Learning Probabilistic Models of Connectivity from Multiple Spike Train Data

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Multi-Neuronal Data

- Neuronal circuits carry out brain function through complex coordinated firing patterns.
- Inferring topology of neuronal circuits from spike train data is challenging and hard.

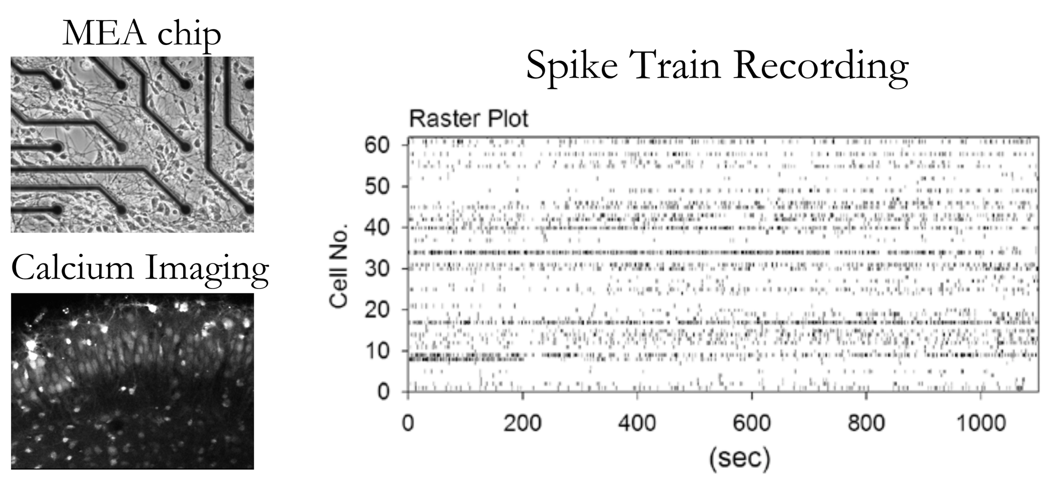
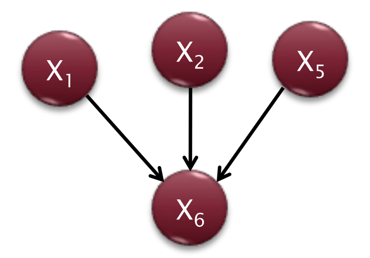


Figure 1: *Simultaneously recorded multi-neuron data*

Probabilistic Models

- Probabilistic Models, such as Bayesian Networks, provide compact factorizations of joint probability distributions.
- The probability of spiking of a neuron is conditioned on the activity of a subset of relevant neurons in recent past (or history window).
- Learning probability models from spike train data is a hard problem. Most efficient methods are heuristic.

$$\text{Prob}(X_1 \ldots X_n = x_1 \ldots x_n) = \prod_{i=1}^{n} \text{Prob}(X_i = x_i | \text{Parent}(X_i))$$



$Parent(X_6) = \{X_1, X_2, X_5\}$

Conditional Probability Table

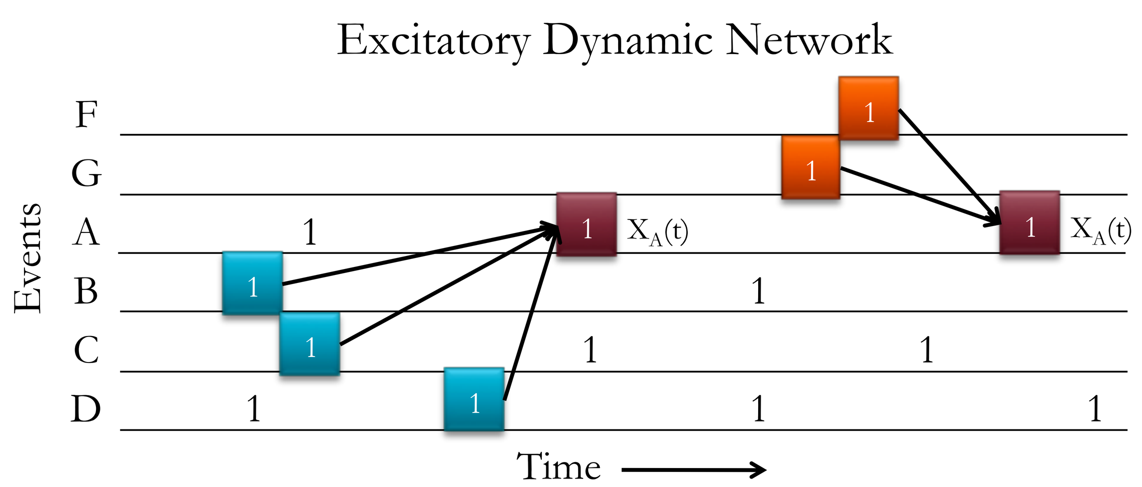
X_1	X_2	X_5	$P[X_6=0]$	$P[X_6=1]$
0	0	0	0.9	0.1
0	0	1	0.9	0.1
.	.	.	.	.
1	1	0	0.9	0.1
1	**1**	**1**	**0.1**	**0.9**

