Automatic Generation of Multi-Core Accelerated Chemical Kinetics for Simulation and Prediction

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Abstract—This work presents KPPA (the Kinetics PreProcessor: Accelerated), a general analysis and code generation tool that achieves significantly reduced time-to-solution for chemical kinetics kernels on three multi-core platforms: NVIDIA GPUs using CUDA, the Cell Broadband Engine, and Intel Quad-Core Xeon CPUs. A comparative performance analysis of three chemical kernels is presented for each platform in double and single precision on coarse and fine grids. We identify the three architectural parameters KPPA requires to generate a chemical kernel for these platforms and describe a code generation system that produces highly-tuned platform-specific code. Compared to state-of-the-art serial implementations, speedups exceeding $25 \times$ are regularly observed, with a maximum observed speedup of $41.1 \times$ in single precision.

Index Terms—KPPA, Multi-core, NVIDIA CUDA, Cell Broadband Engine, OpenMP, Chemical Kinetics, Atmospheric Modeling, Kinetic PreProcessor

1 INTRODUCTION

Chemical kinetics models trace the evolution of chemical species over time by solving large numbers of partial differential equations. The Weather Research and Forecast with Chemistry model (WRF-Chem) [1], the Community Multiscale Air Quality Model (CMAQ) [2], the Sulfur Transport and dEposition Model (STEM) [3], and GEOS-Chem [4] approximate the chemical state of the Earth’s atmosphere by applying a chemical kinetics model over a regular grid. Computational time is dominated by the solution of the coupled equations arising from the chemical reactions, which may involve millions of variables [5]. The stiffness of these equations, arising from the widely varying reaction rates, prohibits their solution through explicit numerical methods.

These models are embarrassingly parallel on a fixed grid since changes in concentration of species $y_i$ at any grid point depend only on concentrations and meteorology at the same grid point. Yet chemical kinetics models may be responsible for over 90% of an atmospheric model’s computational time. For example, a RADM2 kinetics mechanism combined with the SORGAM aerosol scheme (RADM2SORG chemistry kinetics option in WRF-Chem) involves 61 species in a network of 156 reactions. On a typically-sized $40 \times 40$ grid with 20 horizontal layers, the meteorological part of the simulation (the WRF weather model itself) is only $160 \times 10^6$ floating point operations per time step, about 2.5% the cost of the full WRF-Chem with both chemical kinetics and aerosols. As a second example, Figure 1 shows the performance of serial and parallel runs of the global tropospheric model GEOS-Chem. The cost of chemical kinetics dominates in both runs, even as the number of threads increases. A strong-scaling approach is needed to improve performance.

The new generations of multi-core processors mass produced for commercial IT and “graphical computing” (i.e. video games) achieve high rates of performance for highly-parallel applications, such as atmospheric models which contain abundant coarse- and fine-grained parallelism. Successful use of these novel architectures as accelerators on each node of large-scale conventional compute clusters will enable not only larger, more complex simulations, but also reduce the time-to-solution for a range of earth system applications.

Writing chemical kinetics code is often tedious and error-prone work, even for conventional scalar architectures. Emerging multi-core architectures, particularly
heterogeneous emerging architectures such as the Cell Broadband Engine Architecture (CBEA) and General Purpose Graphics Processing Units (GPGPUS), are much harder to program than their scalar predecessors. For these architectures, a deep understanding of the problem domain is required to achieve good performance. General analysis tools like the Kinetic PreProcessor (KPP) [5] make it possible to rapidly generate correct and efficient chemical kinetics code on scalar architectures, but these generated codes cannot be easily ported to strong-scaling emerging architectures.

This work presents KPPA (the Kinetics PreProcessor: Accelerated), a general analysis and code generation tool that achieves significantly reduced time-to-solution for chemical kinetics kernels. KPPA facilitates the numerical solution of chemical reaction network problems and generates code targeting OpenMP, NVIDIA GPUs with CUDA, and the CBEA, in C and Fortran, and in double and single precision. KPPA-generated mechanisms leverage platform-specific multi-layered heterogeneous parallelism to achieve strong scalability. Compared to state-of-the-art serial implementations, speedups as high as 41.1× are observed.

A brief formulation of a chemical kinetics model is given in Section 3. A typical homogeneous multi-core chipset, NVIDIA GPUs, and the CBEA are outlined in Section 4. Preliminary work with the RADM2 kinetics kernel on three multi-core platforms provide generalizations and best practices for multi-core kinetics codes in Section 5. Multi-core code generation is described in Section 6. Performance results from three KPPA-generated codes are given in Section 7, followed by conclusions and future work in Section 8. Related work is discussed in Section 9.

2 OVERVIEW OF KPPA

KPPA (Figure 2) combines a general analysis tool for chemical kinetics with a code generation system for scalar, homogeneous multi-core, and heterogeneous multi-core architectures. It is written in object-oriented C++ with a clearly-defined upgrade path to support future multi-core architectures as they emerge. KPPA has all the functionality of KPP 2.1, the latest version of KPP, and generates mechanism code in C, Fortran, or other platform-specific languages, such as CUDA.

KPPA’s input files and lexical parser are enhanced versions of the same components from KPP. Several new keywords describing the target architecture, loop unrolling parameters, and other new features have been added to the parser, but all original KPP functionality is retained.

The general analysis component was rewritten from scratch but borrows heavily from KPP. It is used to formulate the chemical system as described in Section 3. Many atmospheric models, including WRF-Chem and STEM, support a number of chemical kinetics solvers that are automatically generated at compile time by KPP.

![Fig. 2. Principle KPPA components and its program flow.](image)

Reusing these analysis techniques in KPPA insures its accuracy and applicability. KPPA is backwards-compatible with KPP and can be used as a drop-in replacement in many situations.

The code generation component is written from scratch to accommodate the two-dimensional design space of programming language / target architecture combinations (Table 1). Given the model description from the analytical component and a description of the target architecture, the code generation component produces a time-stepping integrator, the ODE function and ODE Jacobian of the system, and other quantities required to interface with an atmospheric model. It can generate code in several languages, and can be extended to new target languages as desired. It’s key feature is the ability to generate fully-unrolled, platform-specific sparse matrix/matrix and matrix/vector operations which achieve very high levels of efficiency. Code generation is described in Section 6.

