Report for Project H45C
Development of an Adjoint Model for CMAQ-4.5

Prepared for
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1 List of tasks accomplished to date

A. We have generated the KPP code that implements the SAPRC99 chemical mechanism used in CMAQ. The following steps have been taken:

1. Translate the reaction list from CMAQ format (mech.def) to KPP format (saprc99.eqn)

2. Translate the species list from CMAQ format (GC_SPC.EXT) to KPP format (saprc99.spc)

3. Run KPP and generate the Fortran90 code.

B. We have constructed discrete adjoint schemes for the horizontal (HPPM) and vertical (VPPM) advection schemes. Following steps have been taken in order to carry out this task:

1. Parts of the original code (vppm.F / hppm.F) have been rewritten to be understood by the automatic differentiation program. Special functions such as min(), max(), abs() are not compatible with the TAMC interface. These functions are replaced with their corresponding if else statements.

2. The automatic differentiation software TAMC has been applied in forward mode to obtain the tangent linear models, and in reverse mode to obtain the discrete adjoint models of HPPM and VPPM.

   ppm_ftl: Forward Tangent Linear model denotes the sensitivity of the intermediate states with respect to perturbations in the initial state.

   ppm_ad: Discrete Adjoint model denotes the sensitivity of the final state with respect to a change in the intermediate states.

3. Interfaces for the advection scheme files and TAMC generated automatic differentiation codes have been developed.

4. The tangent linear and discrete adjoint code has been validated on the two-dimensional “rotating cone” test problem, as discussed in chapter 7 of CMAQ 4.5 documentation. The validation results are shown next.

5. Using the b-LBFGS optimization method, data-assimilation tests have been performed for Discrete & Continuous adjoint modes on a perturbed cone. The plots are shown next to the validation results.
2 Validation results for the discrete adjoints of horizontal advection HPPM

A simple two-dimensional test problem has been employed for the validation of the direct and adjoint sensitivity code. The test problem follows closely the one described in Chapter 7 of the CMAQ science documentation (Byun et al.). A Gaussian profile is advected by a two-dimensional divergence-free, rotating wind field. The grid size is 33x33. After half period the profile is displaced by 180° clockwise. The CMAQ piecewise parabolic method (ppm) routines are applied in a direction split approach.

The following results are obtained with the nonlinear ppm routine and its tangent linear and adjoint models.

1. Rotating Cone test for the original advection scheme (hppm) for half cycle:

![Image](https://via.placeholder.com/150)

Fig(a) \( t = 0 \)  
Fig(b) \( t = T \)

Figure. The rotation of the Gaussian profile by \( T = \) half period.

2. Rotating Cone test for TAMC generated tangent linear model (hppm_ftl): A spike is placed at the cone location and is advected in forward direction using hppm_ftl.

![Image](https://via.placeholder.com/150)

Fig(a) \( t = 0 \)  
Fig(b) \( t = T \)

Figure. A delta (spike) perturbation at the initial time is advanced forward in time by the tangent linear model.

3. Rotating Cone test for TAMC generated adjoint model (hppm_ad): A spike is
placed at the location where the cone moving in forward direction ends (at t=T) and is advected in reverse direction using the hppm_ad subroutine.

Figure. A delta (spike) perturbation at the final time is advanced backward in time by the discrete adjoint model.

Observation: The wiggles at the boundaries are because of the differentiation of discontinuities at the cone boundaries.

4. Validation of Tangent Linear model by comparing TLM with the finite difference scheme:

A delta perturbation (spike) is considered at the tip of the profile at t=0. This perturbation is advanced forward in time using

(i) Finite Difference formulae[Diff]
(ii) TLM generated TLM
(iii) PPM subroutine(hppm.f)[Direct Conc].

The finite difference formula represents the difference between the solution started with a perturbed initial condition (the Gaussian profile is added the spike at the initial time) and the solution started with the reference initial condition (the Gaussian profile).
Observation: The TAMC generated TLM plot resembling the Finite Difference plot validates the TLM code. The difference between them at any point is $O(0.1)$.

5. Validation of TAMC generated adjoint model using validated tangent linear model:

(a) We first generate the Transfer Matrix for the Tangent Linear & ADJ models (for a 5x5 grid) as follows. Each code is run for one time step multiple times; each run

Fig(a) $t = 0$

Fig(b) $t = T$
uses an initial profile that is equal to zero everywhere, except a single gridpoint where is it equal to one. There are 25 distinct initial conditions, and after one time step they produce the 25 columns of the transfer matrix. We have computed separately the transfer matrices for the tangent linear and for the discrete adjoint ppm models. The result obtained shows that these two matrices are the exact transpose of each other, which validates the adjoint code. Few lines of result from the generated output are as follows:

\begin{align*}
\begin{array}{ccc}
i=1 & j=1 & F=0.543906 & A^\dagger=0.543906 \\
i=4 & j=5 & F=0.207375 & A^\dagger=0.207375 \\
i=10 & j=5 & F=0.141969 & A^\dagger=0.141969 \\
\end{array}
\end{align*}

(b) Next we take the scalar product of the TLM perturbations with the ADJ variables at different times and then find the relative difference of the initial and final steps. This validation procedure is based on the fact that in theory one should have the equality

\[ \lambda^T(t_f) \cdot \delta \lambda^\dagger(t_f) = \lambda^T(t_0) \cdot \delta \lambda^\dagger(t_0) \]

where, \( \lambda \) are the adjoint variables and \( \delta C \) are perturbations of the forward solution (TLM variables). The relative difference obtained is:

\[ \text{rel\_diff} = -0.0944, \]

also reflects the conformation between the tangent linear model and discrete adjoint model.
3 Validation results for the discrete adjoints of vertical advection VPPM

We will follow the similar validation procedure for VPPM as was done for HPPM.

1. Rotating Cone test for the original advection scheme (vppm) for half cycle:

![Fig(a) t = 0](image1)

![Fig(b) t = T](image2)

2. Rotating Cone test for TAMC generated tangent linear model (vppm_ftl):

![Fig(a) t = 0](image3)

![Fig(b) t = T](image4)

3. Rotating Cone test for TAMC generated adjoint model (vppm_ad):

![Fig(a) t = T](image5)

![Fig(b) t = 0](image6)

4. Validation of Tangent Linear model by comparing TLM with the finite
difference scheme:
A spike is placed at the location where forward cone starts (at $t=0$) and then is advected using (i) Finite Difference formulae [Diff] (ii) TAMC generated TLM (iii) PPM subroutine [ppm.f] [Direct Conc]

Fig(a) $t = 0$

Fig(b) $t = T$
Observation: The TAMC generated TLM plot resembling the Finite Difference plot validates the TLM code. The difference between them at any point is $O(0.1)$.

5. Validation of TAMC generated adjoint model using validated tangent linear model:

(a) Comparison of the Transfer Matrices for the Tangent Linear & ADJ models of vppm reflects that these two matrices are the exact transpose of each other and hence conform each other. Few lines of result from the generated output are as follows:

\begin{align*}
  i=2 & \quad j=3 & F=0.198187 & A^=0.198187 \\
  i=9 & \quad j=5 & F=0.055125 & A^=0.055125 \\
  i=13 & \quad j=14 & F=0.175744 & A^=0.175744
\end{align*}

(b) Next the scalar product of the TLM concentrations with the ADJ concentrations is performed and then the relative difference of all the time steps is plotted.

