Developing Model-Based Reasoning in Mathematics and Science

Richard Lehrer and Leona Schauble
University of Wisconsin

It is essential to base instruction on a foundation of understanding of children's thinking, but it is equally important to adopt the longer-term view that is needed to stretch these early competencies into forms of thinking that are complex, multifaceted, and subject to development over years, rather than weeks or months. We pursue this topic through our studies of model-based reasoning. We have identified four forms of models and related modeling practices that show promise for developing model-based reasoning. Models have the fortuitous feature of making forms of student reasoning public and inspectable—not only among the community of modelers, but also to teachers. Modeling provides feedback about student thinking that can guide teaching decisions, an important dividend for improving professional practice.

We study the early emergence and subsequent development of model-based reasoning in mathematics and science. In their book *Fearful Symmetry*, Stewart and Golubitsky (1992) effectively explained the significance of this form of thinking:

> Scientists use mathematics to build mental universes. They write down mathematical descriptions—models—that capture essential fragments of how they think the world behaves. Then they analyze their consequences. This is called 'theory.' They test their theories against observations: this is called 'experiment.' Depending on the result, they may modify the model and repeat the cycle until theory and experiment agree. Not that it's really that simple, but that's the general gist of it, the essence of the scientific method (p. 2).

Like others, we have noted the discrepancy between this description and typical school science and mathematics, especially at the elementary grades. If Stewart and

---

Direct all correspondence to: Richard Lehrer or Leona Schauble, Department of Educational Psychology, University of Wisconsin, Education Sciences 1071, 1025 West Johnson Street, Madison, WI 53706 <rlehrer@facstaff.wisc.edu> or <schauble@facstaff.wisc.edu>.
Both authors contributed equally.
This research was supported by The James S. McDonnell Foundation and by a grant from The U.S. Department of Education, Office of Educational Research and Improvement, to The National Center for Improving Student Learning and Achievement in Mathematics and Science (R305A60007).

Journal of Applied Developmental Psychology 21(1): 39–48
ISSN: 0193-3973

Golubitsky are correct, it is a distortion of mathematics and science to present them as bodies of finished knowledge, rather than as activities centered around the ongoing production of knowledge. Modeling should be central from the earliest years of instruction, not postponed until high school or beyond. Consistent with this perspective, we have been working closely with elementary school teachers in four schools in a district near Madison, Wisconsin, who are reorienting their science and mathematics instruction from computation of algorithms and learning of "facts" to an approach that emphasizes the construction, evaluation, and revision of models.

At least one person has suggested to us that children are natural modelers. We agree—in part. It is certainly true, borne out richly by the developmental literature, that there are myriad ways in which even very young children come to regard one thing as representing another. Consider, for example, deLoache's (1989) work on children's understanding of pictures and scale models, Gentner and Toupin's (1986) work on analogy, and Leslie's (1987) work on pretend play. Collectively, this work illuminates the impressive cognitive achievement that happens when a child pretends that a banana is a telephone, elaborates the function and use of the banana as a telephone, and yet knows very well that after all, it remains just a banana. These same early forms of competence also play out more specifically in the origins of model-based reasoning. Even preschoolers can use counters for "direct modeling" to solve simple early number problems involving grouping and separating. Increasingly, instructional programs are capitalizing on these features of young children's thinking (Carpenter & Fennema, 1992).

But these early foundations do not entail all the features of "model" that are needed for developing a deep understanding of mathematics and science. Using one system to represent another is certainly part of what constitutes modeling. But what we will call Big-M modeling also includes the self-conscious separation of a model and its referent, the explicit consideration of measurement error, and the understanding, based on analysis of model-world residuals, that alternative models are possible and may in fact be preferable. Nor are young children typically aware of the role of rival models in evaluating alternative hypotheses. In sum, it is essential to base instruction on a foundation of understanding of children's thinking, but it is equally important to adopt the longer-term view that is needed to stretch these early competencies into forms of thinking that are complex, multifaceted, and subject to development over years, rather than weeks or months.