3 ATMOSPHERIC CHEMICAL KINETICS

Regardless of computational architecture or modeling application, the solution to the chemical reaction network problem is calculated in the same way. A time-stepping method advances the system of coupled and stiff ODEs through time.

3.1 Forming the Chemical System

Given a reaction network and initial concentrations as input files, KPPA generates code to solve the differential equation of mass action kinetics to determine the chemical system. The chemical system is defined as:

\[ \frac{\text{d}c_i}{\text{d}t} = r_i - \sum_{j \neq i} r_j + \sum_{j \neq i} \text{O}_{ij}c_j \]

1. Lambert defines stiffness in [6]: “If a numerical method is forced to use, in a certain interval of integration, a step length which is excessively small in relation to the smoothness of the exact solution in that interval, then the problem is said to be stiff in that interval.”
concentrations at any future time. The derivation of this equation is given at length in [5] and summarized here. Consider a system of \( n \) chemical species with \( R \) chemical reactions, \( r = [r_1, \ldots, r_R]^T \). Let \( y \) be the vector of concentrations of all species involved in the chemical mechanism, \( y = [y_1, \ldots, y_n]_T \). The concentration of species \( i \) is denoted by \( y_i \). We define \( k_j \in k = [k_1, \ldots, k_R]_T \) to be the rate coefficient of reaction \( r_j \).

The stoichiometric coefficients \( s_{i,j} \) are defined as follows. \( s_{i,j}^- \) is the number of molecules of species \( y_i \) that react (are consumed) in reaction \( r_j \). Similarly, \( s_{i,j}^+ \) is the number of molecules of species \( y_i \) that are produced in reaction \( r_j \). If \( y_i \) is not involved in reaction \( r_j \) then \( s_{i,j}^- = s_{i,j}^+ = 0 \).

The principle of mass action kinetics states that each chemical reaction progresses at a rate proportional to the concentration of the reactants. Thus, the \( j \)th reaction in the model is stated as

\[
(r_j) \sum s_{i,j}^- y_i \xrightarrow{k_j} \sum s_{i,j}^+ y_i, \quad 1 \leq j \leq R,
\]

where \( k_j \) is the proportionality constant. In general, the rate coefficients are time dependent: \( k_j = k_j(t) \).

The reaction velocity (the number of molecules per time unit by

\[
\omega_j(t, y) = k_j(t) \prod_{i=1}^n y_i^{s_{i,j}^-}.
\]

\( y_i \) changes at a rate given by the cumulative effect of all chemical reactions:

\[
\frac{d}{dt} y_i = \sum_{j=1}^R (s_{i,j}^- - s_{i,j}^+) \omega_j(t, y), \quad i = 1, \ldots, n
\]

If we organize the stoichiometric coefficients in two matrices,

\[
S^- = (s_{i,j}^-)_{1 \leq i \leq n, 1 \leq j \leq R}, \quad S^+ = (s_{i,j}^+)_{1 \leq i \leq n, 1 \leq j \leq R},
\]

then Equation 3 can be rewritten as

\[
\frac{d}{dt} y = (S^+ - S^-) \omega(t, y) = S \omega(t, y) = f(t, y),
\]

where \( S = S^+ - S^- \) and \( \omega(t, y) = [\omega_1, \ldots, \omega_R]^T \) is the vector of all chemical reaction velocities.

Equation 4 gives the time derivative function in aggregate form. Depending on the integration method, other forms, such as a split production-destruction form may be preferred. KPPA can produce both aggregate and production-destruction forms. Implicit integration methods also require the evaluation of the Jacobian of the derivative function:

\[
J(t, y) = \frac{\partial}{\partial y} f(t, y) = \begin{bmatrix}
\frac{\partial f_1}{\partial y_1} & \frac{\partial f_1}{\partial y_2} & \cdots & \frac{\partial f_1}{\partial y_n} \\
\frac{\partial f_2}{\partial y_1} & \frac{\partial f_2}{\partial y_2} & \cdots & \frac{\partial f_2}{\partial y_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_R}{\partial y_1} & \frac{\partial f_R}{\partial y_2} & \cdots & \frac{\partial f_R}{\partial y_n}
\end{bmatrix}
\]

Initialize \( k(t, y) \) from starting concentrations and meteorology \((p, t, q, p)\)

Initialize time variables \( t \leftarrow t_{\text{start}}, h \leftarrow 0.1 \times (t_{\text{end}} - t_{\text{start}}) \)

While \( t \leq t_{\text{end}} \)

\[
Fcn_0 \leftarrow f(t, y) \quad \text{Jac}_0 \leftarrow J(t, y)
\]

For \( s \leftarrow 1,2,3 \)

Compute \( \text{Stage}_s \) from \( Fcn \) and \( \text{Stage}_{s-1}(\delta) \)

Solve for \( \text{Stage}_s \) implicitly using \( G \)

Update \( k(t, y) \) with meteorology \((p, t, q, p)\)

Update \( Fcn \) from \( \text{Stage}_{s-1} \)

Compute \( Y_{\text{new}} \) from \( \text{Stage}_{s-1} \)

Compute error term \( E \)

If \( E > \delta \) then discard iteration, reduce \( h \), restart

Otherwise, \( t \leftarrow t + h \) and proceed to next step

Finish: Result in \( Y_{\text{new}} \)

Fig. 3. A general outline of the three-stage Rosenbrock solver for chemical kinetics. \( t \) is the system time, \( h \) is the small time step, \( \text{Stage}_s \) is the result of Rosenbrock stage \( s \), \( \delta \) is an error threshold, \( k(t, y) \), \( f(t, y) \), and \( J(t, y) \) are as given in Section 3.1. \( Y_{\text{new}} \) is the new concentration vector.

### 3.2 Solving the Chemical System

The solution of the ordinary differential equations is advanced in time by a numerical integration method. We focus on the Rosenbrock integrator with three implicitly-solved Newton stages as an example, but KPPA also supports Runge-Kutta methods [7]. The Rosenbrock implementation takes advantage of sparsity as well as trading exactness for efficiency when reasonable. An outline of the implementation is shown in Figure 3.