This plot also reflects the conformation between the tangent linear model and discrete adjoint model.
4 Optimization (data assimilation) tests for advection process

Data assimilation is a process in which one tries to adjust the initial state of a model to best reproduce some observed behavior (Bischof). We conducted data assimilation tests with continuous and discrete adjoints for both horizontal and vertical advection schemes. The optimization code is L-BFGS-B, which implements a quasi-Newton approach.

All tests are done in a twin experiment framework. Periodic snapshots of the reference solution are recorded on an 8x8 “observational grid” to provide artificial observations. Then the initial conditions are perturbed by changing the amplitude and the location of the Gaussian profile. The optimization procedure starts from the perturbed initial profile and attempts to recover the reference initial profile using the pseudo-observations.

The results and observations from this test are as follows. For each case we show the messages from the optimization routine (L-BFGS-B) confirming convergence.

**Vertical Advection:**

(1) Discrete Adjoint

\[ F = 13.422926114834592 \]

CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH

Cauchy time 1.300E-02 seconds.
Subspace minimization time 2.800E-02 seconds.
Line search time 6.773E+01 seconds.

Total User time 6.881E+01 seconds.
Fig(1) Data Assimilation result with discrete adjoint for vertical advection scheme.

Perturbation measures:

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Position</th>
<th>Observation grid</th>
<th>Actual grid size</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0 Left, h=grid size</td>
<td>8x8</td>
<td>33x33</td>
</tr>
</tbody>
</table>

(2) Continuous Adjoint

\[ F = 0.04340421732269162 \]

CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH

Cauchy time 1.299E-02 seconds.
Subspace minimization time 2.700E-02 seconds.
Line search time 1.299E+02 seconds.

Total User time 1.306E+02 seconds.
Fig (2) Data Assimilation result with continuous adjoint for vertical advection scheme.

Perturbation measures:

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Position</th>
<th>Observation grid</th>
<th>Actual grid size</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-h Left, h=grid size</td>
<td>8x8</td>
<td>33x33</td>
</tr>
</tbody>
</table>

Observation: The continuous adjoint produces much better result even with large perturbation. The discrete adjoint does not work well with large perturbations.

**Horizontal Advection:**

(1) **Discrete Adjoint**

\[
F = 222.00656351046362
\]

CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH

Cauchy time 1.953E-03 seconds.
Subspace minimization time 3.021E-03 seconds.
Line search time 8.649E+02 seconds.

Total User time 8.975E+02 seconds.
Fig (1) Data Assimilation result with discrete adjoint for horizontal advection scheme.

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Position</th>
<th>Observation grid</th>
<th>Actual grid size</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0 Left, h=grid size</td>
<td>8x8</td>
<td>33x33</td>
</tr>
</tbody>
</table>

(2) Continuous Adjoint

\[ F = 0.08047599869213462 \]

CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH

Cauchy time 1.778E-02 seconds.
Subspace minimization time 2.197E-02 seconds.
Line search time 2.533E+03 seconds.

Total User time 2.553E+03 seconds.
Fig (2) Data Assimilation result with continuous adjoint for horizontal advection scheme.

Perturbation measures:

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Position</th>
<th>Observation grid</th>
<th>Actual grid size</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-h Left, h=grid size</td>
<td>8x8</td>
<td>33x33</td>
</tr>
</tbody>
</table>

Observation: The continuous versus discrete adjoint properties as described in vertical advection case hold the same for horizontal advection. In addition, horizontal advection results are inferior to those produced by vertical advection. Also, the running time is too high for horizontal advection case.
4D-Var Data Assimilation for CMAQ

Introduction

4D-Var data assimilation allows the optimal combination of three sources of information: an a priori (background) estimate of the state of the atmosphere, knowledge about the physical and chemical processes that govern the evolution of pollutant fields as captured in the chemistry transport model (CTM), and observations of some of the state variables.

Data Assimilation Experiment

In order to integrate the 4D-Var data assimilation with CMAQ, an interface has been developed to integrate the science processes with the optimization routine L-BFGS. A reference run of 6hrs is being performed on the original data that comes with CMAQ in order to generate non-constant concentration field. The concentration field thus obtained acts as an initial condition ($c_0^0$) for our data assimilation test. An observation grid is extracted by performing a forward CMAQ run on the current concentration field ($c_0^0$).

A perturbation is then introduced in $c_0^0$ to produce $c_p^0$.

$$c_p^0 = c_0^0 + \text{perturbation}$$

This perturbed concentration is then transferred to the optimization subroutine in order to obtain the best estimate $c_{op}^0$ of the original concentration $c_0^0$ after several data-assimilation runs.
At iteration 0, \( x_0 = c_p^0 \)

At each subsequent iteration \( k \) \((k \geq 1)\),

\[
 x_{k+1} \leftarrow \text{L-BFGS} \left( x_k, f, g \right) \\
 c_o^0 \leftarrow x_{k+1} \\
 (f, g) \leftarrow \text{reverse_mode} \left( c_o^0, \text{chk}_11 \right)
\]

where, \( f \) is the cost function and \( g \) is the gradient of the cost function.

The cost functions and gradients are defined as follows:

\[
f = \frac{1}{2} \sum (c_o^{k,m} - c_{\text{obs}}^{k,m})^T R_k^{-1} (c_o^{k,m} - c_{\text{obs}}^{k,m}) + \frac{1}{2} \sum (c_o^{k,b} - c_{\text{b}})^T B^{-1} (c_o^{k,b} - c_{\text{b}})
\]

and

\[
g = \sum R_k^{-1} (c_o^{k,m} - c_{\text{obs}}^{k,m}) + \sum B^{-1} (c_o^{k,b} - c_{\text{b}})
\]

where, \( R_k \) is the misfit covariance matrix and \( B \) is the background covariance matrix, \( k = 1,2,\ldots \text{step*steps} \), is the total number of science process iterations in the forward mode and \( m \) is a 4-tuple observation grid. The background forcing is added only at the initial time step. The observation grid specifications and perturbation amounts used are presented as follows:

**Test Parameters:**

<table>
<thead>
<tr>
<th>Observation Grid</th>
<th>Perturbation amount</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Columns</strong></td>
<td><strong>Rows</strong></td>
</tr>
<tr>
<td>Linspace(x1,x2,10) X1=1, x2 = ncols</td>
<td>Linspace(x1,x2,10) x1=1, x2 = nrows</td>
</tr>
</tbody>
</table>

In our observation grid, the columns and rows are linearly spaced among 10 points from \( x1 \) to \( x2 \) using Matlab’s `linspace()` function. The vertical layer slices are double spaced. For background, the cost function is calculated over the whole \( n\text{cols} \times n\text{rows} \) grid.

**Caution:**

The species under consideration are ozone only. In our experiment we are perturbing ozone and retrieving ozone only.