Figure 2: *A typical Bayesian Network*

Excitatory Dynamic Networks (EDNs)

We define a special class of models, *Excitatory Dynamic Networks*:

- Neurons can only exert excitatory influences on one another.



- $\text{Parent}(X_A(t))$
  - Ind. Parent Comp.
  - {B,C,D} excite A
  - {F, G} also excite A

B	C	D	F	G	Prob[A firing]
0	0	0	0	0	LOW
.	.	.	.	.	LOW
.	.	.	1	1	HIGH
1	1	1	.	.	HIGH

Figure 3: *Independent parent components in our Excitatory Dynamic Network (EDN) formulation.*

Method

Our emphasis on excitatory networks enables:

- Learning of connectivity models by exploiting fast and efficient data mining algorithms [2].

EDN Structure Learning

- Structure Learning requires identifying high mutual informal parent sets.
- We formally establish a connection between efficient frequent episode mining algorithms and learning probabilistic models for excitatory connections.
- Frequent Episode Mining is used to identify frequently repeating patterns of spiking activity [3].

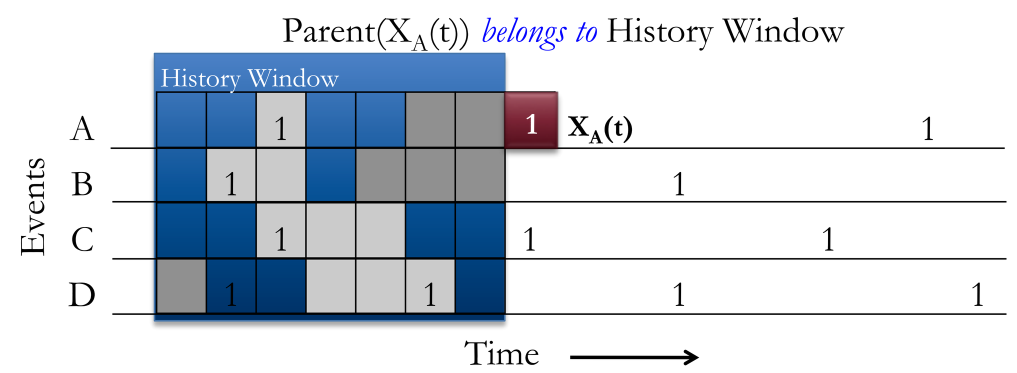


Figure 4: *Search for high mutual information parent set restricted to immediate history window.*

Theorem 1 *Consider node X_A in an excitatory DBN with parent-set Π. Let ϵ^* be an upper-bound for $P[X_A = 1 \mid \Pi = a]$ for all $a \neq a^*(= \mathbf{1})$. $I[X_A\,;\,\Pi] > \vartheta$ implies $P[X_A = 1, \Pi = a^*] \geq P_{min}\Phi_{min}$, where*

$$P_{min} = \frac{P[X_A = 1] - \epsilon^*}{1 - \epsilon^*} \quad (1)$$

$$\Phi_{min} = h^{-1}\left[\min\left(1, \frac{h(P[X_A = 1]) - \vartheta}{P_{min}}\right)\right] \quad (2)$$

and where $h(\cdot)$ denotes the binary entropy function $h(q) = -q\log q - (1-q)\log(1-q)$, $o < q < 1$ and $h^{-1}[\cdot]$ denotes its pre-image greater than $\frac{1}{2}$.