Although children spontaneously use objects to "stand in" for others, they don't necessarily select representations for the purpose of highlighting objects and relations that are theoretically important. They need experience with conventionalizing those representations in inscriptions or notations, that is, with using representational artifacts that are adopted as matters of convention for supporting the reasoning of a practicing community. Modeling must be practiced systematically so that the forms and uses of a variety of models are explored and evaluated. Lesh (personal communication, June, 1995) suggested that being a good modeler is in large part a matter of having a number of fruitful models in your "hip pocket." Acquiring such a collection almost certainly requires sustained work in a context where modeling has a purpose and a payoff.

Because these forms of thinking do not progress very far without explicit instruc-

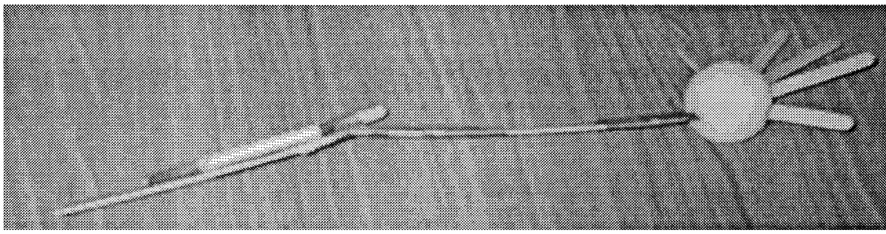

Figure 1. Child's Model of the Elbow.

tion and fostering, the study of model-based reasoning must be pursued in contexts that give it a central place. That is why we are working with teachers simultaneously to foster and study it. Interestingly, the focus on models helps with the teacher agenda, too. Models have the fortuitous feature of making forms of student reasoning public and inspectable—not only among the community of modelers, but also to teachers. Modeling provides feedback about student thinking that can guide teaching decisions, an important dividend for improving professional practice.

Because we are interested in the development of these forms of reasoning, and not their full-blown practice as realized in the professional work of scientists and mathematicians, we collaborate with teachers to establish a variety of modeling practices that educate children about the nature and functions of models. We have identified four forms of models and related modeling practices that show promise for developing model-based reasoning (Lehrer, Schauble, & Penner, 1995).

Forms of Modeling

Physical Models

Physical models, like models of solar systems or elbows, are microcosms of systems that draw heavily on children's intuitions about resemblance to sustain the relationship between the world being modeled and the model itself. Figure 1 displays a child's model of the elbow. Note the inclusion of features like "fingers" (the wooden Popsicle sticks) and the protrusion of the elbow (the Styrofoam ball). Although the model maps iconically from the world to the model, it also embodies hypotheses about unseen function. Note, for instance, the rubber bands that mimic the connective function of ligaments and the wooden dowels that are arranged so that their translation in the vertical plane cannot exceed 180 degrees. Although the search for function is supported by initial resemblance, what counts as resemblance typically changes as children revise their models. For example, attempts to make models exemplify elbow motion often lead to an interest in the way muscles may be arranged.

Representational Models

Representational models, like maps, diagrams, and related forms of display notations, are often based at first on overt resemblance. However, extended work