**Validation Results**

To validate the developed test-bed, two sets of plots are being generated; one for the difference \( c_p^0 - c_0^0 \) and the second for \( c_o^0 - c_0^0 \). The idea is to illustrate the fact that data assimilated concentration fields are the best estimates of the original concentration fields. Figure 1 reflects that \( c_o^0 \) is quite close to \( c_0^0 \). A cost-function and root-mean square vs. model runs plot, Figure 2, is also provided for further validations.
Layer 1 O3c-O3b

\( b - \text{CCTM}_2 \alpha_2 \text{LCHK}_2 \alpha_2, \quad c - \text{CCTM}_2 \alpha_2 \text{LCHK}_2 \alpha_2 \)

Layer 1 O3a–O3b

\( a - \text{CCTM}_2 \alpha_2 \text{CHK}_2 \alpha_2, \quad b - \text{CCTM}_2 \alpha_2 \text{LCHK}_2 \alpha_2 \)

a) Difference between perturbed and reference concentration \((c_p^0 - c_0^0)\)

b) Difference between optimized and reference concentration \((c_{op}^0 - c_0^0)\)

**Figure 1:** Demonstration of recovery of the original concentration field from the perturbed state.

**Figure 2:** Cost-function and root-mean square values vs. model runs.

**Conclusion**

As explained in the last section, Figure 1 shows a good recovery of the original concentration
field from the perturbed field. Figure 2 on the other hand reflects that there is a significant cost function decrease with model runs which means that the solution converges to the original concentration field at the grid points. However, to illustrate an overall convergence of the solution, we calculate the root-mean square (RMS) value given by:

\[
\text{RMS} = \| \mathbf{c}_0 - \mathbf{c}_0^0 \| / \| \mathbf{c}_0^0 \| 
\]

The decrease in this RMS value with model runs signifies that there is an overall convergence of the solution.
Couple/Decouple in CMAQ Adjoint

In CMAQv4.5, the chemistry and transport processes are called in the SCIPROC subroutine. The underlying numerical schemes in the transport processes are conservative in some cases and non-conservative in others. A conservative scheme requires the concentration values to be in molec/cm$^3$ for mass conservation purposes. While, a non-conservative scheme requires them in mole-fractions. The two subroutines COUPLE and DECOUPLE used in SCIPROC are responsible for this unit conversion.

Let,

$$ C = \text{concentration in molec/cm}^3 $$

$$ y = \text{concentration in mole-fraction} $$

Then, couple and decouple operations can be defined in the following way.

**COUPLE**: multiply concentration variable in mole-fraction by air-density to convert it to molec/cm$^3$

$$ y \times \rho \rightarrow C $$

**DECOUPLE**: divide concentration variable in molec/cm$^3$ by air-density to convert it to mole-fraction

$$ C/\rho \rightarrow y $$

1 Horizontal and Vertical Advection

The horizontal and vertical advection processes use Colella and Woodward’s “piecewise parabolic method (PPM)”, which is a conservative scheme. Since the concentration field in the forward mode of CMAQ is in mole-fractions, it undergoes the coupling process to be converted to molec/cm$^3$. This field is advected by the PPM scheme and then converted back to mole-fraction by the decoupling process.(refer SCIPROC subroutine)

The PPM scheme solves the following one-dimensional advection equation:

$$ \frac{dC}{dt} = -\frac{d}{dx}(uC) \quad (1) $$

The air density $\rho$ is also in molec/cm$^3$. Hence, it follows the same transport equation as above.

$$ \frac{d\rho}{dt} = -\frac{d}{dx}(up) \quad (2) $$

The advection equation for decoupled concentrations can be derived from equation (1) as:

$$ \frac{d(yp)}{dt} = -\frac{d}{dx}(up) $$

$$ \Rightarrow y \frac{d\rho}{dt} + \rho \frac{dy}{dt} = -y \frac{d}{dx}(up) - (up) \frac{dy}{dx} $$
Multiplying equation 2 with $y$ and substituting it in the above equation leads to

$$\frac{dy}{dt} = -u \frac{dy}{dx}, \text{ (non-conservative form)} \tag{3}$$

The adjoint equivalent of equation (1) is given by

$$\frac{d\lambda}{dt} = -u \frac{d\lambda}{dx} \tag{4}$$

From the above discussion, it is easy to deduce that the adjoint equivalent (equation (4)) of the advection equation in conservative form is similar to its non-conservative form (equation (3)), and vice-versa. As the concentrations in forward mode of CMAQ are in mole-fractions, the adjoint variables are in molec/cm$^3$. Also, the continuous adjoint of the PPM scheme is conservative. Hence, there is no coupling/decoupling involved in the adjoint mode of CMAQ for advection processes as reflected in SCIPROC_ADJ subroutine.

## 2 Horizontal Diffusion

The horizontal diffusion process in the forward mode solves the following equation:

$$\frac{dy}{dt} = \frac{1}{\rho} \frac{d}{dx} \left( K\rho \frac{dy}{dx} \right) \tag{5}$$

The above equation is in non-conservative form and there is an internal decoupling in the HDIFF subroutine. Thus, the concentration values are coupled before HDIFF subroutine is called in SCIPROC. The adjoint equivalent of the diffusion equation (5) is given by:

$$\frac{d\lambda}{dt} = -\frac{d}{dx} \left( K\rho \frac{d(\lambda/\rho)}{dx} \right)$$

The above adjoint equation is in non-conservative form. Since, the adjoint variables are in molec/cm$^3$ and there is an internal coupling involved in HDIFF_ADJ, there is no external coupling/decoupling required for horizontal diffusion in SCIPROC_ADJ.

## 3 Vertical Diffusion

The vertical diffusion is carried out by the VDIFF subroutine in CMAQ. This subroutine solves the following diffusion equation:

$$\frac{dC}{dt} = \frac{d}{dz} \left( K \frac{dC}{dz} \right) \tag{6}$$

This equation is in conservative form and there is an internal coupling and decoupling involved in VDIFF subroutine. Hence, this subroutine is kept out of the coupling-decoupling loop in SCIPROC. The adjoint equivalent of equation (6) is given by:

$$\frac{d\lambda}{dt} = -\frac{d}{dz} \left( K \frac{d(\lambda/\rho)}{dz} \right)$$

The above adjoint equation is in conservative form. The adjoint variables are in molec/cm$^3$ and the internal coupling and decoupling is switched off in VDIFF_ADJ subroutine. Thus, no external coupling/decoupling is required for VDIFF_ADJ subroutine in SCIPROC_ADJ.
CMAQ v4.5 Adjoint User’s Manual

An adjoint model for CMAQ to perform 4D-Var Data Assimilation and Sensitivity Analysis

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Date: 07/05/2007
1. Installation

The CMAQ_ADJv4.5 adjoint package comes in the same format as CMAQv4.5. To install the adjoint package,

(i) Download the CMAQv4.5 package from CMAS center’s website: http://cmascenter.org/help/documentation.cfm?MODEL=cmaq&VERSION=4.5.

(ii) Replace the tar files M3MODELS.CMAQv4.5.tar and M3SCRIPTS.CMAQv4.5.tar of CMAQv4.5 package with those of the adjoint package.

(iii) Follow the $M3HOME/docs/README file for instructions on installing the package. Instructions for subdirectories icon, bcon, jproc, and others in $M3HOME/scr ipts/ directory remain the same, except for the cctm directory where bldit.cctm.pgf and run.cctm scripts should be executed based on the mode required.

(iv) Follow section 2 on installing and running various modes of adjoint operations provided with this CMAQ_ADJ4.5 package.

The visible changes in the adjoint package as compared to the basic CMAQ package are:

- The addition of an adjoint directory in $M3HOME/models/CCTM/src/ which contains all the adjoint files. These files are extracted by setting some flags in the bldit.cctm.pgf script in $M3HOME/scr ipts/cctm directory, as discussed in Section 2. A brief description about each of these adjoint files is provided in Section 6.

- Makefile provided explicitly in the $M3HOME/scr ipts/cctm directory to produce executables, to be used by the run.cctm script based on user’s mode selection.

