5214 Modeling and Evaluation Fall 2023

Solution to Test #1 (open books/notes)

I pledge that this test has been completed in compliance with the Graduate Honor Code and that I have neither given nor received any aid on this test.

Student Name: ____________________

Signed: ____________________

Part I : Multiple – Choice Problems (60%). Select one correct answer in each problem.

1. A system has three components, 1, 2, and 3. The system is available if component 1 is available and at least one of the two remaining components, 2 and 3, is available. Let A_1, A_2 and A_3 be the availabilities of components 1, 2 and 3, respectively. What is the availability of the system?
   √ (a) $A_1A_2 + A_1A_3 - A_1A_2A_3$
   (b) $A_1A_2 + A_2A_3 - A_1A_2A_3$
   (c) $A_1(1 - A_2)(1 - A_3)$
   (d) $A_1(1 - A_2A_3)$
   (e) $1 - (1 - A_1)(1 - A_2)(1 - A_3)$

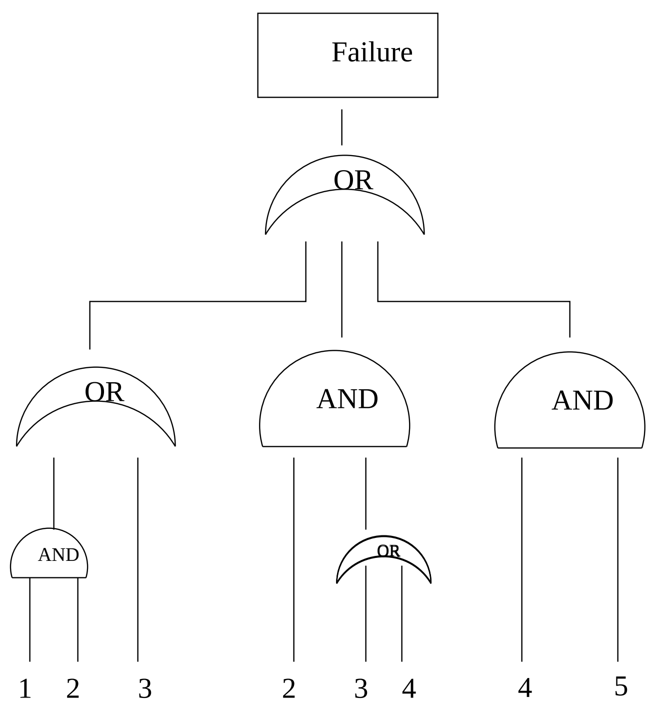
2. Use M/M/1 to solve this problem. Consider a phone booth servicing customers arriving at a rate of 15 customers/hour. Suppose the average duration of a phone conversation is 3 minutes. What is the average wait time (just the queuing time) per customer at the phone booth?
   (a) 5 min
   (b) 6 min
   (c) 7 min
   (d) 8 min
   √ (e) 9 min

3. Suppose we observe a system for 1000 minutes. Within this period, the system is busy for 500 minutes, the total number of customers completed is 5000 and the average customer population is 6. Which of the following is not true?
   (a) the utilization of the system is 0.5
   (b) the throughput of the system is 5 customers/minute
   (c) the response time per customer is 1.2 minute
   √ (d) the wait time per customer is 1 minute

4. A server system obeying exponential failure law has a constant failure rate of 0.01 hr^{-1}. Assume that customers arrive at the server system with a rate of 5 customers/hour and the server is able to service customers at a rate of 6 customers/hour. Which one of the following is true?
   (a) The reliability of the system 1 - $e^{-0.01}$ at time = 1 hour

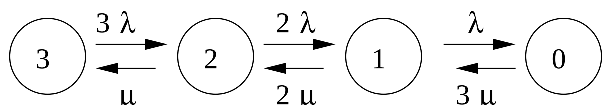
√ (b) The expected number of customers the system can serve before it fails is 500 customers
(c) The average population in the system before it fails is 10 customers
(d) the expected response time per customer before it fails is 0.5 hour.
(e) The throughput of the system before it fails is 6 customers/hour