If the system is autonomous, the reaction rates do not depend on the time variable \( t \) and may be computed only once. Otherwise they must be recomputed during the integration stages as in Figure 3. Autonomicity is largely irrelevant to the implementation of both serial and multi-core solver implementations since the reaction rates are calculated from data external to the integrator. Multi-core architectures with limited per-thread memory (such as the CBEA) can take advantage of this independence by overlaying the integration routine with the reaction rate updates. In our experience however, there is sufficient per-thread memory in any architecture to make this optimization unnecessary.

A chemical mechanism may be so stiff that it will not converge without double precision floating point computation. One example is the SAPRC mechanism [8]. KPPA can generate code in either double or single precision. The choice of target architecture may have a profound impact on performance if double precision is required.
Once an iterative solver has been generated, an atmospheric model applies the solver to every point on a fixed domain grid. Chemical kinetics are embarrassingly parallel between cells, so there is abundant data parallelism (DLP). Generally, within the solver itself, the ODE system is coupled so that, while there is still some data parallelism available in lower-level linear algebra operations, parallelization is limited largely to the instruction level (ILP). Some specific chemical mechanisms are only partially-coupled and can be separated into a small number of sub-components, but such inter-module decomposition is rare under the numerical methods examined in this work. Thus, a three-tier parallelization is generally possible: ILP on each core, DLP using single-instruction-multiple-data (SIMD) features of a single core, and DLP across multiple cores (using multi-threading) or nodes (using MPI). The coarsest tier of MPI and OpenMP parallelism is supplied by the atmospheric model.

4 Multi-core Architectures

Once the general analysis component has formulated the chemical system as in Section 3, the code generation component constructs the architecture- and language-specific implementation of the numerical integration method. KPPA can generate code for homogeneous multi-core chipsets, NVIDIA GPUs with CUDA, and the CBEA. Since power consumption and thermal issues have become the principle limitation to computing power, both the peak gigaflops and the gigaflops per dissipated watt should be considered when investing in a new technology. This section reviews these architectures. Figure 4 shows the gigaflop and gigaflop/watt performance of each architecture.

4.1 Homogeneous Multi-core Chipsets

Homogeneous multi-core design has supplanted single-core design in commercial servers, workstations, and laptops. The Intel Xeon 5400 Series [9] is typical of this design. A quad-core chip at 3GHz has a theoretical peak floating point performance of 48 gigaflops with nominal 90W dissipation. It achieves 40.5 gigaflops (0.5 gigaflops/watt) in the LINPACK benchmark [10]. We include it in this study as an example of the current industry standard and the baseline performance metric.

4.2 GPGPUs and NVIDIA CUDA

Graphics Processing Units (GPUs) are low-cost, massively-parallel homogeneous microprocessors designed for visualization and gaming. Because of their power, these special-purpose chips are being used for non-graphics “general-purpose” applications, hence the term GPGPU. The NVIDIA Tesla C1060 (Figure 5) has 4GB of GDDR3 device memory and 240 1.2GHz processing units on 30 multiprocessors. Each multi-processor has 16KB of fast shared memory and a 16K register file. The C1060’s theoretical peak performance is 933 single precision gigaflops (3.95 gigaflops/watt) or 76 double precision gigaflops (0.38 gigaflops/watt) [11]. GPU performance is often an order of magnitude above that of comparable CPUs, and GPU performance has been increasing at a rate of 2.5x to 3.0x annually, compared with 1.4x for CPUs [12]. GPU technology has the additional advantage of being widely-deployed in modern computing systems. Many desktop workstations have GPUs which can be harnessed for scientific computing at no additional cost. Recent versions of NVIDIA GPUs incorporate a hardware double precision floating point unit in addition to eight SIMD streaming processors on each multi-processor. Since double precision operations must be pipelined through this unit instead of executing on the streaming processors, the GPU has a penalty for double precision beyond just the doubling of data volumes. In practice this is highly application specific. Applications that perform well on the GPU do so by structuring data and computation to exploit registers and shared memory and have large numbers of threads to hide device memory latency.

NVIDIA GPUs are programmed in CUDA [13]. Since CUDA’s release in 2007, hundreds of scalable parallel programs for a wide range of applications, including computational chemistry, sparse matrix solvers, sorting, searching, and physics models have been developed [14].
Many of these applications scale transparently to hundreds of processor cores and thousands of concurrent threads. The CUDA model is also applicable to other shared-memory parallel processing architectures, including multi-core CPUs [15].

4.3 The Cell Broadband Engine Architecture (CBEA)

The CBEA describes a heterogeneous multi-core processor that has drawn considerable attention in both industry and academia (Figure 6). It consists of a multi-threaded Power Processing element (PPE) and eight Synergistic Processing elements (SPEs) [16]. These elements are connected with an on-chip Element Interconnect Bus (EIB) with a peak bandwidth of 204.8 gigabytes/second. The PPE is a 64-bit dual-thread PowerPC processor. Each SPE is a 128-bit SIMD processor with two major components: a Synergistic Processor Unit (SPU) and a Memory Flow Controller (MFC). All SPE instructions are executed on the SPU. The SPE includes 128 registers of 128 bits and 256 KB of software-controlled local storage.

The MFC’s DMA commands are subject to size and alignment restrictions. Data transferred between SPE local storage and main memory must be 8-byte aligned, at most 16 KB large, and in blocks of 1, 2, 4, 8, or multiples of 16 bytes. Multiples of 128 bytes are most efficient. Incontiguous (i.e. strided) data cannot be transferred with a single command and must be handled by DMA lists. A DMA list is a list of commands, each specifying a starting address and size. It is formed in local storage by the SPU and passed to the MFC.

The Cell Broadband Engine (Cell BE) is the game box implementation of the CBEA and may have either six or eight SPEs. It is primarily a single precision floating point processor with a peak single precision FP performance of 230.4 gigaflops (2.45 gigaflops/watt) and a double precision peak of only 21.03 gigaflops (0.22 gigaflops/watt) [17]. The PowerXCell 8i processor is the latest implementation of the CBEA intended for high-performance, double precision floating point intensive workloads that benefit from large-capacity main memory. It has nominal dissipation of 92W and a double precision theoretical peak performance of 115.2 gigaflops (1.25 gigaflops/watt) [18]. Roadrunner at Los Alamos, the first computer to achieve a sustained petaflops, uses the PowerXCell 8i processor [19], and the top seven systems on the November 2008 Green 500 list use the PowerXCell 8i [20].