2. Running CMAQ Adjoint model

There are several adjoint run modes provided to the users to select from, based on their needs. First of all, in “$M3HOME/scr ipts/cctm/bldit.cctm.pgf” file one can choose not to extract the adjoint files if they are planning to run only the forward mode of CMAQ similar to the basic package. To do so, make sure the *ADJOINT MODE* (lines 372-373) is commented out. However, for sensitivity analysis, 4D-Var data-assimilation and finite-difference test modes, adjoint files are required. Executing the bldit.cctm.pgf script creates the BLD_e2a and MOD_DIR directories.

To select various adjoint run modes, follow the instructions provided as follows:
2.1 Mode Selection

*SIMULATION MODES* (lines 190-212) in this run.cctm script provide various adjoint run modes to choose from. Following are the setting details for each individual mode:

2.1.1 Forward mode

Uncomment *BUILD FORWARD* mode (lines 194-195) and comment out rest of the modes. This will create the fwd executable which is for the forward mode of simulations, similar to the basic CMAQ model run.

2.1.2 Sensitivity Test mode

Uncomment *BUILD SENSITIVITY* mode (lines 199-200) and comment out the rest of the modes. This will create the snst executable which is to perform sensitivity analysis.

2.1.3 4D-Var Data Assimilation mode

Uncomment *BUILD 4D-VAR* mode (lines 204-205) and comment out the rest of the modes. This will create the 4dv executable which is to perform 4D-Var data assimilation experiment.

2.1.4 Finite Difference Test mode

Uncomment *BUILD FINITE-DIFF* mode (lines 209-210) and comment out rest of the modes. This will create the fd executable which is to perform finite difference test for adjoint code validation.

2.2 Data Directories and Files

The important output directories and useful data files (including new adjoint files) are listed as follows:

$M3HOME/data/cctm/

----------------------------
CCTM_e2aCONC.e2a  - concentration file for basic CMAQv4.5 run.
----------------------------
CCTM_e2aCHK.e2a   - checkpoint files for sensitivity test and data assimilation.
CCTM_e2aL3CHK.e2a
CCTM_e2aL5CHK.e2a
CCTM_e2aL6CHK.e2a
----------------------------
2.3 Description / Instructions about current settings

The checkpoint files listed above are used to save different datasets in different adjoint run modes. Thus to maintain consistency, all the output data files and executables are deleted each time the run script is executed. It is advisable to save the checkpoint files elsewhere for future use. However, if you are running the same mode and would like to keep the data files, then comment out “make dataclean” (line 188, run.cctm file). Also, if you are making changes in the files for the same adjoint run mode and do not want to recompile everything, then comment out “make clean” (line 186, run.cctm file). It is recommended to clear the data files and executables during transitions from one mode to the other. Individual setting details for each mode will be provided in the following sections.

3. Code Validation – Finite Difference (FD) Test

To obtain consistent data assimilation and sensitivity results, it is necessary to validate all the adjoint science process subroutines. The validation test is designed to compare the adjoint variable with its finite-difference approximation. A detailed description of the test setup is provided as follows.

The CTM model is first run with original concentration values (say $c_0$) and an observation grid is extracted. The original concentration field ($c_0$) is then perturbed at the initial time and a run with the new concentration (say $c_{01}$) in forward and reverse mode is carried out. A similar run with different perturbation on the original concentration field is carried out next. Let $c_{02}$ be the perturbed concentration at initial time for the second run.

In reverse mode of each run, the adjoint variable $\lambda$ is initialized with zero concentration values at the final time and is updated with the gradient ($g$) of the cost function ($f$) at each dynamic time step during the adjoint calculations.

\[
\begin{align*}
  f &= (1/2) \sum (c_{op}^{k,m} - c_{obs}^{k,m})^T R_k^{-1} (c_{op}^{k,m} - c_{obs}^{k,m}) \\
  g &= \sum R_k^{-1} (c_{op}^{k,m} - c_{obs}^{k,m}) \\
  \lambda &= \lambda + g_k
\end{align*}
\]

where, $R_k$ is the misfit covariance matrix, $k=1, 2, \ldots, istep*ntsteps$, is the total number of science process iterations in the forward mode and $m$ is a 4-tuple observation grid.
Based on this setup, adjoint model for each CMAQ science process is validated separately. The validation procedure is based on the fact that the value of adjoint variable (≡ g) should be close enough to its finite difference approximation calculated with the objective cost functions for the two runs with concentrations \( c_{01} \) and \( c_{02} \), i.e.,

\[
f(c_{02}) - f(c_{01}) = \lambda^T (c_{02} - c_{01})
\]

### 3.1 Mode Selection

To test the adjoint of each science process individually, one can choose one of the modes discussed as follows:

**Caution:**

- The files extracted and kept in the BLD_e2a directory are read-only. Thus, to comment/uncomment the science processes one needs to `chmod a+w filenames`. The PING, CLDPROC, AERO and ADJADV processes should remain commented, since the adjoint for these subroutines are not available with this package.

- The results provided for the current test setup are produced with 24 hours of simulation. Hence, make sure that NSTEPS = 240000 in run.cctm script, line 42.

#### 3.1.1 Chemistry

*filenames:*

- `sciproc_cadj.F` – Uncomment *CHEM_ADJ* subroutine (line 178) and comment out rest of the processes (lines 188-220).

- `modsciproc.F` – Uncomment *CHEM* subroutine (lines 334-336) and comment out rest of the processes (lines 252-323).

#### 3.1.2 Advection

*filenames:*

- `sciproc_cadj.F` – Uncomment *rhoj checkpoint reading*, *XADV_CAD*, *YADV_CAD*, *ZADV_CAD* subroutines (lines 193-218) and comment out the rest of the processes (lines 178,188,220).

- `modsciproc.F` – Uncomment *couple*, *decouple*, *XADV*, *YADV*, *ZADV* subroutines (lines 258-300, 317-322) and comment out the rest of the science processes (lines 252-254, 311-313, 334-336).

#### 3.1.3 Diffusion

*filenames:
sciproc_cadj.F – Uncomment *HDIFF_ADJ*, *VIDFF_ADJ* subroutines (lines 188, 220) and comment out the rest of the science processes (lines 178, 193-218).

modsciproc.F – Uncomment *VIDFF*, *DIFF* subroutines (lines 252-254, 311-313) and comment out the rest of the processes (lines 258-300, 317-336).

3.1.4 All science processes

filenames:
sciproc_cadj.F - Uncomment all the science process adjoints, *VDIFF_ADJ*, *HADV_ADJ*, *ZADV_ADJ*, *HDIFF_ADJ*, *CHEM_ADJ*.

modsciproc.F – Uncomment all the science processes, *VDIFF*, *HADV*, *ZADV*, *HDIFF*, *CHEM*.

