# CS-6404 (CRN 20179)

## Inverse problems and data assimilation: Inverse modeling with differential equations

## Spring 2019

### Essential information.

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| Instructor | Adrian Sandu |
| Phone | 231-2193 |
| E-mail | sandu@cs.vt.edu |
| Office | 2224 KnowledgeWorks II, 2202 Kraft Drive. |
| Lecture | Tue-Thu, 11:00AM–12:15PM, 207 McBryde Hall |
| Office hours | by appointment |
| Web Page | `http://www.cs.vt.edu/~asandu/Courses/CS6404/CS6404.html` |
| Prerequisites | Graduate standing. |
| Exam section | 11T. |

### Prerequisites.

Graduate standing; graduate coursework in numerical methods; instructor approval.

### Overview.

Computer/mathematical models encapsulate the physical laws that govern the evolution of a real system, and allow us to predict the evolution of that system.

- The *forward problem* consists of using the model (with prescribed values of system parameters) to simulate the evolution of the physical system. n(As a useful example, consider a pendulum of known length $L$. For a prescribed mass $m$ of the pendulum the model can predict the period of small oscillations).

- The *inverse problem* consists of using data from measurements of reality to infer the values of the parameters that characterize the system. (In this case one measures the period of small oscillations; based on this data, and using the model equations, one infers the mass of the pendulum).

Inverse problems are tremendously important in many fields from biology to nuclear engineering to numerical weather prediction. They are also highly challenging, and special methodologies are required for their solution.

This class introduces different computational methods for solving inverse and data assimilation problems, including ensemble Kalman filters, variational methods, and nonlinear and non-Gaussian techniques. We will build a strong background in inverse problems by covering topics such as statistical estimation theory, ill-conditioning and regularization techniques, differential equations and adjoint modeling, and numerical optimization.

### Textbook.

The instructor will hand in typeset course notes to registered students. There is no single textbook that covers all the material discussed; nevertheless, the following texts may prove useful.

- A. Tarantola: "Inverse Problem Theory and Methods for Model Parameter Estimation", SIAM, 2005. ISBN 0-89871-572-5. Available on-line at `http://www.ipgp.jussieu.fr/~tarantola/Files/Professional/Books/index.html`.
- M. Asch, M. Bocquet, M. Nodet: "Data Assimilation: Methods, Algorithms, and Applications", SIAM, 2017. ISBN ISBN 978-1-611974-53-9.
- K. Law, A., Stuart, K. Zygalakis: "Data Assimilation A Mathematical Introduction". Springer, 2015. ISBN 978-3-319-20325-6. First part available on-line at `https://arxiv.org/abs/1506.07825`.
- R.C. Aster, B. Borchers, and C.H. Thurber: "Parameter Estimation and Inverse Problems", Second ed., Elsevier Academic Press, 2012. ISBN 0-12385-048-7.
- J. Nocedal, S.J. Wright: "Numerical Optimization", Second ed., Springer Series in Operations Research, 2006. ISBN 0-387-30303-0. Available in electronic format from `http://lib.vt.edu`.
- C. T. Kelley: "Iterative Methods for Linear and Nonlinear Equations" and "Iterative Methods for Optimization". Available on-line at `http://www.siam.org/books/textbooks/download.php`.

**Topics.**

This class will introduce different computational methods for solving inverse problems. Topics discussed during the course include:

1. Review of probability theory.
2. Statistical estimation theory, minimum variance, maximum likelihood. Bayesian approach, Kalman filter.
3. Numerical optimization, optimality conditions.
4. Discrete linear inverse problems: linear regression, least squares, least-absolute-values, and minimax criteria. Singular value decomposition.
5. Ill-conditioning and regularization techniques (Tikhonov, total variation, truncated SVD, maximum entropy).
6. Discrete non-linear inverse problems: nonlinear regression, Gauss-Newton, Levenberg-Marquardt.
7. Ensemble-based (Monte Carlo) methods. Sampling, Metropolis-Hastings algorithm, ensemble Kalman filters, particle filters.
8. Direct and adjoint sensitivity analysis for differential equations.
9. Continuous non-linear inverse problems with ordinary and partial differential equations.

**Grading.**

There will be no in-class midterm or final examinations. The grade will be based on homework problems and projects. For those topics for which lecture notes are not available one student may be designated, by rotation, to take notes in class. You are responsible for typesetting them in LaTeX, making sure they are error-free (after consulting with your colleagues), and handing them to the instructor *no later than one week after the lecture*. Any figures drawn on the table during the lecture will be reproduced using any appropriate drawing software. The LaTeX notes will be distributed to the entire class when ready.

## Disclaimer.

The information given to you in class will supersede the information in this syllabus.

## Student Complaints and Academic Misconduct.

The Honor Code guides student conduct in the class. If you have any problems, the first step is to discuss with me directly. Should you need to speak with the Chair of the C.S. Department, you can make an appointment by speaking with the Departmental Secretary in KWII.

## Disabilities.

Please let me know if you have a disability which requires special arrangements.