5. For an M/M/3/10 queueing facility with arrival rate of λ and service rate of μ for each server, which one of the following is false? Assume that P_i stands for the steady-state probability that the system contains i customers (including both queueing and in service).
(a) the average number of customers is $\sum_{i=0}^{10}(i \times P_i)$
(b) the average rejection rate is $P_{10} \times \lambda$
(c) the throughput is $\sum_{i=1}^{3}(i \times \mu \times P_i) + \sum_{i=4}^{10}(3\mu \times P_i)$
(d) the throughput is $\sum_{i=0}^{9}(\lambda \times P_i)$
√ (e) the utilization of the facility is $\sum_{i=0}^{10} P_i$



6. Which of the following statements is false for the fault tree model shown above?
(a) the system is alive when components 1, 3, and 4 are alive
(b) the system fails when components 4 and 5 fail
(c) the system is alive when components 2, 3, and 5 are alive
(d) the system fails when component 3 fails
√ (e) the system fails when component 4 fails

7. Continue from the above question. Suppose that components 1, 2 and 3 are always reliable with probability 1 (say via internal redundancy) while components 4 and 5 have the reliabilities of 0.7 and 0.8, respectively, at 2000 hours. What is the system reliability at 2000 hours?
(a) 0.006
(b) 0.994
(c) 0.504
(d) 0.56
√ (e) 0.94



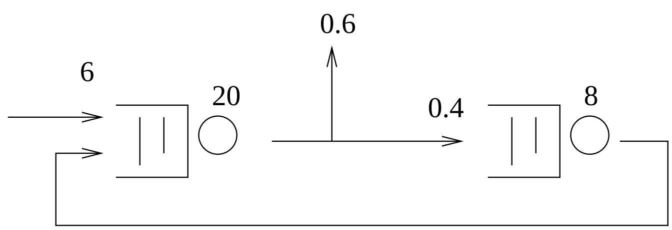
8. Consider a TMR system with each component having an independent failure rate of λ and an independent repair rate of μ. Assume that alive components can still fail regardless of the states of other components. A Markov model for this TMR system is shown below to model the availability of the system. The system is operational when at least 2 out of 3 components are operational. State i means that i components are functional.

Let $P_i(t)$ be the probability that the system is in state i at time t based on the above Markov model. What is the availability of the system at time t?
(a) $e^{-3\lambda t}$
√ (b) $P_2(t) + P_3(t)$
(c) $1 - e^{-3\lambda t} + e^{-\mu t} - e^{-2\lambda t}$
(d) $P_0(t) + P_1(t)$
(e) cannot be calculated from the Markov model.

9. Continue from the same Markov model. What is the reliability of the system at time t?
(a) $e^{-3\lambda t}$
(b) $P_2(t) + P_3(t)$
(c) $1 - e^{-3\lambda t} + e^{-\mu t} - e^{-2\lambda t}$
(d) $P_0(t) + P_1(t)$
√ (e) cannot be calculated from this Markov model.

10. Continue from the same Markov model. Suppose we assign a reward of zero to states 1 and 0, and a reward of one to states 3 and 2. The expected cumulative reward E[Y(t)] under this reward assignment is equal to
(a) the mean down time in [0,t]
√ (b) the mean up time in [0,t]
(c) the mean number of jobs which the system can service during [0,t]
(d) the mean time to failure
(e) the availability of the system at time t

