I pledge that this test has been completed in compliance with the Graduate Honor Code and that I have neither given nor received any aid on this test.

Student Name: ____________________________________________________________

Multiple Choice Problems. Select one correct answer in each problem. Problems #1-#13 are each worth 5 points. Problems #14-#18 are each worth 7 points.

d 1. Suppose we observe a system for 2000 minutes. Within this period, the busy time of the system is 500 minutes, the total number of customers completed is 10000 and the average number of customers in the system is 3. Which of the following is not true?
   (a) The throughput of the system is 5 customers/minute
   (b) the wait time per customer is 0.55 minute
   (c) The service time (not including the wait time) per customer is 0.05 minute
   (d) The utilization of the system is 0.2
   (e) the response time per customer is 0.6 minute

a 2. Consider a phone booth servicing customers arriving at a rate of 10 customers/hour. Suppose the average duration of a phone conversation is 2 minutes. What is the average wait time (just the queuing time) per customer at the phone booth?
   (a) 1 min
   (b) 2 min
   (c) 3 min
   (d) 4 min
   (e) 5 min

e 3. Consider an M/M/2/5 queueing facility with the customer arrival rate being \( \lambda \) and the service rate per server being \( \mu \). Let \( P_i \) stand for the steady-state probability that the system contains \( i \) customers (including both queueing and in service). Which one of the following is not true?
   (a) the average number of customers is \( \sum_{i=1}^{5} (i \times P_i) \)
   (b) the average rejection rate is \( P_5 \times \lambda \)
   (c) the throughput is \( (1 - P_5) \times \lambda \)
   (d) the throughput is \( P_1 \times \mu + (P_2 + P_3 + P_4 + P_5) \times 2\mu \)
   (e) the response time per customer not rejected is \( \frac{1}{\mu} \)
4. A single server queuing system obeying exponential failure law has a constant failure rate of 0.1 hr$^{-1}$. Assume that customers arrive at the server system with a rate of 10 customers/hour and the server is able to service customers at a rate of 12 customers/hour. Which one of the following is true?
(a) The throughput of the system before it fails is 12 customers/hour
(b) The reliability of the system $1 - e^{-0.1}$ at time = 1 hour
(c) The expected number of customers the system can serve before it fails is 100 customers
(d) The average population in the system before it fails is 10 customers
(e) The mean time to failure of the system is 12 hours

5. Consider a perfect parallel system (or a 1 out of 3 system) consisting of three identical components with each component having an independent failure rate of $\lambda$ and an independent repair rate of $\mu$. Which of the following statements is true?
(a) The system reliability at time $t$ is $1 - (1 - R(t))^3$ where $R(t) = e^{-\lambda t}$
(b) The system reliability at time $t$ is $R^3(t)$ where $R(t) = e^{-\lambda t}$
(c) The system availability at time $t$ is $1 - (1 - A(t))^3$ where $A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$
(d) The system availability at time $t$ is $A^3(t)$ where $A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$
(e) The reliability of the system can be calculated by using a fault tree model.
6. For the same 1 out of 3 system in the last problem, consider a Markov model shown above for the purpose of reliability modeling. Let \( P_3(t), P_2(t), P_1(t), \) and \( P_0(t) \) be the probabilities that the system is in states 3, 2, 1 and 0, at time \( t \), respectively. Which one of the following is false for this Markov model?

(a) \( P'_3(t) = -3\lambda P_3(t) + \mu P_2(t) \)
(b) \( P'_2(t) = 3\lambda P_3(t) - (2\lambda + \mu)P_2(t) + 2\mu P_1(t) \)
(c) \( P'_0(t) = \lambda P_1(t) \)
(d) the reliability of the system at time \( t \) is \( P_3(t) + P_2(t) + P_1(t) \)
(e) the availability of the system at time \( t \) is \( 1 - P_0(t) \)

7. Continue from the same Markov model. Suppose we assign a reward of one to state 0, and a reward of zero to states 3, 2 and 1. The expected instantaneous reward rate \( E[Z(t)] \) under this reward assignment is equal to

(a) the reliability of the system at time \( t \)
(b) the unreliability of the system at time \( t \)
(c) the mean time to failure of the system
(d) the mean up time in \([0,t]\)
(e) the availability at time \( t \)

8. Continue from the same Markov model. Suppose we assign a reward of zero to state 0, and a reward of one to states 3, 2 and 1. The expected cumulative reward until absorption \( E[Y(\infty)] \) under this reward assignment is equal to

(a) the mean down time in \([0,t]\)
(b) the mean up time in \([0,t]\)
(c) the mean number of jobs which the system can service during \([0,t]\)
(d) the mean time to failure
(e) the reliability at time \( t \)
9. For the same 1 out of 3 system in the last problem, consider an irreducible Markov model shown above for the purpose of availability modeling. Let $P_3(t)$, $P_2(t)$, $P_1(t)$, and $P_0(t)$ be the probabilities that the system is in states 3, 2, 1 and 0, at time $t$, respectively. Which one of the following is false for this Markov model?
(a) $P'_3(t) = -3\lambda P_3(t) + \mu P_2(t)$
(b) $P'_2(t) = 3\lambda P_3(t) - (2\lambda + \mu)P_2(t) + 2\mu P_1(t)$
(c) $P'_0(t) = \lambda P_1(t) - 3\mu P_0(t)$
(d) the reliability of the system at time $t$ is $P_3(t) + P_2(t) + P_1(t)$
(e) the availability of the system at time $t$ is $1 - P_0(t)$