High Mutual Information Parent set

$$\text{I}(X_j(t); Parent(X_j(t)) \geq \theta$$

$$P[Parent(X_j(t) = \vec{1}; X_j(t) = 1]$$

$$\text{support} > P_{min}\phi_{min}$$

B	C	D	P[A=1|Parents]
0	0	0	$< \epsilon^*$
0	0	1	$< \epsilon^*$
0	1	0	$< \epsilon^*$
0	1	1	$< \epsilon^*$
1	0	0	$< \epsilon^*$
1	0	1	$< \epsilon^*$
1	1	0	$< \epsilon^*$
{B=1, C=1, D=1}			$P(A=1) > \phi$

Figure 5: *Search for high mutual information parent sets translates to finding frequent episodes.*

Frequent Episode Mining

Serial Episodes: Patterns of the form $\langle B \xrightarrow{1} C \xrightarrow{2} D \xrightarrow{2} A \rangle$

Frequent: $\sigma(B \xrightarrow{1} C \xrightarrow{2} D \xrightarrow{2} A) = \frac{count}{T} > \vartheta$ supp. threshold

Efficient Algorithm: Level-wise mining
Candidate generation → Counting → Retain frequent episodes.

Counting: Maximum number of non-overlapped occurrences.

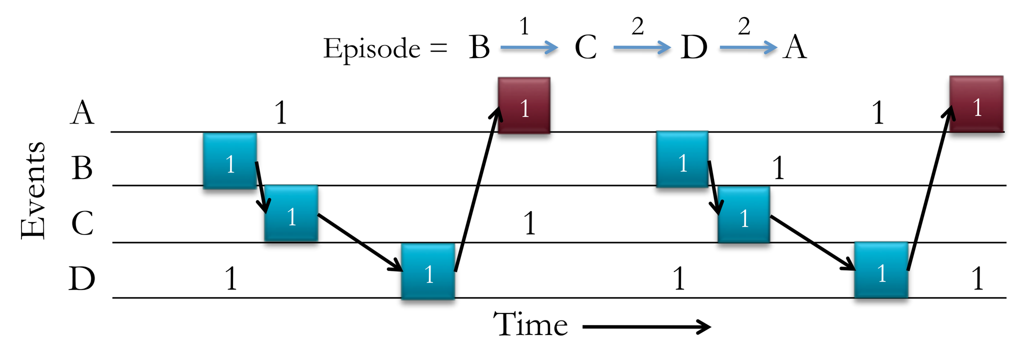


Figure 6: *Frequent Episode Mining - Fast and efficient data mining algorithm.*

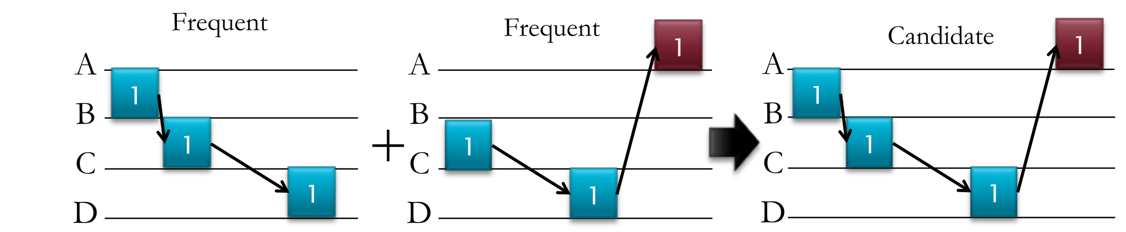


Figure 7: *Level-wise candidate generation in frequent episode mining.*

Results

Synthetic Data Generation

- Synthetic data generation models each neuron as an *Inhomogeneous Poisson Process.*
- Firing rate is modulated by the spikes received by neuron in recent past.

