## Parsing - 1

- What is parsing?
- Shift-reduce parsing
- Shift-reduce conflict
- Reduce-reduce conflict
- Operator precedence parsing


## Parsing

- Parsing is the reverse of doing a derivation
- By looking at the terminal string, effectively try to build the parse tree from the bottom up
- Finding which sequences of terminals and nonterminals form the right hand side of production and reducing them to the left hand side nonterminal


## Shift-reduce Parsing

- Handle- substring which is right hand side of some production; corresponds to the last expansion in a rightmost derivation
- Replacement of handle by its corresponding nonterminal left hand side, results in reduction to the distinguished nonterminal by a reverse rightmost derivation
- Parse works by shifting symbols onto the stack until have handle on top; then reduce; then continue


## Example

$$
\begin{array}{lr}
\hline \mathbf{S} \rightarrow \mathrm{E} & (1) \\
\mathrm{E} \rightarrow \mathrm{E}+\mathrm{T}(2) \\
\mathrm{E} \rightarrow \mathbf{T} & \text { (3) } \\
\mathbf{T} \rightarrow \boldsymbol{i d} \tag{4}
\end{array}
$$



## Example

|  | ept, error |
| :---: | :---: |
|  |  |
| \$ | id1 + id2 + id3 \$ |
| \$ id1 | + id2 + id3 \$ |
| \$ T | + id2 + id 3 \$ |
| \$ E | + id $2+\mathrm{id} 3$ \$ |
| \$ E + | id $2+\mathrm{id} 3$ \$ |
| \$ E + id2 | +id3 \$ |
| \$ E + T | +id3 \$ |
| \$ E | +id3 \$ |
| \$ E + | id3 \$ |
| \$ E + id3 | \$ |
| \$ E + T | \$ |
| \$ E | \$ |
| \$ S | \$ |


|  | $\begin{aligned} & \mathrm{S} \rightarrow \mathrm{E} \\ & \mathrm{E} \rightarrow \mathrm{E}+\mathrm{T}(1) \\ & \text { (2) } \end{aligned}$ |
| :---: | :---: |
|  | $\mathrm{E} \rightarrow \mathrm{T}$ |
| Action | $\mathrm{T} \rightarrow$ id (4) |
| shift |  |
| reduce (4) |  |
| reduce (3) |  |
| shift |  |
| shift |  |
| reduce(4) |  |
| reduce (2) |  |
| shift |  |
| shift |  |
| reduce (4) |  |
| reduce(2) |  |
| reduce (1) |  |
| accept |  |

## Possible Problems

- Can get into conflicts where one rule implies shift while another implies reduce $S \rightarrow$ if $E$ then $S \mid$ if $E$ then $S$ else $S$
On stack: if $E$ then $S$
Input: else
Should shift trying for 2nd rule or reduce by first rule?


## Possible Problems

- Can have two grammar rules with same right hand side which leads to reduce-reduce conflicts
$A \rightarrow \alpha$ and $B \rightarrow \alpha$ both in grammar
When $\alpha$ on top of the stack, how know which production choose? That is, whether to reduce to A or B?
- In both kinds of conflicts, problem is with the grammar, not necessarily the language
- Recall, there can be many context-free grammars corresponding to the same language!


## Shift-Reduce Parsing

- Actions
- Shift - push token onto stack
- Reduce - remove handle from stack and push on corresponding nonterminal
- Accept - recognize sentence when stack contains only the distinguished symbol and input is empty
- Error - happens when none of the above is possible; means original input was not a sentence!


## Handles

- Any string of terminals and nonterminals derived from the distinguished nonterminal is called a sentential form
- If grammar is unambiguous, then each right sentential form has a unique handle
$\mathbf{Z} \underset{\mathrm{rm}}{\stackrel{*}{\rightarrow}} \alpha \mathbf{A} \mathbf{w} \rightarrow \alpha \beta \mathbf{w}$,
where $\alpha$ is a mixture of terminals and nonterminals; $\beta$ is the handle;
and $w$ is a string of terminals


## A Handle in the Parse Tree



## Ambiguity Example

$\mathrm{Z} \rightarrow \mathbf{E}$
$\mathbf{E} \rightarrow \mathbf{E}$ or $\mathbf{E} \mid \boldsymbol{a}$
Two rightmost derivations (handles in red):
$\mathbf{Z} \rightarrow \mathbf{E} \rightarrow \mathbf{E}$ or $\mathbf{E} \rightarrow \mathbf{E}$ or $a \rightarrow \mathbf{E}$ or $\mathbf{E}$ or $a \rightarrow \mathbf{E}$ or a or $a \rightarrow$ aoraora
$\mathbf{Z} \rightarrow \mathbf{E} \rightarrow \mathbf{E}$ or $\mathbf{E} \rightarrow \mathbf{E}$ or $\mathbf{E}$ or $\mathbf{E} \rightarrow \mathrm{E}$ or $\mathbf{E}$ or $a \rightarrow$
Eor a or a $\rightarrow$ a or a or a
Shift $a$, reduce to $E$, shift or, shift $a$, reduce to $E$ (now have $E$ or $\mathbf{E}$ on stack). In deriv1, reduce $\mathbf{E}$ or $\mathbf{E}$ to $\mathbf{E}$. In deriv2 shift or and $a$ onto stack. SHIFT-REDUCE conflict.