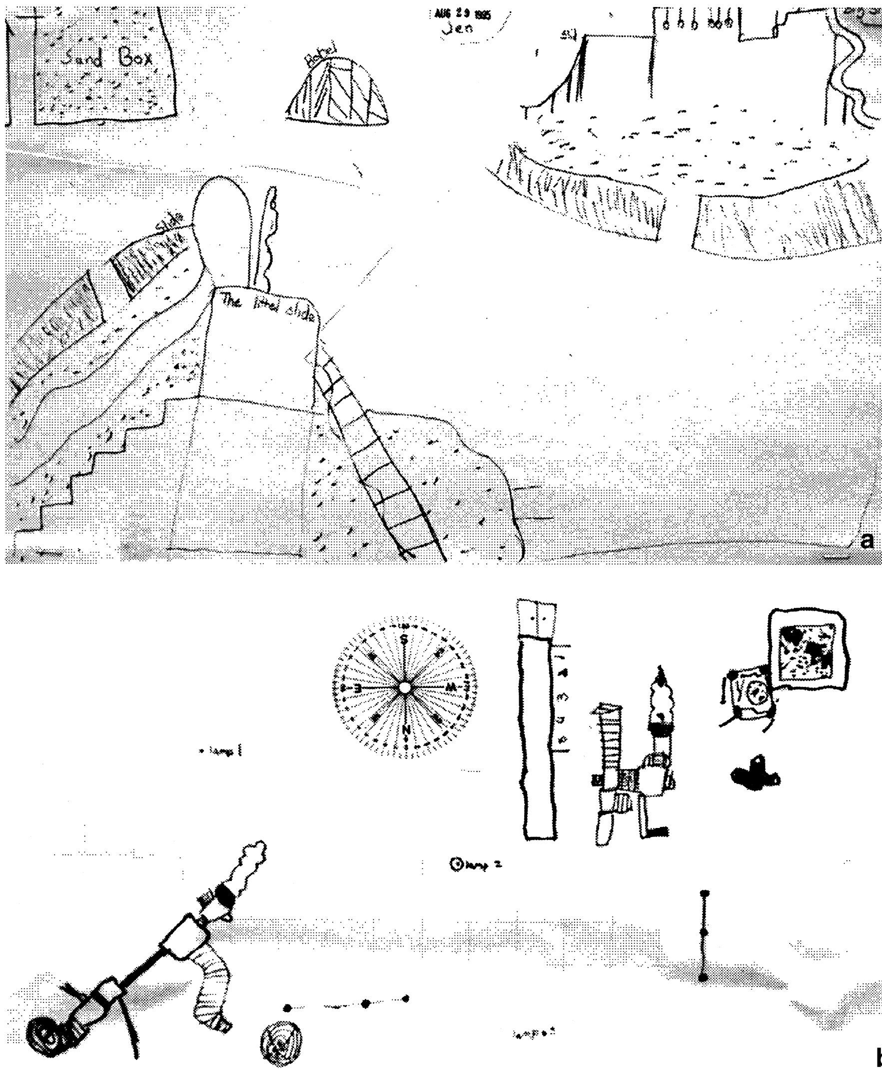

Figure 2. Two "Maps" of the Schoolyard Developed by a Third Grade Child.

with them typically fosters children's understanding of the need to establish conventions that make explicit the relationship between the model and the world (Lehrer, Jacobson, Kemeny, & Strom, 1999). For example, Figure 2 displays two "maps" of the school yard developed by a third-grade child. Figure 2a is more like a drawing. It conveys a picture of the playground anchored in this child's interest in a piece of apparatus. (In this sense, the child's view of the playground echoes the Manhattanite's view of the world first published in *The New Yorker*.) On further consideration of the large-scale structure of the playground, the child revised his map (Figure 2b) to reflect a more accurate but also more conventional view of the configuration of the playground space. Note the use of scale and the establishment of a simple polar coordinate system. Knowledge of these conventions helps establish the map as a microcosm of the space. Without them, we could take the revision as artistic expression, rather than model.

Syntactic Models

Syntactic models exchange resemblance for analogy. In a syntactic model, the epistemological claim is that one system functions much like another. In our classrooms, these analog systems range from Logo (e.g., Does a program describing recursive growth of a quadrilateral provide a good model of the growth of a chambered nautilus?) to coin flips (e.g., Is a bird's preference for a type of seed modeled by a random coin flip, or can the bird be making a decision?). Although this form of modeling may appear to be exotic for elementary grade students, we believe it is necessary for grasping important ideas like biologic diversity, animal foraging, and other natural phenomena in which understanding is dependent on sampling models.

Hypothetical-Deductive Models

Hypothetical-deductive models, the kind referred to by Stewart and Golubitsky, move beyond the realm of describing the observable. These models embody unseen hypothetical entities that interact to produce emergent behavior. This form of modeling is particularly difficult for children, partly because there is no direct connection between any single hypothetical entity and the observed world, and partly because the very idea of emergence may be problematic for people of all ages. For example, the kinetic theory of gases invokes hypothetical entities—molecules—that do not map directly onto the behavior of a gas. Yet, when they interact in the manner of Newtonian billiard balls, molecules account for relationships observed among temperature, pressure, and volume. Interestingly, although we have ways of indexing and measuring these variables, none of them is directly observed, either.

We have begun to approach this form of modeling gingerly but straightforwardly, using Resnick's (1995) ideas of agency. For example, in a classroom of second-grade children investigating termite mound construction, each child acted like a termite. Carrying slips of paper to represent particles and obeying a simple set of rules, children were surprised to find that after a number of cycles, the random interaction of many agents (each child in the classroom) resulted in the emergence of piles of paper analogous to termite mounds.

