Reliability Block Diagram using sharpe

```
block name {( param-list )}
[ < block line >
end]
```

can be one of the following:

P.358 Appendix B
1) comp name exponential-polynomial

\[ F(t) = 1 - e^{-\lambda t} \]

Referring to the cumulative dist function (cdf)

\[ R(t) = e^{-\lambda t} \]

\( \exp \) (lambda) ← meaning

\( \text{cdf} \) (component-name, state, arg-list)

which has been defined before

or

\( \text{gen} \) triple 1, triple 2, ...

in the form of \((a_j, k_j, b_j)\)

\[ F(t) = \sum_{j} a_j \cdot t^{k_j} \cdot e^{b_j t} \]

2) parallel name name-1 name-2 \{name3 name4 \ldots\}

The parallel system is assigned to the first name.

3) series name name-1 name-2 \{name3 name4 \ldots\}

The series system is assigned to the first name.

4) kofn name expression-1, expression-2, component-name

\( k \quad n \) identical components
5) `kofn` name k-expression, n-expression, name1 name2 {name3…}

representing a k-out-of-n system having possibly different components

Components do not have identical failure-time dist.

Ex:

Fig. 2.5 (p. 36)

Sharing memory: a k-out-of-n device

```
block system ( k, n, pfrate, mfrate )
comp CPU exp ( pfrate )
comp mem exp ( mfrate )
parallel CPUs CPU CPU
kofn mems k, n, mem
series subsystem CPUs mems
end
```

**Output**

CDF for system in semi symbolic form

\[
1 - 6e^{-0.00903t} + 3e^{-0.0104t} + \ldots
\]

(1-out-of-3)

\[
\begin{align*}
\text{k=1.00} & \quad \text{mean(system;k,3,…)} = 2.26*10^2 \\
\text{rel (10,k,3…)} & \quad = 9.9981*10^{-1} \\
\text{rel (365,k,3…)} & \quad = 8.33*10^{-1}
\end{align*}
\]

\[
\text{k=2.00}
\]
Comments: any line starting with the symbol “*”

* printing output.
* printing $F(t)$ in symbolic form
* define reliability function at time $t$

```plaintext
func rel(t, k, n, pf, mf) \[ \overline{1-\text{value}}(t; \text{system}; k, n, pf, mf) \]

Returning $F(t)$ at time $t$ numerically

loop k, 1, 3, 1
  expr mean (system; k, 3, 0.00139, 0.00764)
  expr rel (10, k, 3, 0.00139, 0.00764)
  expr rel (365, k, 3, 0.00139, 0.00764)
end
end
```
Fig. 9.1 p. 156

* printing: 5 decimal places
format 5
* cdf
  cdf (block1a)
  expr 1-value(10; block1a)
  end

Print 1-F(t)=R(t) at t=10
## Availability Modeling

\[
A_i(t) = \frac{\mu_i}{\lambda_i + \mu_i} + \frac{\lambda_i}{\lambda_i + \mu_i} e^{-(\lambda_i + \mu_i)t}
\]

To define \(1 - A_i(t)\)

\[
\text{poly unavailable(mu, lambda)}
\]

\[
\text{gen triple of the form (a_j, k_j, b_j)}
\]

\[
F(t) = \sum_{j} a_j \cdot t^{k_j} \cdot e^{b_j t}
\]

See p.354 text on a user-defined distribution syntax:

\[
\text{poly name(param-list) dist.}
\]

\[
\text{gen triple (a_j, k_j, b_j)}
\]

* print steady state
* availability \(A(\infty)\)

expr `pinf(block1a)`

* print instantaneous
* availability at \(t=100\)

expr `1-value(100; block1a)`
Fault Trees  

* A pictorial representation of events that can cause the occurrence of an undesirable event.
* An event at a high level is reduced to a combination of lower level events by means of logic gates

\[ \text{AND: when all fail, the failure event occurs.} \]
\[ \text{OR: when one fails, the failure event occurs.} \]
\[ \text{K out of n: when at least k out of n components fail, the failure event occurs.} \]

**Example:**

```
  failure
   /\   ___
  or  /  /
  /   /  /
P1  P2 M1 M2 M3
```

\[ \equiv \]

```
P1  P2    M1
  |   |    M2
  |   |    M3
```
For a fault tree without repeated components:

\[
Q_{\text{ftree}}(t) = \begin{cases} 
\prod_{i=1}^{n} Q_i(t) & \text{AND gate} \\
1 - \prod_{i=1}^{n} (1 - Q_i(t)) & \text{OR gate} \\
\sum_{i=k}^{n} \binom{n}{i} (Q(t))^i (1 - Q(t))^{n-i} & \text{k-out-of-n gate: for n identically distributed components} \\
\sum_{|J| \geq k} \left( \prod_{j \in J} Q_j(t) \right) \left( \prod_{j \not\in J} (1 - Q_j(t)) \right) & \text{k-out-of-n gate: for n non-identically distributed components}
\end{cases}
\]

Unreliability or failure probability

A set that contains at least k failed components
The above equation cannot be used when there is a repeated component
e.g.