3.2 FD Validation Results

In the current code-validation setup, the test parameters have following values:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eps</td>
<td>0.001</td>
</tr>
<tr>
<td>T_f</td>
<td>24 hrs</td>
</tr>
<tr>
<td>C_01</td>
<td>CGRID*(1.1+9*eps)</td>
</tr>
<tr>
<td>C_02</td>
<td>CGRID*(1.1+10*eps)</td>
</tr>
</tbody>
</table>

Table 1: Test Parameter Values

The test parameters presented in Table 1 can be varied according to individuals need. However for the current setup, the validation results for different modes are as follows:

<table>
<thead>
<tr>
<th>Process</th>
<th>$F(c_{02}) - F(c_{01})$</th>
<th>$\lambda_2^{+}(c_{02} - c_{01})$</th>
<th>Relative Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemistry</td>
<td>587.15439</td>
<td>589.84952</td>
<td>0.456918 %</td>
</tr>
<tr>
<td>Advection</td>
<td>390.95876</td>
<td>390.36101</td>
<td>-0.153130 %</td>
</tr>
<tr>
<td>Diffusion</td>
<td>624.34778</td>
<td>630.00552</td>
<td>0.898046 %</td>
</tr>
<tr>
<td>All</td>
<td>340.66521</td>
<td>329.15051</td>
<td>-3.498310 %</td>
</tr>
</tbody>
</table>

Table 2: CMAQ Adjoint Validation Results

4. Sensitivity Test

Sensitivity analysis is an approach to quantify the changes in output due to different sources of variation. Since adjoint calculations are receptor based, one can calculate the sensitivity of an output with respect to large number of parameters.
In order to perform adjoint sensitivity tests on the validated CMAQ adjoint model, it is required to perform one forward and one reverse mode of simulation. In the forward run, the concentration values and air densities are checkpointed and in the reverse mode they are read to be utilized for adjoint calculations. A receptor (cost-function) measuring certain species (say \(x\)) is defined at a given location at the final time in the reverse mode. At the end of the simulation, we obtain sensitivities of \(x\) with respect to the grid species and emission species.

4.1 Settings

filenames:

sensitivity_driver.F – main driver file to initialize CGRID, open checkpoint files and call subdriver to perform forward and backward runs.

senstdriver_bwd.F – subdriver to perform one forward and one backward run with adjoint variable defined by calling subroutine DEFINE_RECEPTOR and emission adjoint variable initialized with zero concentration.

define_receptor.F – subroutine to initialize the adjoint variable LGRID.

Note: Adjoint variable LGRID(1:ncols, 1:nrows, 1:nlays, 1:spc) has 4 dimensions. In order to change the species number for which sensitivity test has to be performed, change the spc index at the initialization step in define_receptor.F (line 50).

4.2 Test details and Analysis

filename:

CCTM_e2aL5CHK.e2a – checkpoint file for LGRID values for every dynamic time step.
CCTM_e2aL6CHK.e2a – checkpoint file for cumulative emission adjoint variable.

In order to calculate the sensitivity with respect to grid species and emissions, set the parameters according to your needs using the files described in section 4.1. Choose the snst mode in run.cctm for sensitivity calculations and perform the run. The adjoint variable for grid species initialized at the final time undergoes all the science process adjoints while the emission adjoint variable is accumulated with the gradient of emissions over time.

4.2.1 Sensitivity Results

In order to produce the adjoint trajectory for sensitivities with respect to grid species, PAVE the CCTM_e2aL5CHK.e2a checkpoint file. Then from the list of species in formula popup window, add the ones with respect to which sensitivities has to be performed. Then create the tile plot. Similar procedure is followed for sensitivities with respect to emissions. PAVE the CCTM_e2aL6CHK.e2a checkpoint file and select the emission species with respect to which the sensitivities have to be performed. A major difference between the two plots is the time length.
The emissions plot is for the final time step so it has no time series involved, while the grid species plot has NSTEPS*(dynamic time steps) time length.

**Grid-species:**

In the sample test case, we consider a receptor measuring ozone at the final time at a given location (defined in define_receptor.F). We compute its sensitivity with respect to changes in various grid species at earlier times. A tile plot for dO$_3$/dNO$_2$ tile is provided as follows:

In figures 1, the receptor is O$_3$ at location ncols=18:22, nrows=18:22 and nlays=1 at time T=24hrs.

**Emissions:**

Under the same settings, the sensitivity of ozone is calculated with respect to various emission species. Tile plots for dO$_3$/dNO$_x$ emissions are provided as follows:

In figures 2, the receptor is O$_3$ at location ncols=18:22, nrows=18:22 and nlays=1 at time T=47hrs.
4.3 More than one-day simulations:

To perform sensitivity test for N (> 1) days, a set of instructions are provided as follows:

- Set the fwd mode in run.cctm script and run the forward mode for N-1 days. The current setup will run the code for 24 hours. However, for rest of the N-2 days we need to make the following changes:

  (i) Change the STDATE in run.cctm (line 39) for each day’s simulation.
  (ii) Set the EMISfile to the emission file of that day. The current run script has two emission file names.
  (iii) Uncomment set GC_ICpath = $OUTDIR and set GC_ICfile = $EXEC"CONC".$APPL (lines 119,120) and comment out earlier settings with ICON (lines 121,122).

  Caution: Comment out the “make dataclean” mode (line 188, run.cctm file)

Settings (i) and (ii) are done for each day, while (iii) remains the same for all N-1 days including the Nth day.

This will create the CCTM_e2aCONC.e2a file for all the N-1 days which acts as the initial condition file for the Nth day simulation. The adjoint calculations are then done backwards for each day N-1, N-2, ..., 2, 1.

- For the last day, set the snst mode in run.cctm script and follow steps (i),(ii) and (iii) described above. Make sure the flag “LASTDAY” in senstdriver_bwd.F (line 104) is set to TRUE. The simulation hours for the last day must be less than 24 hours, since interpolation for an extra hour is performed inside the model.

  Caution: Keep the “make dataclean” mode (line 188, run.cctm file) commented.

For the rest of the previous days, follow steps (i),(ii) and (iii) and change the flag “LASTDAY” in senstdriver_bwd.F (line 104) to FALSE.

This will create the necessary checkpoint files for all the days required.

5. 4d-Var Data Assimilation

4D-Var data assimilation allows the optimal combination of three sources of information: an a priori (background) estimate of the state of the atmosphere, knowledge about the physical and chemical processes that govern the evolution of pollutant fields as captured in the chemistry transport model (CTM), and observations of some of the state variables.

In order to integrate the 4D-Var data assimilation with CMAQ, an interface has been developed to integrate the science processes with the optimization routine L-BFGS. This routine is used to solve large scale nonlinear optimization problems with simple bounds. It is based on the gradient

The BFGS is a quasi-Newton algorithm which solves

\[ \text{min } f(x), \text{ subject to } l \leq x \leq u. \]

where \( f(x) \) is an objective cost-function in \( x \) which we are trying to minimize. \( l \) and \( u \) are the lower and upper bounds on the values of \( x \). The optimization routine takes as input, an initial value \( x \), the cost-function value \( f \) and the gradient of the cost function \( g \) and gives the next best estimate of \( x \). The same procedure is followed until the solution converges to a single point. The number of iterations can be restricted based on individuals need.

For data assimilation experiments, the cost-function is defined as follows:

\[
f = \frac{1}{2} \sum (c_{opt}^{k,m} - c_{obs}^{k,m})^T R_k^{-1} (c_{opt}^{k,m} - c_{obs}^{k,m}) + \frac{1}{2} \sum (c_{opt} - c_b)^T B^{-1} (c_{opt} - c_b)
\]

where, \( R_k \) is the misfit covariance matrix and \( B \) is the background covariance matrix, \( k=1, 2, \ldots, \text{istep} \times \text{nsteps} \), is the total number of science process iterations in the forward mode and \( m \) is a 4-tuple observation grid.

The gradient of this cost-function is then calculated as follows:

\[
g = \sum R_k^{-1} (c_{opt}^{k,m} - c_{obs}^{k,m}) + \sum B^{-1} (c_{opt} - c_b)
\]

Here \( c_b, c_{obs} \) and \( c_{opt} \) are the background concentration, observations and the best estimate respectively. In case of real data inputs, \( c_b \) acts as apriori concentration and \( c_{obs} \) is the observation. \( c_{opt} \) acts as the current best estimate which is updated by the L-BFGS subroutine every iteration based on the cost-function and its gradient values.

