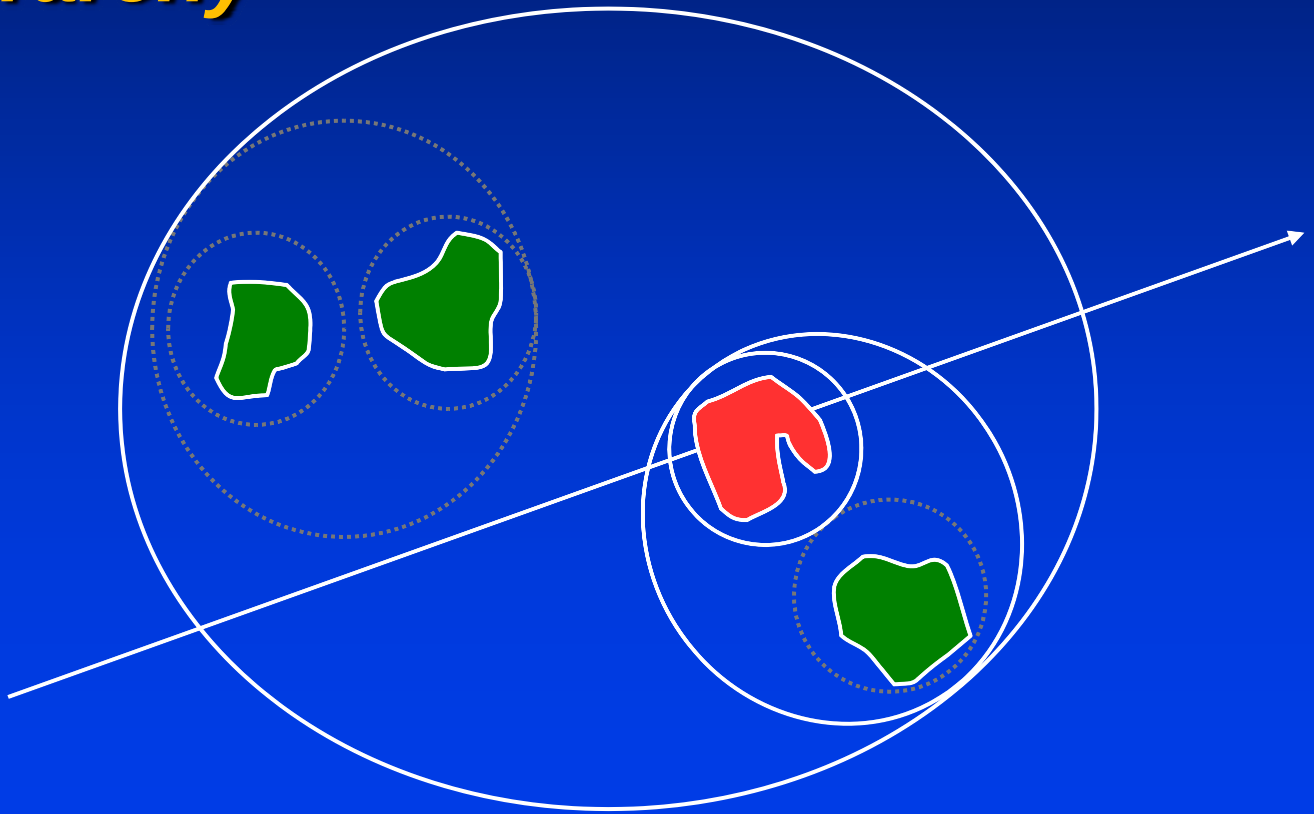
39321600000 intersection tests

100000 tests/second -> 109 days!

Must use an acceleration method!