2 processors: $P_1$ & $P_2$
3 memory modules: $M_1$, $M_2$ & $M_3$

$M_3$ is shared by $P_1$ & $P_2$
$M_1$ is private to $P_1$
$M_2$ is private to $P_2$

the system will operate as long as there is at least one operational processor with access to either a private or shared memory.
i.e. $\overline{P_i} = 1$ if
Processor i fails at time $t$
and $\overline{P_i} = 0$ otherwise.

Let

\[ F_{\overline{P_i}} = E\left[\overline{P_i}\right] \]

(\because F_{\overline{P_i}} = 1\cdot F_{\overline{P_i}} + 0\cdot R_{\overline{P_i}})

Then

\[ \overline{\phi} = \overline{X_{S_1}X_{S_2}} = \left[1 - \left(1 - \overline{P_1}\right)\left(1 - \overline{M_1M_3}\right)\right] \cdot \left[1 - \left(1 - \overline{P_2}\right)\left(1 - \overline{M_2M_3}\right)\right] \]

The subsystem fails when either $P_1$ fails or both $(M_1M_3)$ fail

\[
= \left(\overline{P_1} + P_1\overline{M_1M_3}\right) \ast \left(\overline{P_2} + P_2\overline{M_2M_3}\right)
= \overline{P_1P_2} + \overline{P_1P_2M_2M_3} + P_1\overline{P_2M_1M_3} + P_1\overline{P_2M_1M_2M_3}

\]

\[ E\left[\overline{\phi}\right] = Q_{system} = Q_{P_1}Q_{P_2} + Q_{P_1}R_{P_2}Q_{M_2}Q_{M_3} + R_{P_1}Q_{P_2}Q_{M_1}Q_{M_3} + R_{P_1}R_{P_2}Q_{M_1}Q_{M_2}Q_{M_3} \]
Fault Tree using Sharpe

e.g.

\[ Q(t) = 1 - P e^{-a_1 t} - (1-P) e^{-a_2 t} \]

hyperexponential

\[ Q(t) = 1 - e^{-\lambda t} \]

exponential failure law

\[
\begin{align*}
\text{bind} & \\
\quad a_1 & 0.028 \\
\quad a_2 & 0.25 \\
\quad P & 0.5 \\
\text{end} & \\
\text{ftree} & \text{series (lambda)} \\
\quad \text{basic} & B \quad \text{exp (lambda)} \\
\quad \text{basic} & A \quad \text{gen} \\
\quad & 1, 0, 0 \quad \text{* for 1} \\
\quad & -P, 0, -a_1 \quad \text{* for } -P e^{-a_1 t} \\
\quad & -(1-P), 0, -a_2 \quad \text{* for } -(1-P) e^{-a_2 t} \\
\text{or} & \\
\text{end} & \text{top A B} \\
\text{* print} & \\
\quad cdf'(series; 0.05) & \\
\quad eval (series; 0.05) & 0.5 \quad 1.5 \quad 0.5 \\
\text{end} & \\
\end{align*}
\]
e.g. Aircraft flight control system

This means if 3 out of 4 fail then the subsystem fails

Inertial reference sensors: 3
- IRS
- IRS
- IRS

Pitch rate sensors: 3
- PRS
- PRS
- PRS

Computer systems: 4
- CS
- CS
- CS
- CS

Secondary actuators: 3
- SA
- SA
- SA

* most susceptible to failure

* : . use 4 CS components

* print
  format 8
  expr mean(aircraft)
  eval (aircraft) 1000 10000 1000
  expr value(10; aircraft) * unreliability
end
Fault Trees using SHARPE with repeated components