Lessons Learned

We have been working for 2 years to introduce these four forms of modeling practices into elementary school classrooms. This work provides us with a research context for studying the development of model-based reasoning in young children. Here, briefly, are some of the major things we have learned so far in this work.

New Forms of Mathematics

Modeling relies on forms of mathematics that are not typically taught in elementary school. These include spatial reasoning and geometry, uncertainty and probability, measure, data and data structure, and ways of thinking about function, such as rates of change. Interestingly, the National Council of Teachers of Mathematics has been arguing convincingly that these noncomputational forms of mathematics must be introduced much earlier in the curriculum and in an integrated fashion, not as isolated mathematical topics.

Classroom Roles of Tools, Talk, and Notation

A classroom focus on modeling highlights events and artifacts that embody concrete examples of the theoretical concepts and relations of interest. These events and artifacts may include classroom tools (from rulers to computers), notations and inscriptions (from “number sentences” to graphs and diagrams), and talk (especially modes and means of argumentation). As just one example, students’ use of classroom tools like rulers can be very revealing of their emerging theories of measure. A ruler encapsulates a number of concepts about measure that adults take for granted but that children apparently do not—for example, that measure entails use of equal-size units, iteration without overlap or gaps, and a zero point. The work of Lehrer, Jenkins, and Osana (1998) with primary grade children shows that children’s work with rulers often obscures these fundamental ideas because the power of the tool makes these concepts transparent to users. Interestingly, for all the “measuring” practice that occurs in elementary school, these underlying concepts are rarely explicitly discussed or debated, and many students fail to understand their importance until quite late in elementary school, if then.

Modeling in Mathematics and Science

Modeling in mathematics and modeling in science have some interesting characteristic differences and entail different kinds of hurdles for children. Our work to date suggests that for children, modeling in science raises some additional demands over modeling in mathematics. For one thing, children’s beliefs about cause and mechanism can support effective modeling but can also interfere on occasion with data-based reasoning. Students often have difficulty reasoning about relations between cases, with their local properties, and the larger patterns in which those cases participate (Lehrer & Romberg, 1996). For example, children sometimes focus unduly on one particularly salient object or event, ignoring others that bear equally on a question or issue under investigation. They often interpret patterns of data quite differently when those data confirm or disconfirm their favored beliefs (Kuhn, Amsel, & O’Loughlin, 1988; Schauble, 1990, 1996). Moreover, children find especially challenging the failure of models to fit exactly the phenomena being modeled.

This past year, a class of third graders was working on ideas about geometric similarity. The children constructed a graph on which they plotted the circumference and height of a variety of “families” of cylinders. (Along the way they debated a number of related issues about graphical conventions, such as what the scale should be and where the origin should be located and why.) Each “family” of similar cylinders was eventually represented as a series of points that fell on a straight line, and the third graders argued that the lines could be considered a “system” for generating all possible members of a family of cylinders. Soon after, the children made another graph to plot the relationship between weight and volume for 36 objects made of 4 different materials. Generating values for the points on the graph was not a trivial feat, because these young children had to estimate the volumes of the objects by finding or estimating the surface area of the bases, then considering the three-dimensional solids as “layers” of *N* height. This approach was reasonably precise with rectangular prisms, but generated more error with cylinders and spheres. The children’s original intention was to explore whether the 36 objects could be grouped into “families” of brass, aluminum, Teflon, and wood, but they were highly disconcerted by the fact that some of their plotted points did not fall exactly on the line. In fact, there was some slight mismatch between the expected values and observed values, and some children were prepared to discard the hypothesis of “families” of materials. Several children eventually noted that the discrepancies between the line and the points representing each object were related to measurement error, so the possibility of parsimonious description was preserved. However, the object lesson for us was in considering the potential epistemological divide between modeling in mathematics and modeling in science.

