## Parsing - 2

- LR(0) parsing
- Closures and goto sets
- SLR parsing
- Using FOLLOW sets
- LR(k) parsing
- Using lookaheads


## LR(0) parsing

- LR(k) parsing
- Left-to-right parse, Rightmost derivation, ktoken lookahead
- Recognize virtually all real programming languages
- Detects a syntax error as soon as possible in a left to right scan of the input stream
- Most powerful shift-reduce parsing method, yet efficient to implement


## Building a Parser

- How to build the DFA which is the decision maker for the stack parser in last lecture?
- Need a stack which takes <state,symbol> pairs
- Transition table contains four kinds of actions:
- shift into state $n$ (s n)
- reduce by rule $\boldsymbol{y}$ with lefthandside $X$ and then goto state $m$ ( $r y+$ goto entry when $X$ on top of stack) ; this is where actions occur
- accept
- error


## LR(0) Parsing

- Given parser in state $s$ and token $j$ is next, parser does action $[s, j]$ in transition table
stack with state, symbol pairs


LR parser

## Example

- Start with distinguished symbol rule and build start state

$$
\begin{array}{ll}
\mathbf{S}^{\prime} \rightarrow \mathbf{S} & \text { grammar } \\
\mathbf{S} \rightarrow \mathbf{a S} \mathbf{S} \mid \mathbf{a b} & \\
\mathrm{I}_{0}: \mathrm{S}^{\prime} \rightarrow . \mathrm{S} & \text { start state }
\end{array}
$$

- Then add in closure items

$$
\begin{aligned}
& \mathbf{I}_{\mathbf{0}}: \mathbf{S}^{\prime} \\
& \mathrm{S} \rightarrow . \mathbf{S} \\
& \mathrm{S} \rightarrow . \mathbf{a S b} \\
&
\end{aligned}
$$

- Now look for states to transition to on inputs or to goto if top of stack is a nonterminal


## Example

- Transition from $\mathbf{I}_{0}$ on an $a$ to $\mathbf{I}_{1}$
$\mathrm{I}_{1}: \underset{\mathrm{S}}{\mathrm{S}} \rightarrow \mathrm{a} \rightarrow \mathrm{a} . \mathrm{S} \mathrm{b}$
- Now add in closure items to complete $I_{1}$

$$
\begin{aligned}
\mathbf{I}_{1}: & \mathbf{S} \rightarrow \mathbf{a} \cdot \mathbf{S} \mathbf{b} \\
\mathbf{S} & \rightarrow \mathbf{a} \cdot \mathbf{b} \\
\mathbf{S} & \rightarrow . \mathbf{a S b} \\
\mathbf{S} & \rightarrow . \mathbf{a b}
\end{aligned}
$$

- Continue like this until have all the states and transitions


## Example

$$
I_{3}: S^{\prime} \rightarrow S .
$$

## A very simple grammar: $\mathbf{S}^{\prime} \rightarrow \mathbf{S}$ <br> $\mathbf{S} \rightarrow \mathbf{a S b | a b}$

First, build states from items.

$$
\mathbf{I}_{0}: \mathbf{S}^{\prime} \rightarrow . \mathbf{S}
$$

$$
I_{3}=\operatorname{goto}\left(I_{0}, S\right)
$$

$$
\mathrm{I}_{2}=\operatorname{goto}\left(\mathrm{I}_{1}, \mathrm{~S}\right)
$$

$$
\downarrow \begin{aligned}
& \mathbf{S} \rightarrow . \mathbf{a S b} \\
& \mathbf{S} \rightarrow . \mathbf{a b}
\end{aligned}>\text { closure }(S)
$$

$$
\mathbf{I}_{2}: S \rightarrow \mathbf{S} S . \mathbf{b}^{-}
$$

$$
\mathrm{I}_{4}=\operatorname{goto}\left(\mathrm{I}_{1}, \mathrm{~b}\right)
$$

$$
\mathrm{I}_{4}: \mathbf{S} \rightarrow \mathbf{a} \mathrm{b}
$$

$$
\mathrm{I}_{5}: S \rightarrow \text { a } \mathrm{Sb} .
$$

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$$
I_{5}=\operatorname{goto}\left(I_{2}, b\right)
$$