4.4 Benchmark Systems

The benchmarks presented in this work assume a 200W power budget. Within this envelope, two Quad-Core Xeon chips, two CBEA chips, or one Tesla C1060 GPU can be allocated. Quad-Core Xeon benchmarks are performed on a Dell Precision T5400 workstation with two Intel E5410 CPUs and 16GB of memory on a 667MHz bus. NVIDIA GPU benchmarks are performed on the NCSA’s experimental GPU cluster. Each cluster node has two dual-core 2.4 GHz AMD Opteron CPUs and 8 GB of memory (only one Opteron is used in this study). CBEA benchmarks are performed on an in-house PlayStation 3 system, an IBM BladeCenter QS22 at Forschungszentrum Jülich, and an IBM BladeCenter QS20 at Georgia Tech. The PlayStation 3 and the QS20 use the Cell Broadband Engine and lack hardware support for pipelined double precision arithmetic. The QS22 uses the PowerXCell 8i and includes hardware support for pipelined double precision arithmetic. The PlayStation 3 has 256MB XDRAM,
the QS20 has 1GB XDRAM, and the QS22 has 8GB XDRAM. Both the QS20 and the QS22 are configured as “glueless” dual processors; two CBEA chipsets are connected through their FlexIO interfaces to appear as a single chip with 16 SPEs and 2 PPEs. The PS3 has only six SPEs available for yield reasons.

5 Multi-core Chemical Kinetics

Serial implementations of chemical kinetics mechanisms have been studied for decades. However, the uniqueness of the architectures described in Section 4 makes it difficult to anticipate implementation details of a general mechanism, that is, it was unknown what a highly-optimized multi-core implementation of chemical kinetics “looked like”. In order to develop KPP-A code generation functionality, a detailed exploration of a specific mechanism on every target architecture was therefore required.

5.1 RADM2 on Multi-Core

We begin our investigation by hand-porting and benchmarking the RADM2 chemical kernel from WRF-Chem on three multi-core platforms. RADM2 was developed by Stockwell et al. [21] for the Regional Acid Deposition Model version 2 [22]. It is widely used in atmospheric models to predict concentrations of oxidants and other air pollutants. The RADM2 kinetics mechanism combined with the SORGAM aerosol scheme involves 61 species in a network of 156 reactions. It treats inorganic species, stable species, reactive intermediates and abundant stable species (O₂, N, H₂O). Atmospheric organic chemistry is represented by 26 stable species and 16 peroxy radicals. Organic chemistry is represented through a reactivity aggregated molecular approach [23]. Similar organic compounds are grouped together into a limited number of model groups (HC₃, HC₅, and HC₇) through reactivity weighting. The aggregation factors for the most emitted VOCs are given in [23].

The KPP-generated RADM2 mechanism uses a three-stage Rosenbrock integrator. Four working copies of the concentration vector (three stages and output), an error vector, the ODE function value, the Jacobian function value, and the LU decomposition of \( \frac{1}{h^2} - J_{\text{ODE}} \) are needed for each grid cell. This totals at least 1,890 floating point values per grid cell, or approximately 15KB of double precision data. While porting to each platform, care was taken to avoid any design which was specific to RADM2. Thus, our hand-tuned multi-core RADM2 implementations form templates the KPP-A code generator can reuse when targeting these architectures.

Input data to the RADM2 solver was written to files from WRF-Chem using two test cases: a coarse grid of \( 40 \times 40 \) with 20 layers and a 240 second timestep, and a fine grid of \( 134 \times 110 \) with 35 layers and a 90 second timestep. The unmodified Fortran source files for the RADM2 chemical kinetics solver (generated by KPP during WRF-Chem compilation), along with a number of KPP-generated tables of indices and coefficients used by the solver, were isolated into a standalone program that reads in the input files and invokes the solver. The timings and output from the original solver running under the standalone driver for one time step comprised the baseline performance benchmark.

For the accelerated architectures, it was necessary to translate the serial Fortran code to C. CUDA is an extension of C and C++, and although two Fortran compilers exist for the CBEA (gfortran 4.1.1 and IBM XL Fortran 11.1), these compilers have known issues that make fine-tuning easier in C. Once the C model was developed, Fortran code for the CBEA could be generated automatically following the same design.

Table 2 shows the baseline serial performance in seconds based on ten runs of the benchmark on a single core of an Intel Quad-Core Xeon 5400 series. “Rosenbrock” indicates the inclusive time required to advance chemical kinetics one time step for all points in the domain. It corresponds to the process described in Figure 3 and includes “LU Decomp.”, “LU Solve”, “ODE Function”, and “ODE Jacobian”, which are also reported. “LU Decomp.” and “LU Solve” are used to solve a linear system within the Rosenbrock integrator. “ODE Function” and “ODE Jacobian” are the time spent computing the mechanism’s ODE function \( f(t, y) \), and Jacobian function \( J(t, y) \), respectively.

Table 3 shows the performance in seconds of the RADM2 multi-core ports. The labels are identical to those in Table 2. Several operations in the fine-grid double precision case took so long on the PlayStation 3 that the SPU hardware timer overflowed before they could complete. Accurate timings cannot be supplied in this case. Only the overall solver time was available for the “Tesla (a)” implementation because the entire solver is a single CUDA kernel. The benchmark results are discussed in Section 5.2.

5.1.1 Intel Quad-Core Xeon with OpenMP

Since the chemistry at each WRF-Chem grid cell is independent, the outermost iteration over cells in the RADM2 kernel became the thread-parallel dimension; that is, a one-cell-per-thread decomposition. The Quad-Core Xeon port implements this with OpenMP. Attention had to be given to all data references, since the Rosenbrock integrator operates on pointers to global data.
structures. In some cases, simply declaring variables in the private or shared blocks of the parallel constructs did not prevent unwanted data sharing between threads. In these cases, data was copied from global structures to threadprivate variables.