Cycles of Modeling

It is important to ensure that the cyclical property of modeling is preserved in classroom practice. Modeling does not end with the first model; instead, students must evaluate models, especially when several alternatives are “on the table,” and revise them, an activity that frequently initiates a new round of inquiry. Work in our lab (Penner, Giles, Lehrer, & Schauble, 1997) suggested that even first-grade students can come to understand the value of repeated iterations and revisions of modeling. When they evaluate a model against its referent, children receive explicit feedback about how and whether their ideas are working; inevitably, new questions are raised. Not just within a model-construction cycle, but across grades, models must cumulate and go somewhere. A model that is explored in first grade should have “pay-off” beyond its original context of development. Educators must become aware of the kinds of models most likely to be useful across a variety of situations—in fact, this may be one basis for evaluating and pruning the elementary science curriculum, which is presently too broad and too shallow in its content focus.

Choosing Situations With Interesting Problems

Inscriptions, models, and other symbols are powerful, but if they are not introduced with discretion, their very power can lead to obfuscation. Because models condense a history of cognitive work into a relatively compact inscription, diagram, or formula, they can render invisible the history of cognitive work that created

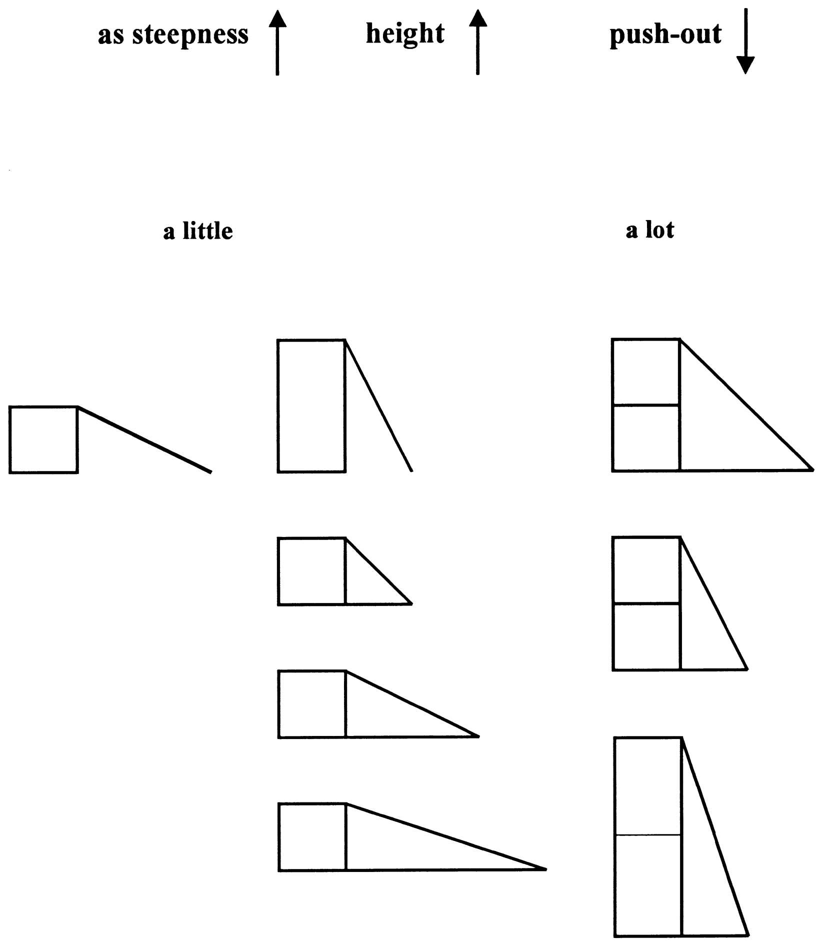

Figure 3. A Series of Right Triangles to Illustrate the Concept of Steepness.

them. An excellent example, mentioned above, is the ruler. Rulers encapsulate a history of decisions about measure. Because these decisions are built into the tool, they also effectively hide from teachers whether children understand the basis for those decisions. For example, it is difficult to tell whether a student understands the need for constant iteration, because rulers incorporate a solution to that problem in their very design. We are not arguing here that children should reinvent civilization, but we have learned the need for careful consideration before providing children with solutions to problems that they do not yet regard as problems. It may be a mistake to give models or other symbolic artifacts to students before the need for them has "ripened" in the classroom and is widely apparent to students; the result may be manipulation of the symbols without understanding.