**Note:**

The background forcing is added only at the initial time step, while the misfit is added at each dynamic time step. Also the misfit is calculated only on the observation grid points, while the background is calculated at all the grid points on a layer.

### 5.1 Experiment details and settings

In the sample test case, the data used is the one which comes with CMAQv4.5 package. A reference run of \( T_f \) hrs is being performed on this data in order to generate non-constant concentration field. The concentration field thus obtained acts as an initial condition \( (c_0^0) \) for our data assimilation test. An observation grid is extracted by performing a forward CMAQ run on the current concentration field \( (c_0^0) \).
Fig 3: Checkpointing procedure during forward run

A perturbation is then introduced in $c_0^0$ to produce $c_p^0$.

$$c_p^0 = c_0^0 + \text{perturbation}$$

This perturbed concentration is then transferred to the optimization subroutine in order to obtain the best estimate $c_{\text{opt}}^0$ of the original concentration $c_0^0$ after several data-assimilation runs.

At iteration 0, $x_0 = c_p^0$

At each subsequent iteration $k$ ($k \geq 1$),

$$x_{k+1} \leftarrow \text{L-BFGS} (x_k, f, g)$$

$$c_{\text{opt}}^0 \leftarrow x_{k+1}$$

$$(f, g) \leftarrow \text{reverse_mode} (c_{\text{opt}}^0, \text{chk}\_\text{obs})$$

where, $f$ is the cost function and $g$ is the gradient of this cost function.

Details about the subroutines involved and the corresponding files where they are defined are as follows:

filenames:
4DVar_driver.F – main driver file to implement 4D-Var data assimilation. Calls sub drivers to lay down observations, checkpoint perturbed data, and perform adjoint calculation to get cost function value and its gradient for the optimization subroutine.
subdriver_fwd.F – holds subroutine to perform forward science process runs. This subdriver is called twice in the main driver. First call produces a non-constant concentration field, required since CMAQv4.5 data is not real data. Second call lays down the observation field.

subdriver_fwdpert.F – contains subroutine to perform forward science process similar to the above subdriver, but lays down checkpoints for perturbed data. This checkpoint is used in the background cost-function calculations.

subdriver_bwd.F – contains subroutine to perform one forward and one reverse run of science processes to calculate the cost-function and its gradient to be used by the L-BFGS subroutine.

calc_obsgrad.F – calculates the cost-function due to misfit and updates the adjoint variables.

calc_bggrad.F – calculates the cost-function due to background errors and updates the adjoint variables.

mask.dat – observation grid index file located in directory: $M3HOME/scripts/run, to be used by OBS_GRAD_UPDATE subroutine (calc_obsgrad.F). In the current test case observation grid, the columns and rows are linearly spaced among 10 points from x1 to x2 using Matlab’s linspace() function. The vertical layer slices are double spaced.

Caution:

- In our experiment we are perturbing ozone and retrieving ozone only (spc=4). To perturb other species, change the spc index from 4 to the species number of interest in 4DVar_driver.F file(line 301, 310). To retrieve other species, change NSPCOPTSTART and NSPCOPTEND variable values in the same file: 4DVar_driver.F (line 325,326).

- The interface for cost function calculations in files: calc_obsgrad.F and calc_bggrad.F are designed for model data input. One should change the setup for real data intake.

In the current experimental setup, the parameter values are set as follows:

<table>
<thead>
<tr>
<th>Observation Grid</th>
<th>Perturbation amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columns</td>
<td>Rows</td>
</tr>
<tr>
<td>Linspace(x1,x2,10)</td>
<td>Linspace(x1,x2,10)</td>
</tr>
<tr>
<td>x1=1, x2 = ncols</td>
<td>X1=1, x2 = nrows</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ c_p^0 = c_0^{0.8}(1.3+9*\text{eps}), \quad \text{eps} = 0.001 \]

Table 3: Data Assimilation experiment parameter values

5.2 4D-Var Data Assimilation Results

filenames:
- CCTM_e2aCHK.e2a – optimized concentration checkpoint file
- CCTM_e2aL3CHK.e2a – observation checkpoint file
- CCTM_e2aL5CHK.e2a – perturbed concentration checkpoint file
To validate the developed test-bed, two sets of plots are being generated; one with the difference $c_p^0 - c_0^0$ (CCTM_e2aL5CHK.e2a - CCTM_e2aL3CHK.e2a) and the second with $c_{opt}^0 - c_0^0$ (CCTM_e2aCHK.e2a - CCTM_e2aL3CHK.e2a). The idea is to illustrate the fact that data assimilated concentration fields are the best estimates of the original concentration fields. Figure 3 reflects that $c_{opt}^0$ is quite close to $c_0^0$.

Layer 1 O3c–O3b

Layer 1 O3a–O3b

Fig 4: Demonstration of recovery of the original concentration field.

A cost-function and root-mean square vs. model runs plot, Figure 4, is also provided for further validations.

Fig 5: Cost-function and root-mean square values vs. model runs.
As explained in the last section, Figure 3 shows a good recovery of the original concentration field from the perturbed field. Figure 4 on the other hand reflects that there is a significant cost function decrease with model runs, which means that the solution converges to the original concentration field at the observation grid points. However, to illustrate an overall convergence of the solution, we calculate the root-mean square (RMS) value given by:

\[
\text{RMS} = \| c_{op}^0 - c_0^0 \| / \| c_0^0 \|
\]

The decrease in this RMS value with model runs signifies that there is an overall convergence of the solution.

Caution:

The data assimilation results presented are generated with \( T_f = 6\text{hrs} \) and 16 iterations.

- \( T_f = 6\text{hrs} \) → set in $M3HOME/scripts/cctm/run.cctm script, NSTEPS = 060000, line 42.
- Number of iterations = 16 → set in $M3HOME/scripts/cctm/BLD_e2a/4DVar_driver.F, N_CMAQ_RUNS to be changed to desired value, line 477.

6. New Subroutines in CMAQ Adjoint

All the adjoint files are CVS archived under $M3HOME/models/CCTM/src/ and when extracted they are placed under $M3HOME/scripts/cctm/

Description:

6.1 Drivers

Main drivers to perform 4D-Var data assimilation, sensitivity analysis and finite difference test.

- **4DVar_driver.F** (Subroutine: DRIVER)

Function:
- CTM driver for 4D-Var DATA ASSIMILATION
- Uses L-BFGS optimization routine
- Exponential Preconditioning option available with this package.
  To invoke it, change EXP_PRECOND to 'TRUE', line = 127

Caution:
- Driver designed to perturb and retrieve ozone only
- To perturb other species change SPC index of CGRID in the Initialization, line = 297 & 302
- To retrieve accordingly, change NSPCOPTSTART & NSPCOPTEND variables, line = 318

Useful Files:
CONC_L3CHK = Reference/Base checkpoint file
CONC_L5CHK = Initial Guess checkpoint file
CONC_CHK   = Next Best Guess concentration

Subroutines and functions called:
SUBDRIVER_FWD     - Forward Run for Reference checkpointing
SUBDRIVER_FWDPERT - Forward Run for Best Guess checkpointing
SUBDRIVER_BWD     - Forward and Backward Run to calculate cost function
& its gradient at each optimization iteration

• sensitivity_driver.F (Subroutine: DRIVER)

Function:
-> CTM driver for SENSITIVITY ANALYSIS
-> SENSITIVITY TEST: Run this driver to generate the adjoint trajectory for species under consideration. To change the specie number or the area under consideration, go to "define_receptor.F" file.