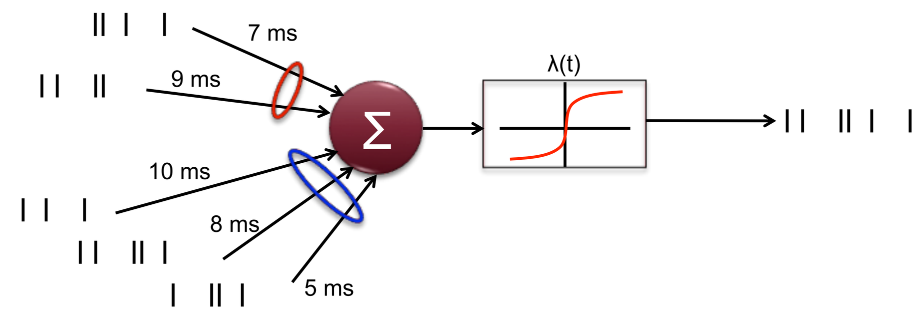


Figure 8: *Simulation Model of a single neuron.*

Synfire Chains

A volley of firing in one group of neurons causes next group to fire and activity propagates over the network. The gray boxes show the MEA view of the activity.

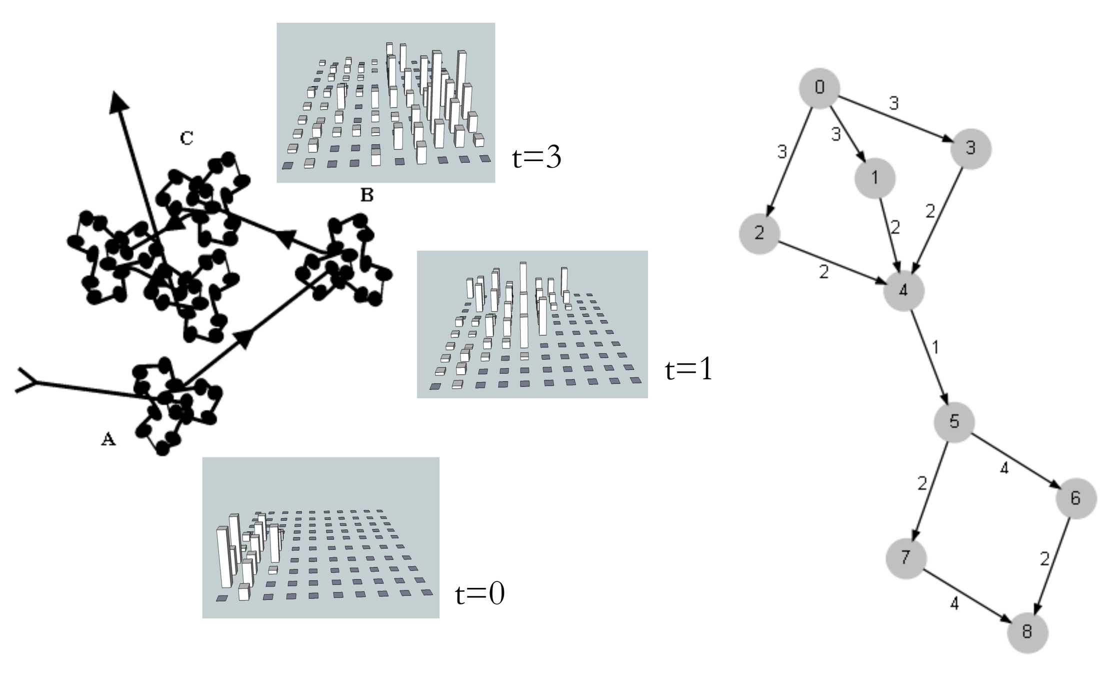


Figure 9: *Discovering Synfire network structure.*

Polychronous Circuit

In polychronous circuits neurons code information through precise spike timing and variable network delays. Complex patterns can be stored and processed by such networks [1].

