Solution to Homework #1
CS5214 Modeling and Evaluation   Fall 2023

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1. (1) The customer turned away probability is **0.279878**.
   (2) The average number of customers waiting in the system is **4.786998**.
   (3) The response time per client for those served by the fast server is **0.642219 s**.
   (4) The response time per client for those served by the two slow servers is **0.7777516 s**.
   (5) The throughput is **10.798395**
   (6) The report confidence accuracy is H/Y = 0.003165/0.642219 = **4.9%**.

Source Code:

```
#include"smpl.h"
#define TOKENS 1000
#define TRUE 1
#define FALSE 0

main()
{
real Ta=1.0/15,Ts_fast=0.2,Ts_slow=1.0/3,mean,hw,nq;
int tk_id=0,customer=0,event,mm310,nb;
int n_fastServed=0,n_slowServed=0; /*number of customers severed by fast server or slow
servers*/
real R_sum_fast=0.0,R_sum_slow=0.0;/*sum of response times served by fast server or slow
servers*/
real n_arrived=0; /* number of customers arrived at the system*/
int n_rejected=0; /*number of customers turned away by the system*/
int n_waiting; /*number of tokens waiting in queue*/
int fastBusy=FALSE,slow1Busy=FALSE,slow2Busy=FALSE; /*server busy or not */
real ts[TOKENS];
int cont=TRUE;
smpl(0, "M/M/3/10 Quene with BMA");
init_bm(200,20000);
mm310=facility("mm310",3);
schedule(1,0.0,tk_id);
while (cont)
{
        cause(&event,&customer);
        switch(event)
        {
                case 1:/*arrival*/
                n_arrived++;
                if(++tk_id>=TOKENS) tk_id=0;
                schedule(1,expntl(Ta),tk_id);
                n_waiting=inq(mm310);
                if (n_waiting>=7) n_rejected++;
```

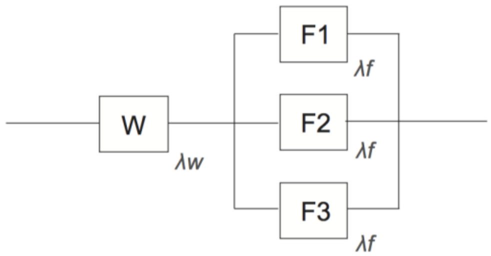
```
		else{
			schedule(2,0.0,customer);
			ts[customer]=time();
		  }
		break;
		case 2:/*request server*/
		if (request(mm310,customer,0)==0){
			if (!fastBusy){
					schedule(3,expntl(Ts_fast),customer);
					fastBusy=TRUE;
			}else if (!slow1Busy){
					schedule(4,expntl(Ts_slow),customer);
					slow1Busy=TRUE;
			}else{
				schedule(5,expntl(Ts_slow),customer);
					slow2Busy=TRUE;
			    }
		}
		break;
		case 3:/*departure from the fast server*/
		release(mm310,customer);
		fastBusy=FALSE;
		n_fastServed++;
		R_sum_fast+=time()-ts[customer];
		if(obs(time()-ts[customer])==1)
			cont=FALSE;
		break;
		case 4:/*departure from the first slow server*/
		release(mm310,customer);
		slow1Busy=FALSE;
		n_slowServed++;
		R_sum_slow+=time()-ts[customer];
		break;
		case 5:/*departure from the second slow server*/
		release(mm310,customer);
		slow2Busy=FALSE;
		n_slowServed++;
		R_sum_slow+=time()-ts[customer];
		break;
	}
} /*end while*/
civals(&mean,&hw,&nb);
/*answer for Q1*/
printf("Customer turned away probability: %f\n", n_rejected/n_arrived);
/*answer for Q2*/
printf("Average number of customers waiting in the system: %f\n", Lq(mm310));
```

```
  /*answer for Q3*/
  printf("Response time per client served by fast server: %f\n", mean);
  /*answer for Q4*/
  printf("Response time per client served by slow servers: %f\n", R_sum_slow/n_slowServed);
  /*answer for Q5*/
  printf("Throughput: %f\n", (n_fastServed + n_slowServed)/time());
  /*answer for Q6*/
  printf("Y=%f,H=%f after %d batches\n",mean,hw,nb);
}
```