CAUTION:
-> Driver designed to perform Sensitivity test with ozone

CHECKPOINT FILES:
CONC_CHK   = Current concentration

Subroutines and functions called:
INITSCEN, ADVSTEP, M3EXIT, WRITE3
STDRIVER_BWD -> performs adjoint model run to create the trajectory

• fd_driver.F (Subroutine: DRIVER)

Function:
-> CTM driver for FINITE DIFFERENCE TESTS
-> FD TEST: Introduce different perturbations and run the adjoint model twice to calculate respective cost funtions f1 and f2 and validate the model with the following eqn:
"Costfunc2-Costfunc1 = LGRID2'.(CGRID2-CGRID1);"

CAUTION:
-> Driver designed to perform FD test with ozone
-> To perturb other species change SPC index of CGRID in Finite-Difference Initialization, line = 327

CHECKPOINT FILES:
->Relevant output Files
CONC_L3CHK = Reference/Base checkpoint file
CONC_CHK   = Current CGRID

->Irrelevant intermediate checkpoint files
CONC_L2CHK = CGRID before starting chemistry
CONC_L4CHK = Checkpoint air density in forward science process

Subroutines and functions called:
SUBDRIVER_FWD -> lay down observation grid
FDDRIVER_BWD -> calculate cost function
6.2 Subdrivers

Called from the main drivers, these subdrivers are responsible for forward and reverse science process runs and checkpoint CGRID and LGRID concentrations.

- **subdriver_fwd.F** (Subroutine: SUBDRIVER_FWD)

  Function:
  Subroutine to perform forward run and lay down the observation grid.

  CHECKPOINT FILES:
  CONC_L3CHK = Reference/Base checkpoint file

  INPUT:
  CGRID

  Subroutines and functions called:
  science processes -> SCIPROC
  write checkpoint files -> WR_L3CHK

- **subdriver_fwpert.F** (Subroutine: SUBDRIVER_FWD)

  Function:
  Subroutine to perform forward run to lay down the best initial guess/background.

  CHECKPOINT FILES:
  CONC_L5CHK = initial guess/background conc checkpoint file

  INPUT:
  CGRID

  Subroutines and functions called:
  science processes -> SCIPROC
  write checkpoint files -> WR_L5CHK

- **subdriver_bwd.F** (Subroutine: SUBDRIVER_BWD)

  Function:
  Subroutine to perform one forward and one backward run to calculate the observation and background parts of cost function and update LGRID.

  CHECKPOINT FILES:
  CONC_L3CHK = Reference/Base checkpoint file
  CONC_CHK = Current concentration CGRID
  CONC_L5CHK = Initial conc checkpoint file for First perturbation

  INPUT:
  CGRID

  OUTPUT:
  f, LGRID
Subroutines and functions called:

- science processes → SCIPROC, SCIPROC_CADJ
- OBS_GRAD_UPDATE,BG_GRAD_UPDATE → calculate cost function & update LGRID
- WR_CHK,RD_CHK,RD_L5CHK → Read and write checkpoint files

- **fddriver_bwd.F** (Subroutine: FDDRIVER_BWD)

  Function: Subroutine to perform one forward and one backward run to calculate the cost function and update LGRID for observation misfit only.

  CHECKPOINT FILES:
  - CONC_L3CHK = Reference/Base checkpoint file
  - CONC_CHK = Current concentration

  INPUT: CGRID

  OUTPUT: f,LGRID

  Subroutines and functions called:
  - SCIPROC, SCIPROC_CADJ → science processes and their adjoint
  - OBS_GRAD_UPDATE → calculate cost function & update LGRID
  - WR_CHK,RD_CHK,RD_L5CHK → Read and write checkpoint files

- **senstdriver_bwd.F** (Subroutine: STDIVER_BWD)

  Function: Subroutine to perform one forward and one backward run to generate adjoint trajectory.

  CHECKPOINT FILES:
  - CONC_CHK = CGRID concentration
  - CONC_L5CHK = LGRID values
  - CONC_L6CHK = EMGRID (emission grid) values

  Subroutines and functions called:
  - science processes → SCIPROC, SCIPROC_CADJ
  - Initializing LGRID → DEFINE_RECEPTOR
  - WR_CHK,RD_CHK,WR_L5CHK → Read and write checkpoint files

6.3 Transport Processes

Subroutines involved in reverse transportation of the adjoint concentrations – horizontal diffusion, vertical diffusion, horizontal advection and vertical advection.

- **hdiff_adj.F** (Subroutine: HDIFF_ADJ)

  Function: Subroutine to perform discrete horizontal diffusion adjoint calculations
INPUT:
LGRID

OUTPUT:
LGRID

Subroutines and functions called:
RHO_J, HCDIFF3D

• vdifffim_adj.F (Subroutine: VDIFF_ADJ)

Function:
-> Discrete adjoint of VDIFF subroutine that comes with CMAQv4.5
-> calculates discrete vertical diffusion adjoint
controlled by flag THETA, using Crank-Nicolson difference scheme
THETA : Crank-Nicolson index [ 1, fully implicit | 0, fully explicit ]

Associated tri-diagonal system is stored in 3 arrays
DI: diagonal
LI: sub-diagonal
UI: super-diagonal
BI: right hand side function
XI: return solution from tridiagonal solver

\[
\begin{bmatrix}
DI(1) & LI(2) & 0 & 0 & \ldots & 0 \\
UI(1) & DI(2) & LI(3) & 0 & \ldots & . \\
0 & UI(2) & DI(3) & LI(4) & 0 & \ldots & . \\
. & . & . & . & \ldots & . \\
. & . & . & . & \ldots & . \\
0 & . & . & . & \ldots & UI(n-1) & DI(n) \\
\end{bmatrix} \quad XI(i) = BI(i)
\]

where \( n = NLAYS \)

• modvdifffim_adj.F (Subroutine: VDIFF_ADJ)

Function:
-> Discrete adjoint of VDIFF subroutine that comes with CMAQv4.5
-> calculates discrete vertical diffusion adjoint
controlled by flag THETA, using Crank-Nicolson difference scheme
THETA : Crank-Nicolson index [ 1, fully implicit | 0, fully explicit ]
-> designed specially for sensitivity tests

Associated tri-diagonal system is stored in 3 arrays
DI: diagonal
LI: sub-diagonal
UI: super-diagonal
BI: right hand side function
XI: return solution from tridiagonal solver

\[
\begin{bmatrix}
DI(1) & LI(2) & 0 & 0 & \ldots & 0 \\
UI(1) & DI(2) & LI(3) & 0 & \ldots & . \\
0 & UI(2) & DI(3) & LI(4) & 0 & \ldots & . \\
. & . & . & . & \ldots & . \\
. & . & . & . & \ldots & . \\
0 & . & . & . & \ldots & UI(n-1) & DI(n) \\
\end{bmatrix} \quad XI(i) = BI(i)
\]
\[
\begin{bmatrix}
. & . & . & . & . \\
0 & UI(n-1) & DI(n) & . & .
\end{bmatrix}
\]

where \( n = NLAYS \)

INPUT:
LGRID, EMGRID

OUTPUT:
LGRID, EMGRID

- **xadvppm_cad.F** (Subroutine: XADV_CAD)

Function:
Advection CONTINUOUS ADJOINT in the horizontal plane; \( x_1 \)-direction:
The process time step is set equal to \( TSTEP(2) \). Boundary concentrations are coupled in RDBCON with \( SqRDMDT = \text{Sq. Root} \left[ \text{det} \left( \text{metric tensor} \right) \right] = \text{Jacobian} / \left( \text{map scale factor} \right)^2 \)
where Air Density \( \times \) \( SqRDMDT \) is loaded into last BCON slot for advection.