10. Continue from the same Markov model. Suppose we assign a reward of zero to state 0, and a reward of one to states 3, 2 and 1. The expected cumulative reward $E[Y(t)]$ under this reward assignment is equal to
(a) the mean down time in $[0,t]$  
(b) the mean up time in $[0,t]$  
(c) the mean number of jobs which the system can service during $[0,t]$  
(d) the mean time to failure  
(e) the availability of the system at time $t$
11. Which of the following statements is false for the fault tree model shown above.
   (a) The system fails when components 4 and 6 fail
   (b) The system fails when components 1 and 3 fail
   (c) The system fails when components 3 and 4 fail
   (d) The system fails when component 2 fails
   (e) The system is alive when components 2 and 4 are alive.
12. The client-server system shown above has $m$ clients in the client subsystem and $n$ servers in the server subsystem with $m > n$. The arrival rate of each of the $m$ clients is $\lambda$ and the service rate of each of the $n$ servers is $\mu$. Let $P_i$ be the steady state probability of $i$ clients in the server subsystem. Which of the following is not true?

(a) The system throughput is $n - \sum_{i=0}^{n-1} i \mu P_i + m \sum_{i=n}^{n} n \mu P_i$

(b) The system throughput is $m \lambda$

(c) The population in the server subsystem is $\sum_{i=0}^{n} i P_i$

(d) The response time in the client subsystem is $\frac{1}{\lambda}$.

(e) The response time in the server subsystem is the population in the server subsystem divided by the system throughput.

13. Suppose that a closed system has two classes of jobs A and B. It is known that the response time of a class A job is $R_A = 15$, the response time of a class B job is $R_B = 20$, the throughput for class A jobs is $X_A = 6$ and the throughput for class B jobs is $X_B = 4$. What is the average response time per job regardless of the job class? (a) 15 (b) 16 (c) 17 (d) 18 (e) 19
14. Consider an open queueing network system shown below has a product-form solution. As shown in the figure, the external arrival rate to the system is 2 jobs/sec and the service rates at the two centers are 20 jobs/sec and 12 jobs/sec, respectively. When a job departs center 1, it goes to center 2 with probability 1. When a job departs center 2, it completes its service (and thus leaves the system) with probability 0.2 or conversely goes to center 1 with probability 0.8. Which of the following is false for the system described above?
(a) throughput at center 1 is 10 jobs/sec
(b) throughput at center 2 is 10 jobs/sec
(c) response time at center 1 per visit is 0.1 sec
(d) response time at center 2 per visit is 0.6 sec
(e) response time is 3 sec
15. Consider the closed QNM below containing a terminal subsystem consisting of a single terminal center T with M terminals and a central subsystem consisting of centers F, C, D and P. Suppose that the visit counts to centers T, F, C, D, P by a client are \( v_T \), \( v_F \), \( v_C \), \( v_D \), and \( v_P \), respectively, with \( v_T = 1 \). The average response times per job per visit at centers T, F, C, D, P are \( r_T \), \( r_F \), \( r_C \), \( r_D \), and \( r_P \), respectively. Which one of the following is not true? 

(a) the throughput of the terminal center is \( \frac{M}{v_T r_T + v_F r_F + v_C r_C + v_D r_D + v_P r_P} \) 
(b) the throughput of center D is \( v_D \times \frac{M}{v_T r_T + v_F r_F + v_C r_C + v_D r_D + v_P r_P} \) 
(c) the throughput of the central subsystem is \( P_0 \times \frac{M}{v_T r_T + v_F r_F + v_C r_C + v_D r_D + v_P r_P} \) 
(d) the average amount of time a terminal job stays in the central subsystem is \( v_F r_F + v_C r_C + v_D r_D + v_P r_P \) 
(e) the population of center F is \( r_F \times v_F \times \frac{M}{v_T r_T + v_F r_F + v_C r_C + v_D r_D + v_P r_P} \)

\[ \begin{array}{c} 1 \\ 2 \\ \vdots \\ M \end{array} \]
16. Consider a system with 8 components connected in a structure as shown in the above block diagram. What are the minimal path sets for this system?
(a) (1,4,7), (2,5,8)
(b) (1,3,5,8), (1,4,6,8), (2,3,4,7), (2,5,6,7)
(c) (1,4,7), (2,5,8), (1,3,5,8), (1,4,6,8), (2,3,4,7), (2,5,6,7)
(d) (1,4,7), (2,5,8), (1,3,5,8), (1,4,6,8), (2,3,4,7), (2,5,6,7), (1,3,5,6,7), (2,3,4,6,8)
(e) (1,3,5,8), (1,4,6,8), (2,3,4,7), (2,5,6,7), (1,3,5,6,7), (2,3,4,6,8)
17. Refer to the Markov model for HW #2, problem #3 as posted on the class website. If we assign rewards to states of the Markov model based on the above table, the expected reward at steady state $E[Z(\infty)]$ under this reward assignment is equal to:

(a) the average number of low-priority, high-QoS clients
(b) the average number of low-priority, low-QoS clients
(c) the average number of high-priority clients
(d) the rejection probability of low-priority clients
(e) the rejection probability of high-priority clients
b 18. Refer to the $\text{M/M}/3/8$ Markov model for HW #2, problem #1 as posted on the class website. If we assign rewards to states of the Markov model based on the above table (where $\mu$ is the service rate per server), the expected cumulative reward over $[0,t]$, i.e., $E[Y(t)]$, under this reward assignment is equal to:

(a) expected throughput at time $t$
(b) expected number of customers completed over $[0,t]$
(c) reliability at time $t$
(d) mean time to failure
(e) expected system up time during $[0,t]$