## Example

## Recognizes prefixes of right sentential forms



## Encoding the parser

| stateslinputs |  |  |  | actions |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | gotos |  |  |  |
|  | a | b | $\$$ | S |  |
| 0 | s1 | - | - | 3 |  |
| 1 | s1 | s4 | - | 2 |  |
| 2 | - | s5 | - |  |  |
| 3 | - | - | accept |  |  |
| 4 | r(iii) | r(iii) | r(iii) |  |  |
| 5 | r(ii) | r(ii) | r(ii) |  |  |


| If $\mathrm{A} \rightarrow \alpha . a \beta$ in $\mathrm{I}_{\mathrm{k}}$ and $\operatorname{goto}\left(I_{k}, a\right)=I_{j}$ table entry for ( $k, a$ ) is sj for $a$ terminal symbol. |
| :---: |
| If $\mathbf{A} \rightarrow \alpha$. in $I_{k}$ then table entry for ( $k, b$ ) is $r$ (rule\#) where $b$ is any input symbol. |
| If $S^{\prime} \rightarrow S$. in $I_{k}$ then table entry for $(k, \$)$ is accept. |

## Example

| states/inputs |  | actio |  | gotos |
| :---: | :---: | :---: | :---: | :---: |
|  | a | b | \$ | S |
| 0 | s1 |  |  | 3 |
| 1 | s1 | s4 | - | 2 |
| 2 |  | s5 |  |  |
| 3 |  |  | accept |  |
| 4 | r(iii) | r(iii) | r(iii) |  |
| 5 | r(ii) | r(ii) | r(ii) |  |

stack
\$ 0
\$ 0 a 1
\$0a1a1
\$0a1a1b4
\$0a1S2
\$0a1S2b5
\$0S3
input
aabb\$
abb\$
bb\$
b\$
b\$
\$
\$
action
s1
s1
s4
$r(i i i), \operatorname{goto}(1, b)=2$
s5
$r(i i), \operatorname{goto}(0, \$)=3$
accept

## SLR(1)

- Previous parser is called $L R(0)$ because we used no knowledge of the input
- SLR(1) is a somewhat stronger parser that adds knowledge about next input symbol
- Sometimes needed to break shift-reduce conflicts
- Need to precompute information about the grammar (from the rules) to use in parsing


## SLR(1)

- Follow set: the set of terminals which can follow a specific nonterminal in a rightmost derivation
- New rule for reduce: only reduce when next input symbol is an element of Follow set of the resulting nonterminal
- In the previous example, we would eliminate reductions in states 4,5 on $a$ because this can't be followed by $a$
- Follow sets are used also in top down parsing


## Shift/Reduce Conflict

$$
\begin{aligned}
& \mathbf{S}^{\prime} \rightarrow \mathbf{S} \\
& \mathbf{S} \rightarrow \mathbf{A} \mathbf{b}|\mathbf{d} \mathbf{c}| \mathrm{b} \text { A c } \\
& \mathbf{A} \rightarrow \mathbf{d}
\end{aligned}
$$

A very simple language $=\{\mathrm{db}, \mathrm{dc}, \mathrm{bdc}\}$
$\operatorname{Follow}(S)=\{\$\}, \operatorname{Follow}(A)=\{b, c\}$
Form part of the SLR(1) parser:

$$
\begin{aligned}
\mathrm{I}_{0}: & \mathrm{S}^{\prime} \\
& \rightarrow . \mathrm{S} \\
\mathrm{~S} & \rightarrow . \mathrm{A} b \\
\mathrm{~S} & \rightarrow . \mathrm{dc} \\
\mathrm{~S} & \rightarrow . \mathrm{b} \text { A c } \\
& \mathrm{A}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{I}_{1}: & \mathbf{S} \rightarrow \mathbf{d} . \mathbf{c} \\
& A \rightarrow \mathbf{d} .
\end{aligned}
$$

But since $c$ is in Follow(A), we don't know whether to reduce or shift in state $I_{1}$ if $\boldsymbol{c}$ is next input symbol!
Deriv1: $\mathbf{S}^{\prime} \rightarrow \mathbf{S} \rightarrow$ dc; Deriv2; $\mathrm{S}^{\prime} \rightarrow \mathrm{S} \rightarrow$ bAc $\rightarrow$ bdc