Ex: also see the example in Fig. 9.22

Ex:

\[
\text{ftree system} \begin{aligned}
\text{basic} & \quad P_1 & \quad \exp (\lambda P_1) \\
\text{basic} & \quad P_2 & \quad \exp (\lambda P_2) \\
\text{basic} & \quad M_1 & \quad \exp (\lambda M_1) \\
\text{basic} & \quad M_2 & \quad \exp (\lambda M_2) \\
\text{repeat} & \quad M_3 & \quad \exp (\lambda M_3) \\
\text{AND} & \quad M_1 M_3 & \quad M_1 & \quad M_3 \\
\text{AND} & \quad M_2 M_3 & \quad M_2 & \quad M_3 \\
\text{OR} & \quad \text{system}_1 & \quad P_1 & \quad M_1 M_3 \\
\text{OR} & \quad \text{system}_2 & \quad P_2 & \quad M_2 M_3 \\
\text{AND} & \quad \text{top} & \quad \text{system}_1 & \quad \text{system}_2 \\
\end{aligned}
\]

\[\equiv \text{kofn system}_1 \ 1, \ 2, \ P_1 \ M_1 M_3\]

\[\equiv \text{kofn top} \ 2, \ 2, \ \text{system}_1 \ \text{system}_2\]

\[\text{end}\]

* print reliability at time t

\text{expr} \ 1\text{-value} \ (t; \ \text{system})
2.6 Series-Parallel Block Diagrams with Components in common
(Also called Network Reliability Models)

e.g.

Can be arranged as

A parallel connection of series structures

A series connection of parallel structures

Definition: a minimal path (set) is a minimal set of components whose functioning ensures the functioning of the system.

Definition: a minimal cut (set) is a minimal set of components whose failure ensures the failure of the system.
Q: how many minimal paths?
A: 4: {1,3,5}, {1,4}, {2,5}, {2,3,4}

Q: how many minimal cuts?
A: 4: {1,2}, {4,5}, {1,3,5}, {2,3,4}

There are series-parallel diagrams with common components
Another example: TMR is a 2-out-of-3 system

3 minimal paths:
{1,2} {2,3} & {1,3}

Let $\phi(t)$ be a boolean variable indicating if the system is alive at time $t$, i.e.,

\[
\begin{align*}
\phi(t) &= 1 \quad \text{if alive} \\
\phi(t) &= 0 \quad \text{if dead}
\end{align*}
\]

Let $X_i(t)$ be a boolean variable indicating if component $i$ is alive at time $t$, i.e.,

\[
\begin{align*}
X_i(t) &= 1 \quad \text{if alive} \\
X_i(t) &= 0 \quad \text{if dead}
\end{align*}
\]

\[
\phi(t) = 1 - \left[ (1 - X_1 X_2)(1 - X_2 X_3)(1 - X_1 X_3) \right] \\
= 1 - \left( 1 - X_1 X_2 - X_2 X_3 - X_1 X_3 + X_1 X_2^2 X_3 + X_1 X_2 X_3^2 + X_1^2 X_2 X_3 - X_1^2 X_2^2 X_3 \right) \\
= 1 - \left( 1 - X_1 X_2 - X_2 X_3 - X_1 X_3 + 2X_1 X_2 X_3 \right)
\]

Due to $X_i^2 = X_i$ from $X_i$’s definition
Now

\[ E[\phi(t)] = R(t) \]

\[ \therefore E[\phi(t)] = 1 \cdot \{ \text{prob. it is alive at } t \} + 0 \cdot \{ \text{prob. it fails at } t \} = R(t) \]

\[ \Rightarrow E[\phi(t)] = E[X_1X_2 + X_2X_3 + X_1X_3 - 2X_1X_2X_3] \]

Terms each contain only independent components

\[ = E[X_1X_2] + E[X_2X_3] + E[X_1X_3] - 2E[X_1X_2X_3] \]

\[ \therefore R_{\text{system}} = R_1R_2 + R_2R_3 + R_1R_3 - 2R_1R_2R_3 \]

For identical components, \( R_1 = R_2 = R_3 \), \( R_{\text{system}} = 3R^2 - 2R^3 \)
Modeling with a Reliability Graph

* A reliability graph consists of nodes & directed arcs.
  - source node — no arcs enter it
  - target (sink) node — no arcs leave it

* A system represented by a reliability graph fails when there is no path from the source to the sink.

* arcs are associated with failure time distribution (in cdf)
The underlying technique for solving the model is minimal path set & minimal cut set.
What is the reliability graph corresponding to the fault tree model on the left?

Ans:

```
relgraph P 2M3shared
src 1 exp(lambda_P_1)
src 2 exp(lambda_P_2)
1 sink exp(lambda_M1)
2 sink exp(lambda_M2)
share sink exp(lambda_M3)
1 share inf
2 share inf
end

* print reliability at time t
expr 1-value(t; P2M3shared)
end
```

This means
1 → share &
2 → share
links never fail

Specifying
\[ F(t) = 1 - e^{-\lambda_r t} \]
for P_1

See p.353 Appendix B specifying a component having all its mass at \( \infty \), i.e., \( F(t)=0 \) except at \( F(\infty)=1 \)