## Machine Independent Compiler Optimization

- What is classical machine independent optimization?
- Control flow graph, basic blocks, local opts
- Control flow abstractions: loops, dominators
- Four classical dataflow problems
- Reaching definitions
- Live variables
- Available expressions
- Very busy expressions


## Phases of Compilation



Optimization is a semantics-preserving transformation
MachineIndepOpt-1, Sp06 © BGRyder

## Example

- To define classical optimizations using an example loop from Fortran scientific code
- Opportunities for these optimizations result from table-driven code generation
...
sum $=0$
do $10 \mathrm{i}=1$, n
10 sum $=s u m+a(i) * a(i)$


## Three Address Code




## Local Common Subexpression Elimination (CSE)

1. $\operatorname{sum}=0$
2. $i=1$
3. if i $>\mathrm{n}$ goto 15
4. $\mathrm{t} 1=\operatorname{addr}(\mathrm{a})-4$
5. $\mathrm{t} 2=\mathrm{i} * 4$
6. sum $=0$
7. $i=1$
8. $t 3=t 1[t 2]$
9. if i $>\mathrm{n}$ goto 15
10. $\mathrm{t} 1=\operatorname{addr}(\mathrm{a})-4$
11. $t 4=\operatorname{addr}(\mathrm{a})-4$
12. $\mathrm{t} 2=\mathrm{i} * 4$
13. $t 3=t 1[t 2]$
14. $t 5=i * 4$
15. $t 4=$ addr(a) -4
16. $\mathrm{t} 6=\mathrm{t} 4$ [t5]
17. $\mathrm{t} 5=\mathrm{i}$ * 4
18. $t 7=t 3$ * t6
19. $\mathrm{t} 8=\mathrm{sum}+\mathrm{t} 7$
20. $\mathrm{sum}=\mathrm{t} 8$
21. $i=i+1$
22. goto 3
23. Blue code eliminated; Red code added
24. $\mathrm{t7}=\mathrm{t} 3$ * t 6
10a t7 $=t 3$ * t3
25. $\mathrm{t} 8=\mathrm{sum}+\mathrm{t} 7$
11a sum $=$ sum + t7
26. sum $=$ t8
27. $i=i+1$
28. goto 3
29. 

## Invariant Code Motion

1. sum $=0$
2. $i=1$
3. if i $>\mathrm{n}$ goto 15
4. $\mathrm{t} 1=\operatorname{addr}(\mathrm{a})-4$
5. $t 2=i * 4$
6. $t 3=t 1[t 2]$

10a t7 $=t 3$ * t3
11a sum $=$ sum + t7
13. $i=i+1$
14. goto 3
15.

1. sum $=0$
2. $i=1$

2a $\mathrm{t} 1=$ addr(a) -4
3. if i > n goto 15
4. $\mathrm{t} 1=\operatorname{addr}(\mathrm{a})-4$
5. $t 2=i * 4$
6. $t 3=t 1[t 2]$

10a $t 7=t 3$ * $t 3$
11a sum $=$ sum $+t 7$
13. $i=i+1$
14. goto 3
15.

## Reduction in Strength

```
1. sum = 0
2. i = 1
2a t1 = addr(a) - 4
3. if i > n goto 15
5. t2 = i * 4
6. t3 = t1[t2]
10a t7 = t3 * t3
11a sum = sum + t7
13. i = i + 1
14. goto 3
15.
2. \(i=1\)
```

1. sum $=0$
2. $i=1$

2a t1 = addr(a) - 4
2b t2 = i * 4
3. if i > n goto 15
5. t2 = i * 4
6. $t 3=t 1[t 2]$
$10 a t 7=t 3$ * $t 3$
11a sum $=$ sum + t7
11b t2 $=$ t2 +4
13. $i=i+1$
14. goto 3
15.

## Test Elision and Induction Variable Elimination

1. sum $=0$
2. $i=1$

2a $\mathrm{t} 1=\operatorname{addr}(\mathrm{a})-4$
2b t2 = i * 4
3. if i $>\mathrm{n}$ goto 15
6. $t 3=t 1[t 2]$

10a t7 $=t 3$ * t3
11a sum $=$ sum + t7
$11 b$ t2 $=t 2+4$
13. $i=i+1$
14. goto 3

15

1. sum $=0$
2. $i=1$

2a t1 = addr(a) - 4
2b t2 $=$ i * 4
2c t9 = 4 * $n$
3. if i $>\mathrm{n}$ goto 15

3a if t2 > t9 goto 15
6. $t 3=t 1[t 2]$

10a t7 = t3 * t3
11a sum $=$ sum $+t 7$
11b $\mathbf{t 2}=\mathbf{t 2}+4$
13. $i=i+1$
14. goto 3a

15

## Constant Propagation and Dead Code Elimination

1. sum $=0$
2. $i=1$

2a $t 1=$ addr(a) - 4
2b t2 $=$ i * 4
2ct9 $=4$ * $n$
3a if t2 > t9 goto 15
6. $t 3=t 1[t 2]$

10a t7 = t3 * t3
11a sum $=$ sum + t7
11 b t2 $=\mathrm{t} 2+4$
14. goto 3a

15

1. sum $=0$
2. $i=1$

2a $\quad$ t1 $=\operatorname{addr}(\mathrm{a})-4$
2 b t2 $=$ i * 4
2d $\mathrm{t} 2=4$
2c t9 = 4 * n
3a if t2 > t9 goto 15
6. $t 3=t 1[t 2]$

10a t7 = t3 * t3
11a sum $=$ sum $+t 7$
$11 b$ t2 $=t 2+4$
14. goto 3a

15


## How to build CFG?

- Need to find basic blocks and possible branches between them
- Basic block leader statements
- First program statement
- Targets of conditional or unconditional goto's
- Any statement following a conditional goto
- For each leader $s$, construct basic block $B_{s}$ as all statements $t$ reachable from $s$ through straight-line code
- Eventually, any statements not included in some basic block are unreachable from program entry dead code