5.1.2 NVIDIA CUDA

The CUDA implementation takes advantage of the very high degree of parallelism and independence between cells in the domain, using a straightforward cell-per-thread decomposition. The first CUDA version (“a” in Table 3) implemented the entire Rosenbrock mechanism (Figure 3) as a single kernel. This presented some difficulties and performance was disappointing (Table 3). The amount of storage per grid cell precluded using the fast but small (16kB per multi-processor) shared memory to speed up the computation. On the other hand, the Tesla GPU has 384K registers which can be used to good effect, since KPP generated fully unrolled loops as thousands of assignment statements. The resulting CUDA-compiled code could use upwards of one hundred registers per thread, though this severely limited the number of threads that could be actively running, even for large parts of the Rosenbrock code that could use many more.

The second CUDA implementation addressed this by moving the highest levels of the Rosenbrock solver back onto the CPU. The lower levels of the Rosenbrock call tree were coded and invoked as separate kernels on the GPU. This design was also easier to debug and benchmark since the GPU code was spread over many smaller kernels with control returning frequently to the CPU, and it compiled considerably faster. Most importantly, it limited the impact of resource bottlenecks to only those affected kernels. Performance critical parameters such as the size of thread blocks and shared-memory allocation were tuned kernel-by-kernel without subjecting the entire solver to worst-case limits.

The principle disadvantage of moving time and error control logic to the CPU is that all cells are forced to use the same minimum time step and iterate the maximum number of times, even though only a few cells required that many to converge. For the benchmark workloads, 90 percent of the cells converge in 30 iterations or fewer. The last dozen or so cells required double that number. While faster by a factor of 3 to 4 on a per-iteration basis, the increase in wasted work limited performance improvement to less than a factor of two. On the Tesla, the improvement was only 9.5 seconds down to about 5 seconds for the new kernel. The “(b)” results in Table 3 were for the multi-kernel version of the solver, but with an additional refinement: time, step-length, and error were stored separately for each cell and vector masks were used to turn off cells that were converged. The solver still performed the maximum number of iterations; however, beyond the half-way mark, most thread-blocks did little or no work and relinquished the GPU cores very quickly, resulting in a reduced wall-clock time.

5.1.3 Cell Broadband Engine Architecture

The heterogeneous Cell Broadband Engine Architecture forces a carefully-architected approach. The PPU is capable of general computation on both scalar and vector types, but the SPUs diverge significantly from general processor design [16]. Recognizing this, we chose a master-worker approach for the CBEA port. The PPU, with full access to main memory, is the master. It prepares grid data for the SPUs which process them and

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**TABLE 3**

RADM2 timings (seconds) of the multi-core kernels in a 200 watt envelope. Each time shown is the minimum over several successive runs. Category labels are explained in detail at the end of Section 5.1. For a socket-to-socket comparison, double the time of all systems except Tesla C1060.

<table>
<thead>
<tr>
<th></th>
<th>Single</th>
<th>Double</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>16 SPEs</td>
<td>16 SPEs</td>
<td>6 SPEs</td>
</tr>
<tr>
<td>LU Decomp.</td>
<td>1.645</td>
<td>1.065</td>
<td>2.633</td>
</tr>
<tr>
<td>LU Solve</td>
<td>0.199</td>
<td>0.198</td>
<td>1.597</td>
</tr>
<tr>
<td>ODE Fun.</td>
<td>0.040</td>
<td>0.040</td>
<td>0.496</td>
</tr>
<tr>
<td>ODE Jac.</td>
<td>0.036</td>
<td>0.036</td>
<td>0.409</td>
</tr>
</tbody>
</table>

**Fine** | **Coarse** | **Fine** | **Coarse** | **Fine** | **Coarse** | **Coarse**

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*a.* The poor fine grid PlayStation 3 performance due to the grid data (380MB) being larger than main memory.  
*b.* Timing detail was not available with single kernel (first implementation) RADM2 on GPU.
returns them to the PPU. In order to comply with size and alignment restrictions (see Section 4.3), the PPU buffers the incontiguous WRF-Chem data into an aligned and padded array so that it can be safely accessed by the MFC.

The SPU’s floating point SIMD ISA operates on 128-bit vectors of either four single precision or two double precision floating point numbers. To take advantage of SIMD, the PPU interleaves the data of four (or two) grid points into an array of 128-bit vectors, which is padded, aligned, buffered and transferred exactly as in the scalar case. This achieves a four-cells-per-thread (two-cells-per-thread in double precision) decomposition. Only one design change in the Rosenbrock integrator was necessary to integrate a vector cell. As shown in Figure 3, the integrator iteratively refines the Newton step size \( h \) until the error norm is within acceptable limits. This will cause an intra-vector divergence if different vector elements accept different step sizes. However, it is numerically sound to continue to reduce \( h \) even after an acceptable step size is found. The SIMD integrator reduces the step size until the error for every vector element is within tolerance. Conventional architectures would require additional computation under this scheme, but because all operations in the SPU are SIMD this actually recovers lost flops. This enhancement doubled (quadrupled for single precision) the SPU’s throughput with no measurable overhead on the SPU.

5.2 RADM2 Performance Analysis and Discussion

Table 3 shows the performance of all the parallel implementations. The benchmark systems are described in Section 4.4. Of the three platforms investigated, Quad-Core Xeon with OpenMP was by far the easiest to program. A single address space and single ISA meant only one copy of the integrator source was necessary. Also, OpenMP tool chains are more mature than those for GPUs or the CBEA. As shown in Figure 7, this port achieved nearly linear speedup over eight cores. However, it may be unreasonable to expect this trend to continue for similar architectures with hundreds of cores. Current interconnect designs cannot provide the necessary memory bandwidth in this paradigm.

The CBEA implementation achieves the best performance. On a fine-grain double precision grid, two PowerXCell 8i chipsets are 11.5 \( \times \) faster than the serial implementation, and 32.9 \( \times \) faster in single precision. Coarse grained single precision grids see a speedup of 28.0 \( \times \). The CBEA’s explicitly-managed memory hierarchy and fast on-chip memory provide this performance. Up to 40 grid cells can be stored in SPE local storage, so the SPU never waits for data. Because the memory is explicitly managed, data can be intelligently and asynchronously prefetched. However, the CBEA port was difficult to implement. Two optimized copies of the solver code, one for the PPU and one for the SPU, were required. On-chip memory must be explicitly managed and careful consideration of alignment and padding are the programmer’s responsibility.