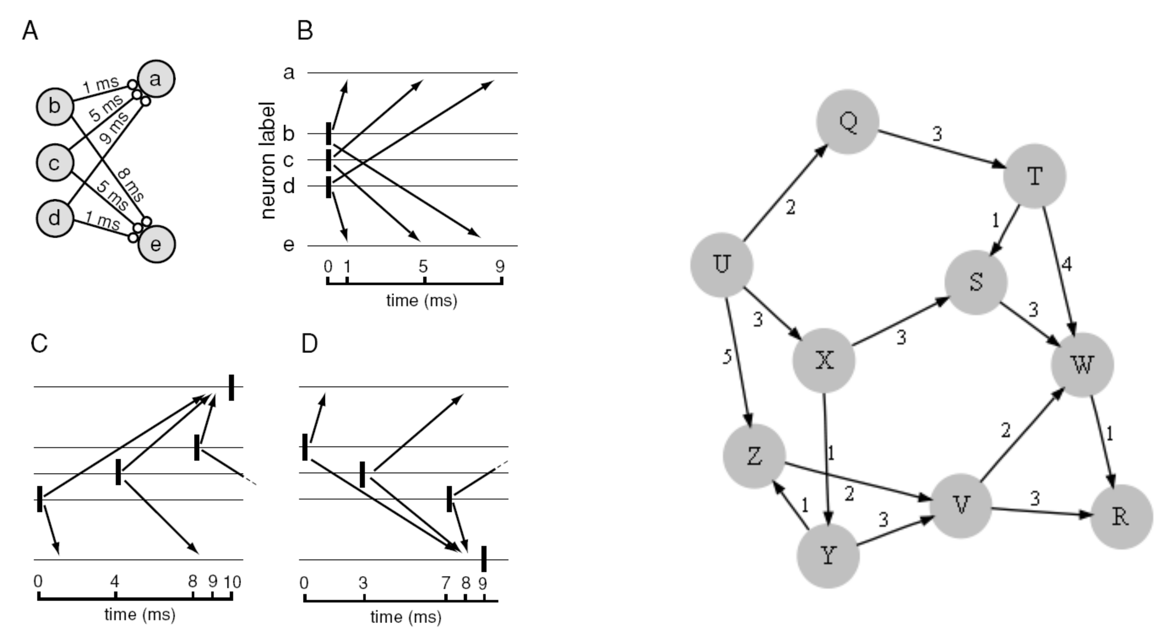


Figure 10: *Discovering Polychronous network structure.*

Real MEA Data

Application of our method on multi-electrode arrays recordings from dissociated cortical cultures gathered by Steve Potter's laboratory at Georgia Tech [4].