Recently we asked second graders to look carefully at the inclined planes they were using for experiments with rolling objects (Lehrer, Schauble, Carpenter, & Penner, in press). One of us asked if the planes of various lengths and steepness reminded the children of a geometric figure. Although most of the children looked bewildered, one child announced that the planes reminded him of triangles. Around the room, light bulbs went on: "Triangles! That's what I thought, too!" We next asked the children to draw what an inclined plane looks like when it "isn't very steep." One by one, children came to the blackboard and drew right triangles to illustrate the concept of steepness. As Figure 3 shows, over a series of drawings, children increasingly emphasized the distance between the bottom edge of the plane and the crates propping up the board—the dimension that the children called "the amount of push-out."

This discussion seemed very fruitful, so we tried to repeat it with a new instructional group. However, in the course of doing so, we made an error that turned out to be disastrous. Instead of asking children what geometric figure the planes looked like, as we had with the first group, we asked these children to "draw us a triangle that shows what the plane looks like when it isn't very steep." They drew triangles, but all of them were prototypical equilateral or isosceles triangles sitting on their bases. In our view, these triangles do not represent anything in the situation at all; they are just triangles. Lacking a clear understanding of the problem that we intended these inscriptions to solve, the children instead defaulted to an activity that they understood: drawing triangles.

In sum, we believe that although children readily adopt many kinds of symbolic systems for purposes of representing, these early forms of competence are only the beginning of a long journey. To understand the journey, we must attend not only to recognizing the kinds of work accomplished by the travelers, but also to mapping out appropriate roadways, and especially to engineering forms of teaching and other supports that keep the trajectory on course. Perhaps amid this talk of highway and trajectory, we must keep in mind the back roads and byways, for our work may well lead to diversity and growth in forms of model-based reasoning not easily encapsulated in curriculum traditions that assume there is one "best sequence" to support children's learning. Better that than the road not taken.

References

Carpenter, T. P., & Fennema, E. (1992). Cognitive guided instruction: Building on the knowledge of students and teachers. In W. Secada (Ed.), Curriculum reform: The case of mathematics education in the U.S. *International Journal of Educational Research, 57*, 470.

DeLoache, J. S. (1989). The development of representation in young children. In H. W. Reese (Ed.), *Advances in child development and behavior* (Vol. 22, pp. 1–39). New York: Academic Press.

Gentner, D., & Toupin, C. (986). Systematicity and similarity in the development of analogy. *Cognitive Science, 10*, 277–300.

Kuhn, D., Amsel, E. D., & O'Loughlin, M. (1988). *The development of scientific thinking skills.* San Diego: Academic Press.

Lehrer, R., Jacobson, C., Kemeny, V., & Strom, D. (1999). Building on children's intuitions to develop mathematical understanding of space. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 63–87). Mahwah, NJ: Erlbaum.

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal study of children's conceptions of geometry.

In R. Lehrer & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 137–167). Hillsdale, NJ: Erlbaum.

Lehrer, R., & Romberg, T. (1996). Exploring children's data modeling. *Cognition and Instruction, 14*, 69–108.

Lehrer, R., Schauble, L., Carpenter, S., & Penner, D. (in press). *The inter-related development of inscriptions and conceptual understanding.* In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 325–360). Mahwah, NJ: Erlbaum.

Lehrer, R., Schauble, L., & Penner, D. (1995). *Modeling in mathematics and science.* Paper prepared for teachers, Wisconsin Center for Education Research, Madison, WI.

Leslie, A. M. (1987). Pretense and representation: The origins of "theory of mind." *Psychological Review, 94*, 412–426.

Penner, D., Giles, N. D., Lehrer, R., & Schauble, L. (1999). Building functional models: Designing an elbow. *Journal of Research in Science Teaching, 34*, 1–20.

Resnick, M. (1995). *Turtles, termites, and traffic jams.* Cambridge, MA: MIT Press.

Schauble, L. (1990). Belief revision in children: The role of prior knowledge and strategies for generating evidence. *Journal of Experimental Child Psychology, 49*, 31–57.

Schauble, L. (1996). The development of scientific reasoning in knowledge-rich contexts. *Developmental Psychology, 32*, 102–119.

Stewart, I., & Golubitsky, M. (1992). *Fearful symmetry: Is God a geometer?* London: Penguin Books.