The NVIDIA CUDA implementation was straightforward to program, however it proved to be the most difficult to optimize. CUDA’s automatic thread management and familiar programming environment improve programmer productivity: our first implementation of RADM2 on GPU was simple to conceive and implement. However, a deep understanding of the underlying architecture is still required in order to achieve good performance. For example, memory access coalescing is one of the most powerful features of the GPU architecture, yet CUDA neither hinders nor promotes program designs that leverage coalescing. In our case, the GPU required the most effort to achieve acceptable performance. On a single-precision coarse grid, this im-
plementation achieves an 8.5× speedup over the serial implementation. The principal limitation is the size of the on-chip shared memory and register file, which prevent large-footprint applications from running sufficient numbers of threads to expose parallelism and hide latency to the device memory. The entire concentration vector must be available to the processing cores, but there is not enough on-chip storage to achieve high levels of reuse, so the solver is forced to fetch from the slow GPU device memory. Because the ODE system is coupled, a per-species decomposition is generally impossible. However, for certain cases, it may be possible to decompose by species groups. This will reduce pressure on the shared memory and boost performance. The alternative is to wait until larger on-chip shared memories are available.

6 Multi-Core Code Generation

Having demonstrated the feasibility and value of multi-core implementations of one chemical mechanism, we proceeded to generalize our hand-tuned RADM2 kernels into KPP’s code generation module. Initially we sought to extend KPP’s code generation routines to support multi-core architectures, however KPP’s design does not facilitate such an approach. KPP is a highly-tuned procedural C code that uses a complex combination of function pointers and conditional statements to translate chemical kinetics concepts into a specific language. While efficient, this design is not extensible. Adding new features to KPP requires careful examination of many hundreds of lines of complex source code. Targeting multiple architectures in multiple languages creates a two-dimensional design space (Table 1), increasing the complexity of the KPP source code to unmaintainable levels. Furthermore, the design space will only continue to grow as future architectures are developed.

KPP’s code generation routines follow object-oriented design principles to enable extensibility in multiple design dimensions. Abstract base classes for Language objects and Architecture objects define the interfaces a Model object uses to produce an optimized chemical code. The Bridge Pattern [24] connects an Architecture to a Language, facilitating future architecture or language implementations. Adding a new language or architecture is as simple as inheriting the abstract class and implementing the appropriate member functions, however some special case functions may be required for specific language/architecture pairs.

6.1 Language-specific Code Generation

KPPA generates code in two ways: complete function generation using lexical trees, and template file specification. Complete function generation builds a language-independent expression tree describing a sparse matrix/matrix or matrix/vector operation. For example, the aggregate ODE function of the mechanism is calculated by multiplying the left-side stoichiometric matrix by the concentration vector, and then adding the result to the elements of the stoichiometric matrix. KPPA performs these operations symbolically at code generation time, using the matrix formed by the analytical component and a symbolic vector, which will be calculated at run-time. The result is an expression tree of language-independent arithmetic operations and assignments, equivalent to a rolled-loop sparse matrix/vector operation. The language-independent lexical tree is translated to a specific language by an instance of the abstract Language class. A concrete Language object specifies how assignments, arithmetic operations, and type casts are performed in a given language.

KPPA uses its knowledge of the target architecture to generate highly-efficient function code. Language-specific vector types are preferred when available, branches are avoided on all architectures, and parts of the function can be rolled into a tight loop if KPPA determines that on-chip memory is a premium. An analysis of three KPPA-generated ODE functions and ODE Jacobians targeting the CBEA showed that, on average, both SPU pipelines remain full for over 80% of the function implementation. Pipeline stalls account for less than 1% of the cycles required to calculate the function. For example, in the SAPRCNOV mechanism on CBEA, there are only 20 stalls in the 2989 cycles required by the ODE function (0.66%), and only 24 stalls in the 5490 cycles required for the ODE Jacobian (0.43%). Clever use of the volatile keyword reduces the number of cycles to 2778 and eliminates stalls entirely.

Figure 8 shows a visual representation of the SPU pipelines during the SAPRCNOV ODE function execution as given by the IBM Assembly Visualizer for CBEA. No stalls are observed for all 2778 cycles. The pattern repeats with minor variations for 2116 of 2778 cycles.
smooth chevrons of 'X's are formed, just as shown. Code of this caliber often requires meticulous hand-optimization, but KPPA is able to generate this code automatically in seconds.

When template file specification is used, source code templates written in the desired language are copied from a library and then “filled in” with code appropriate to the chemical mechanism being generated and the target platform. This is the method used to generate the outer loop of the Rosenbrock integrator, BLAS wrapper functions, and other boilerplate methods. The platform-specific codes from our RADM2 implementations, such as the vectorized Rosenbrock solver and its associated BLAS wrapper functions, were easily converted to templates and added to the KPPA library. Boilerplate code for platform-specific multi-core communication and synchronization was also imported from the RADM2 implementations.

Template file specification enables the rapid reuse of pre-tested and debugged code, and it allows the easy generation architecture-specific code (such as the producer-consumer integrator for CBEA) without making KPPA overly complex. However, it necessitates a copy of every template file translated in every language. In practice, this has not hindered the addition of new languages to KPPA, since most languages are in some way descended from Fortran and/or C. This fact, combined with the small size of the template files, makes by-hand translation of a C or Fortran template to the new language straightforward. Another alternative is to generate code from the original templates and link against the new language-specific code, if possible.

6.2 Multi-core Support

In order to target a specific multi-core architecture, some knowledge of architecture-specific features is required. General computing approaches require a fairly sophisticated machine parameterization, perhaps describing the memory hierarchy of the machine, the processor topology, or other hardware details [25]. Fortunately, because KPPA targets a specific problem domain, only a few architectural parameters are required, and because KPPA has domain-specific knowledge, it is able to generate highly-optimized platform-specific code.

Our experiences in Section 5 show that only four parameters are required to generate a chemical mechanism with multi-layered heterogeneous parallelism (see Table 4). The target architecture name must be specified. The architecture name is passed as a string to the code generator and is used to locate the correct code template files and determine the scalar word size of the architecture. The architecture’s instruction cardinality must be known. Instruction cardinality specifies how many (possibly heterogeneous) instructions of a given precision can be executed per processor per cycle. For the CBEA, this is dictated by the size of a vector instruction, i.e. two in double precision and four in single. CUDA-enabled devices execute instructions in fixed sizes called warps. A single-precision warp is 32 threads on the GTX 200 architecture, making its instruction cardinality 32. In double-precision, one DP unit is shared between 32 threads, so the cardinality is 1, however this architectural detail is hidden from the CUDA developer. Therefore, we still consider the cardinality to be 32 in this case. Scalar cores have an instruction cardinality of 1. Keeping the instruction cardinality independent of the architecture name allows for improvements in existing architectures.