## Justification of Handle Use

- How can we be sure that the handle will always be at the top of the stack?
- Conventions: Greek letters for strings of terminals and nonterminals. Arabic letters for strings of terminals only. Capital letters are nonterminals.
- The following is a rightmost derivation:

Case 1: A's production contains a rightmost nonterminal $B$.

$$
\underset{\mathbf{Z}}{\mathbf{B} \xrightarrow{*} \boldsymbol{m}} \alpha \underset{\sim}{A} \mathbf{q} \underset{\mathrm{rm}}{\rightarrow} \alpha \beta \mathbf{B} \mathbf{y} \mathbf{q} \underset{\mathrm{rm}}{\rightarrow} \alpha \beta \gamma \mathbf{y} \mathbf{q}, \text { where }
$$

## Justification, cont.

Stack will contain $\$ \alpha \beta \gamma$ with $\mathbf{y q}$ in the input. This will be reduced to $\$ \alpha \beta B$ with yq still in the input.
Handle can't be below B in the stack or else the derivation would have to have been:
...X...B $\rightarrow$... $\delta . . . B$ with $\delta$ in the $\alpha \beta$ on the stack. But this isn't a rightmost derivation, because $B$ is to the right of $X$ and $X$ is being expanded first! \#CONTRADICTION

## Justification, cont.

Therefore handle must contain $B$ and it is not "buried" in the stack.

Assume the handle is $\beta B y$ ( $\beta$ or y may be empty)
Case 2: A's production does not contain a nonterminal
$\mathbf{Z} \underset{\mathrm{rm}}{\stackrel{*}{\rightarrow}} \alpha \mathbf{C} \times \mathbf{A} \underset{\mathrm{rm}}{\rightarrow} \alpha \mathbf{C} \times \mathbf{y} \underset{\mathrm{rm}}{\rightarrow} \alpha \gamma \times \mathbf{y} \mathbf{r}$ where $\mathrm{A} \rightarrow \mathrm{y}$ and $\mathrm{C} \rightarrow \gamma$

## Justification, cont.

- Stack will contain $\$ \alpha \gamma$ with input $x y r$. This will be reduced to $\$ \alpha C$, and then $x$ and $y$ will be shifted onto stack. Then $\$ \alpha C x y$ will be reduced to $\$ \alpha$ CxA on the stack with $r$ remaining in the input.
- So the handle is not buried in the stack.


## Operator Precedence Parsing

 ASU, Ch 4.6- A simplified bottom up parsing technique used for expression grammars
- Requires
- No right hand side of rule is empty
- No right hand side has 2 adjacent nonterminals
- Drawbacks
- Small class of grammars qualify
- Overloaded operators are hard (unary minus)
- Parser correctness hard to prove


## Operator Precedence

- Define three precedence relations
$-\mathrm{a}<\mathrm{b}$, a yields in precedence to $b$
$-\mathbf{a}>\mathbf{b}$, a takes precedence over $b$
$-\mathbf{a}=\mathbf{b}$, $\mathbf{a}$ has same precedence as $b$
- Find handle as <====> pattern at top of stack;
- Check relation between top of stack and next input symbol
- Basically, ignore nonterminals


## Example

$\mathbf{Z} \rightarrow \mathbf{E}$
$\mathbf{E} \rightarrow \mathrm{E} * \mathrm{E}|\mathrm{E}+\mathrm{E}|$ id
Define precedence relations between + and $*$.
$+\langle *, *\rangle+,+\rangle+, *\rangle$ (last 2 ensure left associativity)
Form table of precedences. Now parse using the table, and keep track of the operand nonterminals, too. Sometimes can embed error

|  | id | + | $*$ | $\$$ |
| :--- | :--- | :--- | :--- | :--- |
| id |  | $>$ | $>$ | $>$ |
| + | $<$ | $>$ | $<$ | $>$ |
| $*$ | $<$ | $>$ | $>$ | $>$ |
| $\$$ | $<$ | $<$ | $<$ |  | handling in matrix.

## Example

Compare top of stack token to next input token.

Stack
$\$$
$\$<\mathbf{i d} 1$
$\$ \mathbf{E}$
$\$ \mathbf{E}+$
$\$ \mathbf{E}+<\mathrm{id} 2$
$\$ \mathbf{E}+\mathbf{E}$
$\$ \mathbf{E}+\mathbf{E} *$
$\$ \mathbf{E}+\mathbf{E} *<\mathbf{i d} 3$
$\$ \mathbf{E}+<\mathbf{E} * \mathbf{E}$
$\$<\mathbf{E}+\mathbf{E}$
$\$<\mathbf{E}$
accept

Compares Input
id1 + id2 * id3 \$
+id2 * id3 \$
+id2 * id3 \$
id2 * id3 \$

* id3 \$
* id3 \$
id3 \$
\$
\$
\$
\$


## Making OP parsing practical

- How to store these precedences compactly?
- Precedence functions
- Find functions f(), g() such that
- $f($ token 1$)>g($ token 2$)$ means token $1>$ token 2
- $f($ token 1$)=g($ token 2$)$ means token $1=$ token2
- f(token1) < g(token2) means token1 < token2
- Graph partitioning algorithm to find $f(), g()$ if possible.


## Precedence Functions

- Form graph from table of precedences
- Nodes formed by f(token1),f(token2),...g(token1) etc.
- Form equivalence classes of nodes based on the $=$ relation (equal precedence, e.g., */)
- Edges show required relations between function values
- If token1 > token2, then $f($ token 1$)-->g(t o k e n 2)$
- If token1 < token2, then $f($ token1)<--g(token2)
- If the graph is acyclic, then can find integer value assignments for the range values of $f, g$.
- Let value of $f($ token 1$)$ be the length of the longest path from the node representing $f($ token1)


## Example