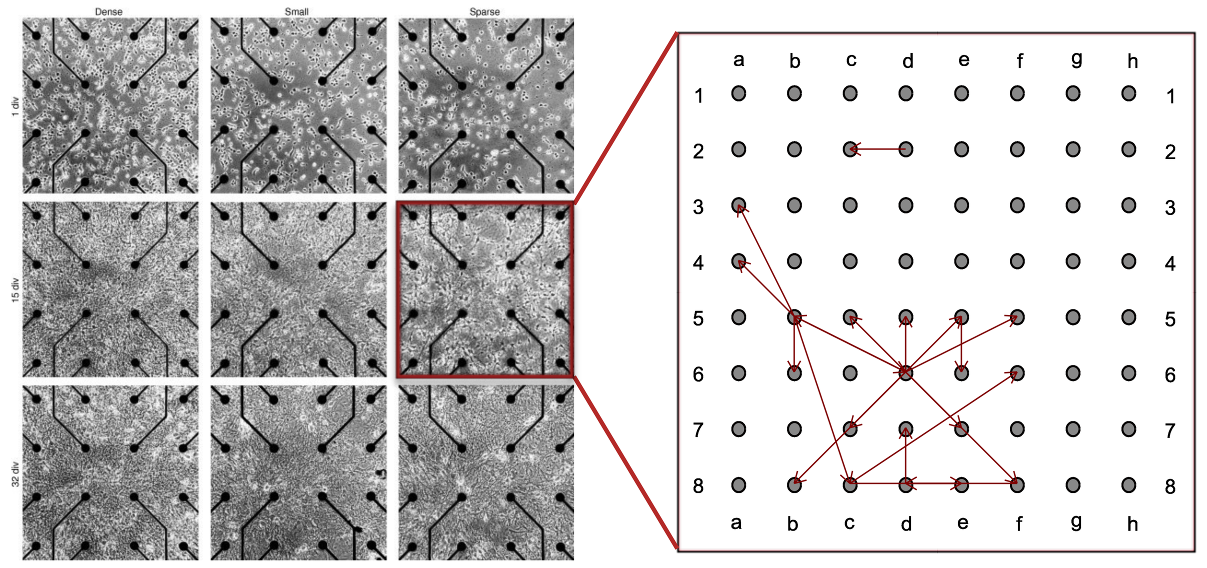


Figure 11: *Network structure discovered from first 15 min of spike train recording on day 35 of culture 2-1.*

Conclusion

Excitatory Dynamic Networks: We provide a formal basis for learning a special class of models from spike train data.

Efficient Learning: Excitatory network assumption allows the use of connect fast frequent episode mining algorithms to learn network structures.

Application to Spike Train analysis: We show that network dynamics like Synfire Chains, Polychrony etc. can be modeled as excitatory networks and can be unearthed using EDN Learning.

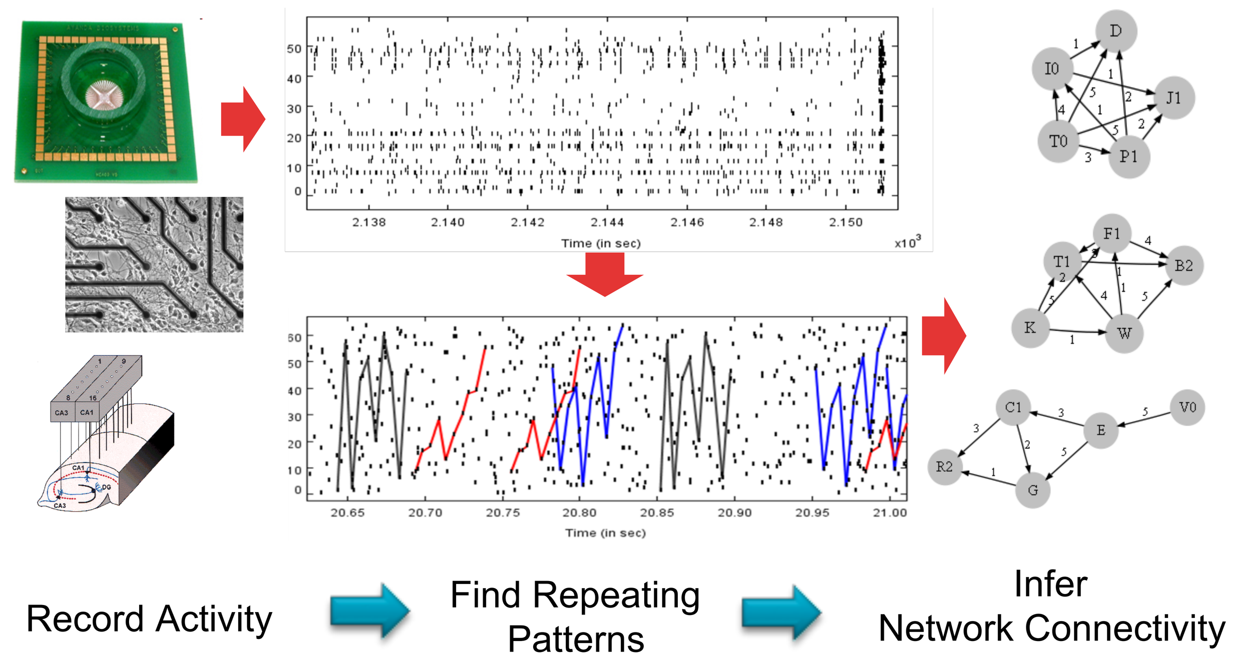


Figure 12: *Framework for discovering Excitatory Dynamic Networks.*

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