## Reduce/Reduce Conflict

$$
\begin{aligned}
& \mathbf{S}^{\prime} \rightarrow \mathbf{S} \\
& \mathbf{S} \rightarrow \mathbf{b} \text { A e } \mid \text { b B d } \mid \text { Ac } \\
& \mathbf{A} \rightarrow \mathbf{d} \\
& \mathbf{B} \rightarrow \mathbf{E} \mathbf{c} \\
& \mathbf{E} \rightarrow \mathbf{d}
\end{aligned}
$$

Deriv1: $\mathbf{S}^{\prime} \rightarrow \mathbf{S} \rightarrow \mathbf{A c} \rightarrow$ dc
Deriv2: $\mathbf{S}^{\prime} \rightarrow \mathbf{S} \rightarrow$ bBd $\rightarrow$ bEcd $\rightarrow$ bdcd Deriv3: $\mathbf{S}^{\prime} \rightarrow \mathbf{S} \rightarrow$ bAe $\rightarrow$ bde

$$
\begin{gathered}
\mathbf{I}_{0}: \mathbf{S}^{\prime} \rightarrow . \mathbf{S} \\
\mathrm{S} \rightarrow . \mathrm{b} \text { A e } \\
\mathrm{S} \rightarrow . \mathrm{b} \text { B d } \\
\mathrm{S} \rightarrow \text {. A c } \\
\mathbf{A} \rightarrow . \mathbf{d}
\end{gathered}
$$

$$
\begin{aligned}
& \mathbf{I}_{1}: \mathrm{S} \rightarrow \mathrm{~b} \cdot \mathrm{~A} \mathrm{e} \\
& \mathrm{~S} \rightarrow \mathrm{~b} \cdot \mathrm{~B} \mathbf{d} \\
& \mathrm{~A} \rightarrow . \mathrm{d} \\
& \mathrm{~B} \rightarrow . \mathbf{E} \mathbf{c} \\
& \mathbf{E} \rightarrow . \mathbf{d}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{I}_{2}: & \mathbf{A} \rightarrow \mathbf{d} \\
\mathbf{E} & \rightarrow \mathbf{d} .
\end{aligned}
$$

Which reduction to take?
Follow set too imprecise here to decide.

## LR(k)

- Solution: keep more information about what next input symbol can be on any parse
- Idea: keep an input lookahead as part of each item
- More precise than Follow sets which essentially union these lookaheads for nonterminal A over all sentential forms in which the $A$ appears
- Potentially gives rise to much bigger parsers than SLR(1) (more states)


## LR(k)

- LR(k) looks $k$ symbols ahead into the input
- There are some grammars which are not parsable with only $k$ lookahead symbols
- Most computer programming languages are LR(k)


## LR(1) Idea

$\mathbf{S}^{\prime} \rightarrow \mathbf{S}$
$\mathbf{S} \rightarrow \mathbf{b A e | b B d | A c}$
$\mathbf{A} \rightarrow \mathbf{d}$
$\mathbf{B} \rightarrow \mathbf{E} \mathbf{c}$
$\mathbf{E} \rightarrow \mathbf{d}$

Deriv1: $\mathbf{S}^{\prime} \rightarrow \mathbf{S} \rightarrow \mathrm{Ac} \rightarrow$ dc
Deriv2: $\mathbf{S}^{\prime} \rightarrow \mathbf{S} \rightarrow$ bBd $\rightarrow$ bEcd $\rightarrow$ bded
Deriv3: $\mathbf{S}^{\prime} \rightarrow \mathbf{S} \rightarrow$ bAe $\rightarrow$ bde

| $\mathrm{I}_{0}: S^{\prime} \rightarrow$. S, \$ |
| :---: |
| S $\rightarrow$. b A e, \$ |
| $\mathbf{S} \rightarrow$. b B d, \$ |
| $\mathbf{S} \rightarrow$. A c, \$ |
| $\mathrm{A} \rightarrow$.d,c |

$$
\begin{aligned}
& \mathrm{A} \rightarrow \text {.d, e Now can distinguish } \\
& B \rightarrow \text {. E c, d derivations by next } \\
& \mathrm{E} \rightarrow \text {.d, } \mathrm{c} \text { expected input symbol. }
\end{aligned}
$$

However potential to generate more states.

## LR(1) Example



Fill in the 8 missing states.
$\mathrm{I}_{3}$ is for ace and acd; $\mathrm{I}_{4}$ is for $b c d$ and $b c e$