An architecture’s integrator cardinality is closely related to it’s instruction cardinality. It is the optimal number of grid cells that are processed simultaneously by one instance of the integrator. In theory, integrator cardinality could be arbitrarily large for any platform, but in practice the optimal number of cells per integrator is dictated by the instruction cardinality. In single precision, the CBEA is most efficient when processing four cells per integrator instance. On the other hand, the sophisticated thread scheduling hardware in CUDA devices encourages a very large integrator cardinality to hide latency to device memory, so CUDA’s integrator cardinality is limited only by the size of device memory.

The scratch size is the amount of (usually on-chip) memory which can be explicitly controlled by a single accelerator process. KPPA uses this information to generate multi-buffering and prefetching code. For an SPE process on the CBEA, the scratch size is equivalent to the size of SPE local storage. CUDA devices share 16KB of on-chip memory with every thread in a block (up to 512 threads for current architectures). Traditional architectures have an implicitly-managed memory hierarchy, so there is no explicitly-controlled cache.

In addition to the architecture parameterization, KPPA must be aware of the language features specific to the target architecture. Compilers targeting the CBEA have 128-bit vector types as first-level language constructs, and CUDA introduces several new language features to simplify GPU programming. In KPPA, a language is encapsulated in a C++ class that exposes methods for host- and device-side function declaration, variable manipulation, and other foundational operations. Support for platform-specific language features is achieved by inheriting a language class (i.e. C for CUDA) and then overriding member functions as appropriate.

<table>
<thead>
<tr>
<th>Architecture Name</th>
<th>Xeon 5400</th>
<th>Tesla C1060</th>
<th>CBEA</th>
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<td>OpenMP</td>
<td>CUDA</td>
<td>32</td>
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<td>Scratch Size</td>
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<td>Integrator Cardinality</td>
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<td>Scratch Size</td>
<td>0KB</td>
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</tbody>
</table>
7 KPPA MECHANISM PERFORMANCE

To verify that an automatically-generated mechanism can achieve the same performance as the hand-tuned RADM2 implementations, we used KPPA to explore the performance of two chemical mechanisms on the CBEA. (An exploration of mechanism performance on the GPU will also be conducted. We are working with two engineers at NVIDIA to better tap the power of NVIDIA GPUs for chemical kinetics, and until that work is finalized, these results are not available. However, GPU results will be added to the final version of this article.) SAPRCNOV is a SAPRC mechanism with 93 species in a network of 235 reactions. It’s extreme stiffness necessitates a double-precision solver, and its size makes it an excellent stress test for on-chip memories. On the other end of the spectrum, SMALL_STRATO is a stratospheric mechanism with only 7 species in a network of 10 reactions. It represents both exceptionally small mechanisms and mechanisms with a separable Jacobian permitting domain decomposition within the mechanism itself. Both mechanisms were applied to the coarse grid from Section 5.1 using a KPPA-generated Rodas4 Rosenbrock integrator with six stages. SMALL_STRATO was calculated in single precision; SAPRCNOV in double precision.

SAPRCNOV’s large size puts severe pressure on on-chip memories. The program text alone is over 225KB large, and each grid cell comprises over 19KB of data. The Quad-Core Xeon system, with it’s large L2 cache is not notably affected, however the performance of the CBEA is impacted drastically. In order to accommodate the large program text, the ODE Function, ODE Jacobian, LU decomposition, and LU solution functions are overlayed in local storage by the linker. When any of these functions are called, the SPE thread pauses and downloads the program text from main memory to an area of local storage shared by the overlayed functions. LU solve and the ODE function are called several times per integrator iteration, which multiplies the pause-and-swap overhead. Even with overlays, there is only enough room for four grid cells in local store, so at most two cells can be prefetched in our triple-buffering scheme.

Figure 9 shows the speedup of the SAPRCNOV mechanism on the CBEA and the Quad-Core Xeon. The baseline performance is the serial performance of the original code on a single core of an Intel Quad-Core Xeon. For less than seven threads (six SPEs + one PPE) speedup is approximately linear, but there is a clear drop in performance if more than six SPEs are used. This is because contention on the on-chip Element Interconnect Bus (EIB) is low for fewer than six SPEs, so overlayed functions download quickly to local storage and the time a SPE thread spends waiting is manageable. However, for more than six SPEs, bus contention increases the time an SPE thread must wait. These findings are consistent with previous studies of the EIB [26]. After six SPEs, speedup continues to increase, but under penalty. In spite of this, the CBEA achieves the same performance as eight Xeon cores with only seven SPEs (over twice the power efficiency), or twice the performance of eight Xeon cores when all SPEs are used.

The exceptionally small size of SMALL_STRATO makes it representative of small atmospheric kernels. It’s program text and over 120 grid points can be held in a single SPE’s local store, resulting in a maximum speedup of 29×. The speedup curve shown in Figure 10 is more ragged than the RADM2 curves in Section 5.2 because the PPE processes any grid remainder which could not be divided evenly among the SPEs, and the performance gap between SPE and PPE is exacerbated by the small mechanism size. SMALL_STRATO achieves the same coarse-grid performance as RADM2.

8 CONCLUSIONS AND FUTURE WORK

We have presented KPPA (KPP: Accelerated), a general analysis tool and code generator for serial, homogeneous multi-core, and heterogeneous multi-core architectures.
KPPA generates time-stepping codes for general chemical reaction networks in several languages, and is well-suited for use in atmospheric modeling. Optimized ports of the RADM2 chemical kinetics mechanism from WRF-Chem for three multi-core platforms, NVIDIA CUDA, the Cell Broadband Engine Architecture (CBEA), and OpenMP, were presented. Performance benchmarks of three KPPA-generated chemical kernels were also presented.