11. Consider an open queueing network system shown below that has a product-form solution. As shown in the figure, the external arrival rate to the system is 6 jobs/sec and the service rates at the two centers are 20 jobs/sec and 8 jobs/sec, respectively. When a job departs center 1, it completes its service (and thus leaves the system) with probability 0.6 or conversely goes to center 2 with probability 0.4. When a job departs center 2, it goes to center 1 with probability 1. Which of the following is false for the system described above?
(a) throughput at center 2 is 4 jobs/sec
(b) population at center 2 is 1
√ (c) response time at center 1 per visit is 1/5 sec
(d) response time at center 2 per visit is 1/4 sec
(e) system response time per job is 1/3 sec



12. Consider the closed QNM below consisting of a terminal center (T) with M terminals and a central subsystem (F, C, D and P). Suppose that the visit counts to centers T, F, C, D, P by a client are v_T, v_F, v_C, v_D, and v_P, respectively, with $v_T = 1$. The average response times per job per visit at centers T, F, C, D, P are r_T, r_F, r_C, r_D, and r_P, respectively. Which one of the following is not true?

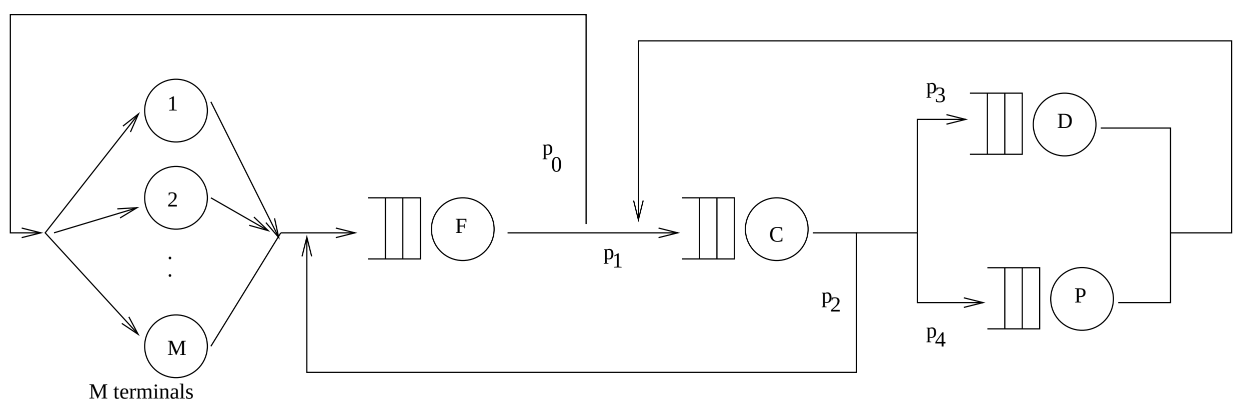
(a) the throughput of the terminal center is $M/(v_T r_T + v_F r_F + v_C r_C + v_D r_D + v_P r_P)$

√ (b) the throughput of the central subsystem is $P_0 \times M/(v_T r_T + v_F r_F + v_C r_C + v_D r_D + v_P r_P)$

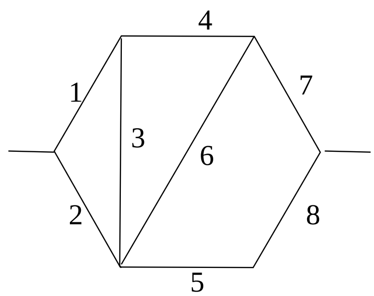
(c) the throughput of center D is $v_D \times M/(v_T r_T + v_F r_F + v_C r_C + v_D r_D + v_P r_P)$

(d) the average amount of time a terminal job stays in the central subsystem is $v_F r_F + v_C r_C + v_D r_D + v_P r_P$

(e) $v_C = P_1 v_F + v_D + v_P$



Part II : Modeling (40%)



1. (20 points.) Consider a system with 8 components connected in a structure as shown in the above block diagram.

   (a) Identify the minimal cut sets for this system.

   (b) Draw a fault tree model based on the minimal cut sets identified above for the purpose of calculating the reliability/availability of the system. Label your fault tree nodes and component names properly.