INPUT:
CGRID

OUTPUT:
CGRID

- **yadvppm_cad.F** (Subroutine: YADV_CAD)

Function:
Advection CONTINUOUS ADJOINT in the horizontal plane; \( x_2 \)-direction:
The process time step is set equal to \( TSTEP(2) \). Boundary concentrations are coupled in RDBCON with \( SqRDMDT = \text{Sq. Root} \left[ \text{det} \left( \text{metric tensor} \right) \right] = \text{Jacobian} / \left( \text{map scale factor} \right)^2 \)
where Air Density \( \times \) \( SqRDMDT \) is loaded into last BCON slot for advection.

INPUT:
CGRID

OUTPUT:
CGRID

- **zadvppm_cad.F** (Subroutine: ZADV_CAD)

Function:
Advection CONTINUOUS ADJOINT in the vertical, \( x_3 \)-direction:
The process time step is set equal to \( TSTEP \)

INPUT:
CGRID

OUTPUT:
CGRID

6.4 Chemistry
Subroutines responsible for forward and reverse chemistry processes. These files are generated using Kinetic PreProcessor (KPP). (courtesy: Amir Hakami)

- **kppdriver.F** – main driver to perform forward chemistry.
- **kppdriver_adj.F** – main driver to perform backward chemistry process on adjoint variable.
- **kppcalcks.F** - computes thermal and photolytic reaction rate coefficients for each reaction.
- **KPP_Data_mod.F** - mechanism & solver data for EBI solver.
- **KPP_Init.F** - to initialize species tolerances, arrays, and indices.
- **KPP_Util.F** – provides function to copy concentrations from USER to KPP and vice-versa.
- **KPP_HessianSP.F** - Hessian Sparse Data.
- **KPP_JacobianSP.F** - Sparse Jacobian Data.
- **KPP_JacobianTE.F**- provides subroutines: Jac_SP - the Jacobian of Variables in sparse matrix representation, and JacTR_SP_Vec - sparse multiplication: sparse Jacobian transposed times vector.
- **KPP_Function.F** - time derivatives of variables - Aggregate form.
- **KPP_Glob.F** - declaration of global variables.
- **KPP_Pars.F** – model parameter definitions.
- **KPP_Precision.F** - Definition of different levels of accuracy for REAL variables using KIND parameterization.
- **kppModel.F** - Completely defines the model CMAQ_CB4 by using all the associated modules.
- **KPP_HessianTE.F** – provides subroutines: Hessian - function for Hessian (Jac derivative w.r.t. variables) and Hess_Vec - Hessian times user vectors.
- **KPP_LinAlg.F** – Sparse Linear Algebra subroutines.

### 6.5 Checkpoint Files

Subroutines used for reading and writing forward (CGRID) and adjoint (LGRID) concentrations.

  
  **Function:**
  
  Subroutine to perform reading from checkpoint file CONC_*CHK

  **INPUT:**
  
  CGRID

  
  **Function:**
  
  Subroutine to perform writing to the checkpoint file CONC_*CHK

  **INPUT:**
6.6 Others

- **modsciproc.F** (Subroutine: SCIPROC)

  Function:
  Controls all of the physical and chemical processes for a grid
  Operator splitting symmetric around chemistry

  CAUTION:
  This is a modified SCIPROC subroutine with some of the physical processes
  such as ADJADV, PING, CLDPROC and AERO switched off. For the adjoint model
  only chemistry and transport processes are considered with this package.

  INPUT:
  CGRID

  OUTPUT:
  CGRID

  Subroutines and functions called:
  All physical and chemical subroutines,
  DECOUPLE, COUPLE, VDIFF, XADV, YADV, ZADV, HDIFF

- **sciproc_cadj.F** (Subroutine: SCIPROC_CADJ)

  Function:
  Controls all of the physical and chemical adjoint processes for a grid
  Operator splitting symmetric around chemistry

  INPUT:
  LGRID, CGRID

  OUTPUT:
  LGRID

  Subroutines and functions called:
  All physical and chemical subroutines,
  VDIFF_ADJ, XADV_CAD, YADV_CAD, ZADV_CAD, HDIFF_ADJ

- **modsciproc_cadj.F** (Subroutine: SCIPROC_CADJ)

  Function:
  - Controls all of the physical and chemical adjoint processes for a grid
  - Operator splitting symmetric around chemistry
  - Designed for use with sensitivity analysis

  INPUT:
  LGRID, EMGRID

  OUTPUT:
  LGRID
Subroutines and functions called:
   All physical and chemical subroutines,
   VDIFF_ADJ, XADV_CAD, YADV_CAD, ZADV_CAD, HDIFF_ADJ

- **calc_bggrad.F** *(Subroutine: BG_GRAD_UPDATE)*

  Function:
  Calculates the background cost-function value and updates
  adjoint variable for initial time

  CAUTION:
  ->This subroutine is being constructed for current test problem.
  ->One needs to modify the formulas according to his/her needs.

  INPUT:
  CGRID - Current concentration field (read from CONC_CHK)
  LGRID - Adjoint variable to be updated
  CF    - Cost-Function update variable
  JDATE,JTIME - Current date and time step values

  DATA READ FROM FILES:
  CBGRID – background (perturbed) conc from file: CONC_L5CHK

  OUTPUT:
  LGRID,CF

- **calc_obsgrad.F** *(Subroutine: OBS_GRAD_UPDATE)*

  Function:
  Calculates the observation cost-function update value and updates
  adjoint variable for the current dynamic time-step

  CAUTION:
  This subroutine uses model-observations to calculate cost function
  and its gradient. One needs to modify the formulaes and I/O procedures
  to incorporate real-observations.

  INPUT:
  CGRID - Current concentration field
  LGRID - Adjoint variable to be updated
  CF    - Cost-Function update variable
  JDATE,JTIME - Current date and time step values

  DATA READ FROM FILES:
  CHKGRID - observed(reference) conc from file: CONC_L3CHK

  OUTPUT:
  LGRID,CF

- **tridiag_adj.F** *(Subroutine: ADTRIDIAG)*

  FUNCTION:
  -> Discrete adjoint of TRIDIAG subroutine that comes with CMAQv4.5
  -> Solves tridiagonal system by Thomas algorithm. Algorithm fails
(M3ERR) if first pivot is zero. In that case, rewrite the equation as a set of order KMAX-1, with X(2) trivially eliminated.

Associated tri-diagonal system is stored in 3 arrays
D: diagonal
L: sub-diagonal
U: super-diagonal
B: right hand side function
X: return solution from tridiagonal solver

\[
\begin{bmatrix}
  D(1) & L(2) & 0 & 0 & \cdots & 0 \\
  U(1) & D(2) & L(3) & 0 & \cdots & . \\
  0 & U(2) & D(3) & L(4) & 0 & \cdots \\
  . & . & . & . & . & . \\
  . & . & . & . & L(n) & . \\
  0 & . & . & . & D(n) & \\
\end{bmatrix}
\]

\[X(i) = B(i)\]

where \(n = \text{NLAYS}\)

**routines.f** (Subroutine: setulb)

Function:
This subroutine partitions the working arrays wa and iwa, and then uses the limited memory BFGS method to solve the bound constrained optimization problem by calling mainlb. (The direct method will be used in the subspace minimization.)

**INPUT:**
\(x,f,g\)

**OUTPUT:**
\(x\)