A detailed performance analysis for each platform was given. The CBEA achieves the best performance due to its fast, explicitly-managed on-chip memory: RADM2 achieves a maximum speedup of 41.1× as compared to the serial implementation. The GPU’s performance is severely hampered by the limited amount of on-chip memory. OpenMP implementations achieve almost linear speedup for up to eight cores. Additional information on the multi-core RADM2 kernel with updated results and codes is maintained here: http://www.mmm.ucar.edu/wrf/WG2/GPU.

The results and analysis presented here are a snapshot in time; for example, newer versions of homogenous multi-core processors (e.g. Intel’s i7, IBM’s Power6, and others) are already showing significant improvement in speed and multi-core efficiency over their previous generations. Similarly, NVIDIA is moving forward with their 300-series GPUs. Currently none has a clear advantage. However, given that chemical kinetics is a challenging benchmark in terms of sheer size and complexity per grid cell, performance and cost performance (both monetary and electrical efficiency) of new homogenous and heterogeneous multi-core architectures will be important gating factors for climate-chemistry, air-quality, wild-fire, and other earth science simulation in the coming decade. KPPA’s object-oriented design will allow it to incorporate future architectures and languages as they emerge.

Future work will concentrate on alternate numerical methods, frameworks for calling KPPA-generated mechanisms remotely, and extending KPPA to other multi-core platforms. The numerical methods presented have strong data dependencies and cannot adequately use fast on-chip memories. Other methods, such as Quasi-Steady-State-Approximation [27], may be appropriate if a high level of accuracy is not required. Installing multi-core chipsets as accelerators in traditional clusters is viable [28], however not every cluster can be easily upgraded. Clusters of ASIC nodes, such as IBM BlueGene [29], do not support accelerator cards, or thermal dissipation and power issues may prohibit additional hardware in the installation rack. These systems can still benefit from accelerated multi-core chipsets by offloading computationally-intense kernels to remote systems. If a kernel is 20× or 30× faster on a specific system, as demonstrated in this work, then the overhead of a remote procedure call may be acceptable, even for large data sets. We are investigating existing solutions, such as IBM Dynamic Application Virtualization [30], and will extend KPPA to generate model interfaces for remote kernels.

The recent acceptance of the OpenCL standard [31] makes OpenCL support an obvious next-step for KPPA. Chemical codes targeting OpenCL will be more portable than CUDA- or CBEA-specific mechanisms, and may be supported on as-yet undeveloped architectures.

9 RELATED WORK

Implementing chemical kinetics models on emerging multi-core technologies can be unusually difficult because expertise in kinetics and atmospheric modeling must be combined with a strong understanding of various multi-core paradigms. For this reason, existing literature tends to focus on the model sub-components, such as basic linear algebra operations [32], [33], [34], and does not comprehensively address this problem domain for chemical kinetics and related problems on real-world domains.

9.1 Atmospheric Modeling and Linear Algebra

Research into using GPUs to accelerate the transport and diffusion of atmospheric constituents is ongoing. Fan et al. [28] used a 35-node cluster to simulate the dispersion of airborne contaminants in the Times Square area of New York City with the lattice Boltzmann model (LBM). For only $12,768, they were able to add a GPU to each node and boost the cluster’s performance by 512 gigaflops to achieve a 4.6x speedup in their simulation. Perumalla [35] used an NVIDIA GeForce 6800 Go GPU to explore time-stepped and discrete event simulation implementations of 2D diffusion. Large simulations saw a speedup of up to 16x on the GPU as compared to the CPU implementation. As previously mentioned, transport forms a relatively small fraction of the computational cost of comprehensive atmospheric simulation with chemistry.

Like many scientific codes, linear algebra operations are a core component of chemical kinetics simulations. The literature of the last four years abounds with examples of significantly improved linear algebra performance for both GPUs and the CBEA. Williams et al. [34], [36] achieved a maximum speedup of 12.7× and power efficiency of 28.3× with double precision general matrix multiplication, sparse matrix vector multiplication, stencil computation, and Fast Fourier Transform kernels on the CBEA as compared to AMD Opteron and Itanium2 processors. Dongarra et al. [37] report up to 328 single-precision gigaflops when computing a left-looking block Cholesky factorization on a pre-released NVIDIA T10P. Bolz et al. [38] implemented a sparse matrix conjugate gradient solver and a regular-grid multigrid solver on NVIDIA GeForce FX hardware. The GPU performed 120 unstructured (1370 structured) matrix multiplies per second, while an SSE implementation achieved only 75 unstructured (750 structured) matrix multiplies per second on a 3.0GHz Pentium 4 CPU. Krüger and Westermann [39] investigated solvers for Navier-Stokes equations on
GPUs. They represented matrices as a set of diagonal or column vectors, and vectors as 2D texture maps to achieve good basic linear algebra operator performance on NVIDIA GeForce FX and ATI Radeon 9800 GPUs. More recent efforts include the MAGMA project [40] which is developing a successor to LAPACK but for heterogeneous/hybrid architectures.

Our work uses the heterogeneous parallelism of the CBEA in a way similar to that introduced by Ibrahim and Bodin in [41]. They introduced runtime data fusion for the CBEA which dynamically reorganizes finite element data to facilitate SIMD-ization while minimizing shuffle operations. Using this method, they achieved a sustained 31.2 gigaflops for an implementation of the Wilson-Dirac Operator. Runtime data fusion is not applicable to chemical kinetic models, but we use the PPU in a similar manner to reorganize data from the WRF-Chem model to facilitate SIMD-ization in a way specific to WRF-Chem.

9.2 General Analysis, Code Generation and Tuning
Auto-tuning techniques like those found in ATLAS [42] and FFTW [43] are being applied to linear algebra on GPUs. Li et al. [44] designed a GEMM auto-tuner for NVIDIA CUDA-enabled GPUs that improved the performance of a highly-tuned GEMM kernel by 27%.

KPP [5] is a general analysis tool that facilitates the numerical solution of chemical reaction network problems. It automatically generates Fortran C code that computes the time-evolution of chemical species, the Jacobian, and other quantities needed to interface with numerical integration schemes. KPP has been successfully used to treat many chemical mechanisms from tropospheric and stratospheric chemistry, including CBM-IV [45], SAPRC [8], and NASA HSRP/AESA. The Rosenbrock methods implemented in KPP typically outperform backward differentiation formulas, like those implemented in SMYGEAR [46, 47].

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