   Ans:

   (a) the minimal cut sets are: {1,2}, {5,7}, {7,8}, {2,3,4}, {4,5,6}, {4,6,8}, {1,3,5,6}, {1,3,6,8}, {2,3,6,7}.

   (b) A diagram is not shown here but the fault tree is an OR gate on top with 9 AND gates at the bottom each covering a mimimal cut set above.

2. (20 points.) Suppose that a network switch center has $n = 3$ slots to accommodate incoming high and low-priority clients, with arrival rates of λ_h and λ_l and departure rates of μ_h and μ_l, respectively. All clients will each occupy one full slot, regardless of class. Draw a Markov state transition diagram for modeling the following resource control policy:

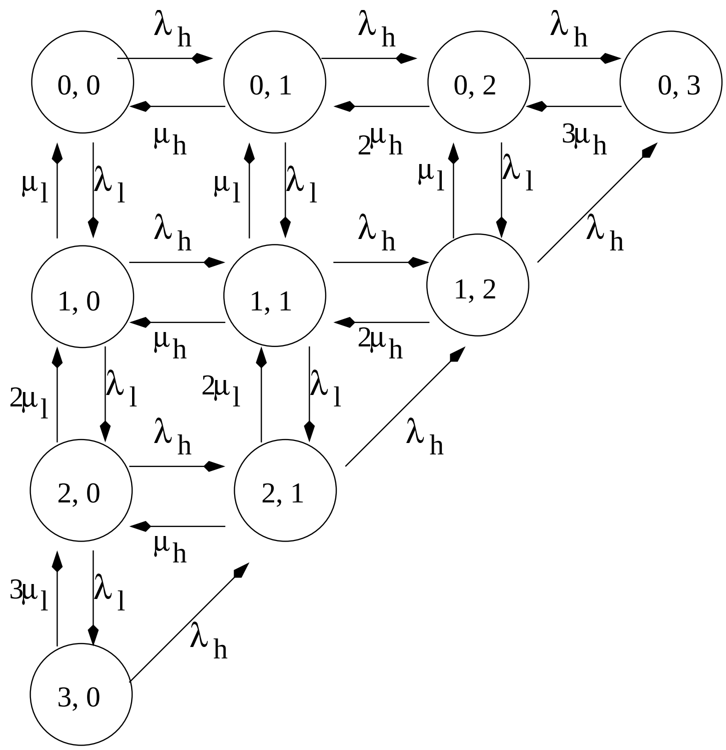
   - If there is at least one slot available, an incoming client always occupies an empty slot, regardless of its priority class.
   - If case all slots are filled, an incoming high-priority client can preempt a low-priority client if it is available, after which the high-priority client occupies one slot and the preempted low-priority client is forced to abort prematurely. If no low-priority client is available, the incoming high-priority client will be rejected.
   - If case all slots are filled, an incoming low-priority client will be rejected.

   Use the representation (a, b) where a stands for the number of low-priority clients, and b stands for the number of high-priority clients. Organize the Markov model so that when a high priority client arrives, the transition goes right; and when a low priority client arrives, the transition goes down. Label the transition rate of each transition clearly.

   Suppose $P_{(a,b)}$ is the steady state probability that the system is in state (a, b). Give mathematical expressions using $P_{(a,b)}$ for calculating the following performance metrics:

   - rejection probability of high-priority clients;
   - abortion probability of low-priority clients;
   - throughput of low-priority clients.

Ans:



The rejection probability of high-priority clients is $P_{(0,3)}$.

The abortion probability of low-priority clients is given by:

$$P_{(3,0)} + P_{(2,1)} + P_{(1,2)}$$

The throughput of low-priority clients is given by:

$$\mu_l P_{(1,0)} + 2\mu_l P_{(2,0)} + 3\mu_l P_{(3,0)} + \mu_l P_{(1,1)} + 2\mu_l P_{(2,1)} + \mu_l P_{(1,2)}$$