Output:

```
batch  1 mean = 0.646
batch  2 mean = 0.641
batch  3 mean = 0.647
batch  4 mean = 0.638
batch  5 mean = 0.646
batch  6 mean = 0.643
batch  7 mean = 0.633
batch  8 mean = 0.639
batch  9 mean = 0.644
batch 10 mean = 0.644, rel. HW = 0.005
Customer turned away probability: 0.279878
Average number of customers waiting in the system: 4.786998
Response time per client served by fast server: 0.642219
Response time per client served by slow servers: 0.777516
Throughput: 10.798395
Y=0.642219,H=0.003165 after 10 batches
```

2. The system structure is shown as follows:



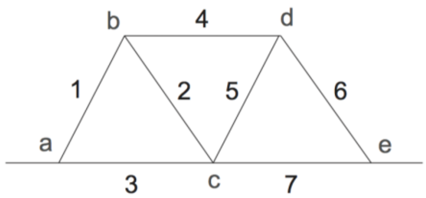
$$R(t) = R_w(t)\left[1 - F_{f1}(t)F_{f2}(t)F_{f3}(t)\right] = R_w(t)\left[1 - (1 - R_f(t))^3\right]$$

$$= e^{-\lambda_\omega t}\left[1 - \left(1 - e^{-\lambda_f t}\right)^3\right] = 3e^{-\lambda_\omega t - \lambda_f t} - 3e^{-\lambda_\omega t - 2\lambda_f t} + e^{-\lambda_\omega t - 3\lambda_f t}$$

$$\text{MTTF} = \int_0^\infty R(t)d_t = \frac{3}{\lambda_\omega + \lambda_f} - \frac{3}{\lambda_\omega + 2\lambda_f} + \frac{1}{\lambda_\omega + 3\lambda_f}$$

3.

(a) The vertices of the reliability graph model are marked with a, b, c, d, and e as follows:



Source Code:

```
bind
r1 0.00001
r2 0.00002
r3 0.00003
r4 0.00004
r5 0.00005
r6 0.00006
r7 0.00007
end

relgraph bridge
a b exp(r1)
a c exp(r3)
b d exp(r4)
d e exp(r6)
c e exp(r7)
bidirect
b c exp(r2)
c d exp(r5)
end

expr 1-value(2016;bridge)
pqcdf (bridge)
end
```

Output:

```
1-value(2016;bridge):   9.8279e-001

CDF for system bridge:

1 - ([P(1:a,c)*P(4:c,e)]+
[P(0:a,b)*P(2:b,d)*P(3:d,e)*(1-P(1:a,c)*P(4:c,e))]+
[P(0:a,b)*Q(1:a,c)*P(4:c,e)*P(5:b,c)*(1-P(2:b,d)*P(3:d,e))]+
[P(1:a,c)*P(3:d,e)*Q(4:c,e)*P(6:c,d)*(1-P(0:a,b)*P(2:b,d))]+
[P(0:a,b)*Q(1:a,c)*P(2:b,d)*Q(3:d,e)*P(4:c,e)*Q(5:b,c)*P(6:c,d)]+
[P(0:a,b)*Q(1:a,c)*Q(2:b,d)*P(3:d,e)*P(5:b,c)*P(6:c,d)*(1-P(4:c,e))]+
[Q(0:a,b)*P(1:a,c)*P(2:b,d)*P(3:d,e)*Q(4:c,e)*P(5:b,c)*Q(6:c,d)])
```

The system reliability after 12 weeks of operation is 0.98279.

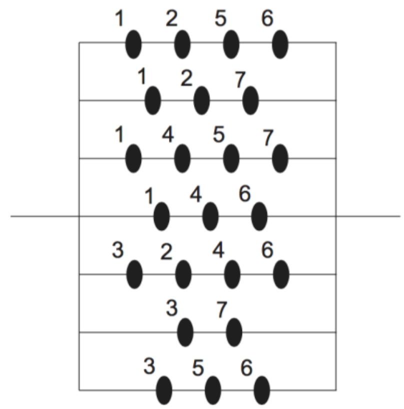
(b) There are 7 minimal path sets:

{1,2,5,6}, {1,2,7}, {1,4,5,7}, {1,4,6}, {2,3,4,6}, {3,5,6}, {3,7}

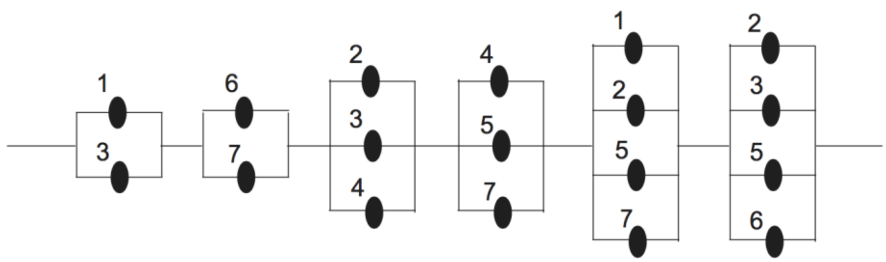
There are 6 minimal cut sets:

{1,3}, {6,7}, {2,3,4}, {4,5,7}, {1,2,5,7}, {2,3,5,6}

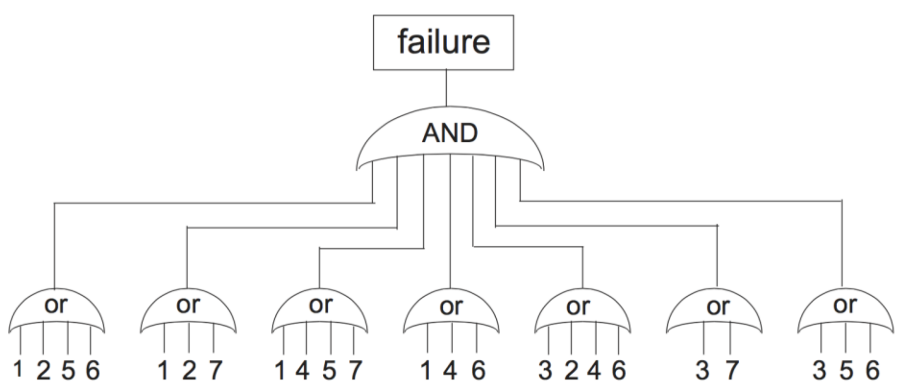
The structure using a parallel connection of the **minimal path sets** is shown as follows:



The structure using a series connection of the **minimal cut sets** is shown as follows:



(c) The fault tree model built based on the **minimal path sets** is shown as follows:



Source Code: **Output:**

```
bind
r1 0.00001
r2 0.00002
r3 0.00003
r4 0.00004
r5 0.00005
r6 0.00006
r7 0.00007
end

ftree network
repeat p1 exp(r1)
repeat p2 exp(r2)
repeat p3 exp(r3)
repeat p4 exp(r4)
repeat p5 exp(r5)
repeat p6 exp(r6)
repeat p7 exp(r7)
OR fork1 p1 p2 p5 p6
OR fork2 p1 p2 p7
OR fork3 p1 p4 p5 p7
OR fork4 p1 p4 p6
OR fork5 p2 p3 p4 p6
OR fork6 p3 p7
OR fork7 p3 p5 p6
AND top fork1 fork2 fork3 fork4 fork5 fork6 fork7
end

expr 1-value(2016;network)
pqcdf (network)
end
```

```
Output

1-value(2016;network):   9.8279e-001

CDF for system network:

[Q(p1)*Q(p3)]+
[Q(p6)*Q(p7)*(1-Q(p1)*Q(p3))]+
[P(p1)*Q(p2)*Q(p3)*Q(p4)*(1-Q(p6)*Q(p7))]+
[P(p2)*Q(p3)*Q(p4)*Q(p5)*P(p6)*Q(p7)*(1-Q(p1))]+
[Q(p4)*Q(p5)*P(p6)*Q(p7)*(1-Q(p3))]+
[Q(p1)*Q(p2)*P(p3)*P(p4)*Q(p5)*P(p6)*Q(p7)]+
[P(p1)*Q(p2)*Q(p3)*P(p4)*Q(p5)*Q(p6)*P(p7)]
```

The system reliability after 12 weeks of operation is **0.98279**.