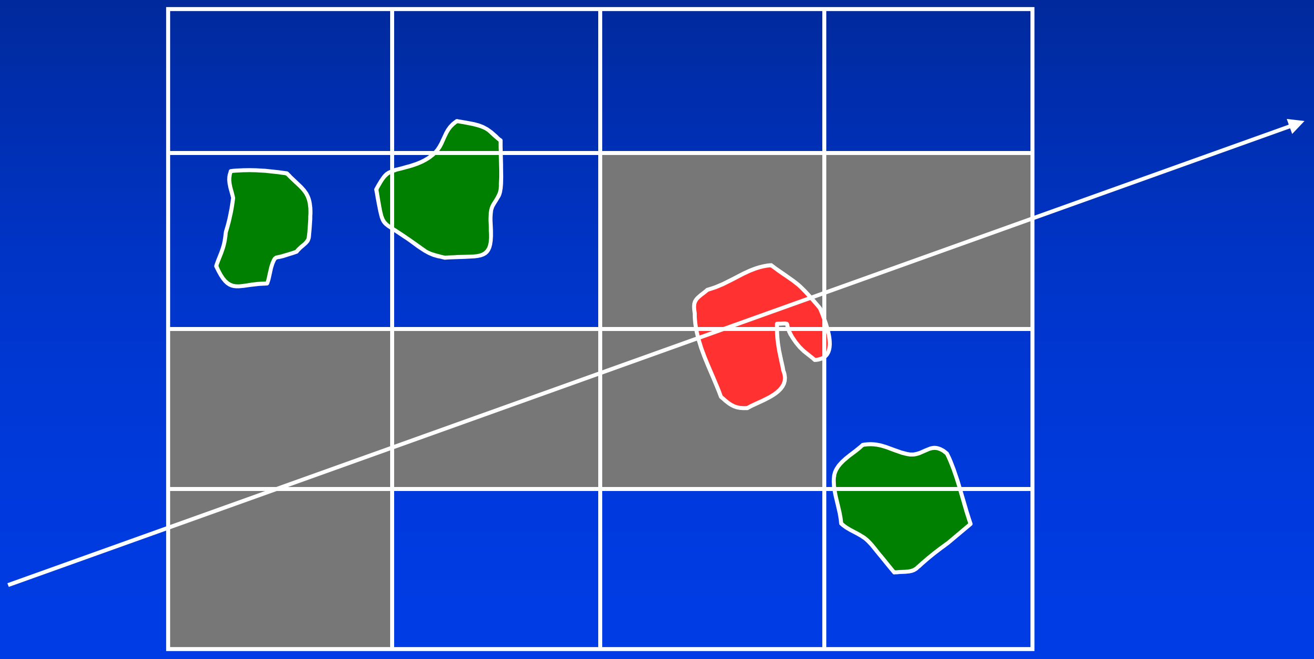
Bounding volumes

- ***Use simple shape for quick test, keep a hierarchy***



Space Subdivision

- ***Break your space into pieces***
- ***Search the structure linearly***

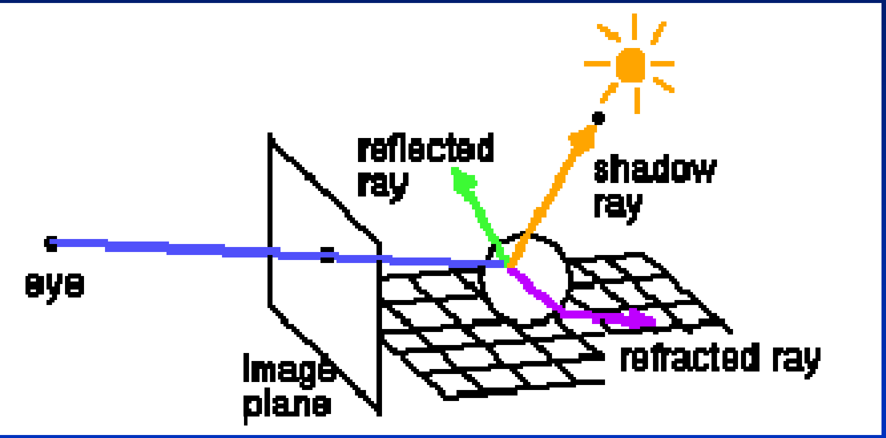


Parallel Processing

- ***You can always throw more processors at it.***

Summary: Raytracing

- ***Recursive algorithm***



Function Main

for each pixel (c,r) on screen

determine ray $r_{c,r}$ from eye through pixel

color(c,r) = raytrace($r_{c,r}$)

end for

end

function raytrace(r)

find closest intersection P of ray with objects

clocal = Sum(shadowRays(P,Lighti))

c_{re} = raytrace(r_{re})

c_{ra} = raytrace(r_{ra})

return c = clocal+k_{re}*c_{re}+k_{ra}*c_{ra}

end

Advanced concepts

- ***Participating media***
- ***Transculency***
- ***Sub-surface scattering (e.g. Human skin)***
- ***Photon mapping***

Raytracing summary

- *View dependent*
- *Computationally expensive*
- *Good for refraction and reflection effects*