## Leader Statements

```
1. sum = 0 first program statement
2. i = 1
3. if i > n goto 15 conditional goto statement
4. t1 = addr(a) - 4 statement following
5. t2 = i * 4 conditional goto
6. t3 = t1[t2]
7. t4 = addr(a) - 4
8. t5 = i * 4
9. t6 = t4[t5]
10. t7 = t3 * t6
11. t8 = sum + t7
12. sum = t8
13. i = i + 1
14. goto 3
15. branch
                        target
```


## Local CSE

- Accomplished while translating into three address code
- For each statement, form expression DAGs (for operand sharing)
- Operands are children of operator nodes
- Operand nodes can be used by more than one operator node
- Intermediate results that must be stored cause creation of compiler temporaries
- Multiple labels on same node mean CSE


## Expression DAG construction



## Expression DAG construction

```
t1 = addr[a]-4
t2 = i * 4
t3 = t1[t2]
t4 = addr[a]-4
t5 = i * 4
t6 = t4 [t5]
t7 = t3 * t6
t8 = sum + t7
sum = t8
i = i + 1
```



## DAG construction

- How to add a subexpression into a partially constructed DAG? A = B + C
- Is there a node already for $\mathrm{B}+\mathrm{C}$ ?
- If so, add A to its list of labels
- If not,
- Is there a node labeled $B$ already? If not, create a leaf labeled B
- Is there a node labeled $C$ already? If not, create a leaf labeled C
- Create a node labeled A for + with left child B and right child $C$


## Flow of Control Abstractions

- Dominator A node $\boldsymbol{x}$ dominates a node $\boldsymbol{y}$ if and only if all paths from the control flow graph (CFG) entry node to $y$ pass through $x$.
- (Natural) Loop Let ( $y, x$ ) be a CFG edge such that $x$ dominates $y$. Then all nodes on paths from $x$ to $y$ are in the loop defined by $(y, x)$.
- $(y, x)$ is called a back edge
- For reducible graphs, the set of back edges is unique
- CFG is reducible if each loop can be entered through a single node
- Irreducible means contains a subgraph



## Loops

Back edges: $(5,3),(4,3),(6,2)$
Loop $(5,3)=\{3,4,5\}\}$ combined Loop $(4,3)=\{3,4\}$
Loop (6,2) $=\{2,3,4,5,6\}$


## General Step in Strength Reduction

 operation into a less expensive one

## General Code Motion

```
n := 1; k := 0; m := 3; read x;
while n \leq 10 do
        if 2 + x < 5 then k := 5;
        if 3 + k = 3 then m := m + 2;
        n := n + k + m;
    endwhile;
```

definitions within loop are barriers to code motion

## General Code Motion

```
    n := 1; k := 0; m := 3; read x;
    if 2 + x < 5 then k := 5;//move first
    t1 := 3 + k = 3 //move second
    while n \leq 10 do
        if 2 + x < 5 then k := 5;
        if 3 + k = 3 then m := m + 2;
        if t1 then m := m + 2;
        n := n + k + m;
    endwhile;
        Why can't we move any
        more code out of the loop?
```


## Program Analysis

- Performed at compile-time, deriving something about semantics of program
- Termed flow analysis
- Control flow analysis reveals possible execution paths
- Cannot tell actual feasibility of path. Why not?
- Dataflow analysis determines information about modification, preservation, and use of data entities in a program


## Four Classical Data Flow Problems

- Reaching definitions, Live uses of variables, Available expressions, Very Busy Expressions
- Def-use and Use-def chains, built from Reach and Live, used for many optimizations
- Avail enables global common subexpression elimination
- VeryB was used for conservative code motion


## Reaching Definitions

- Definition A statement which may change the value of a variable
- A definition of a variable $x$ at node $k$ reaches node $n$ if there is a definition-clear path from $k$ to $n$.



## Live Uses of Variables

- Use Appearance of a variable as an operand of a 3 address statement
- A use of a variable $\boldsymbol{x}$ at node n is live on exit from node $k$ if there is a definition-clear path for $x$ from $k$ to $n$.



## Def-use Relations

- Use-def chain links an use to a definition that reaches that use --
- Def-use chain links a definition to an use that it reaches $\longrightarrow$



## Optimizations Enabled by Def-use

- Dead code elimination (Def-use)
- Code motion (Use-def)
- Strength reduction (Use-def)
- Test elision (Use-def)
- Constant propagation (Use-def)
- Copy propagation (Def-use)



## Constant Propagation



## Classical Dataflow Problems

- How to formulate analysis from CFG to dataflow equations?
- Forward and backward dataflow problems
- May and must dataflow problems



# Reaching Definitions Equations 

$\operatorname{Reach}(\mathbf{j})=\cup \quad\{\operatorname{Reach}(\mathbf{m}) \cap \operatorname{pres}(\mathbf{m}) \cup \operatorname{dgen}(\mathbf{m})\}$ $\mathrm{m} \in \operatorname{Pred}(\mathbf{j})$
where:
pres( $m$ ) is the set of defs preserved through node $m$ dgen $(\mathrm{m})$ is the set of defs generated at node $m$
$\operatorname{Pred}(\mathbf{j})$ is the set of immediate predecessors of node $j$


## Live Uses Equations

```
\(\operatorname{Live}(\mathbf{j})=\bigcup \quad\{\operatorname{Live}(\mathbf{m}) \cap \operatorname{pres}(\mathbf{m}) \cup\) ugen \((\mathbf{m})\}\)
\(\mathbf{m} \in \operatorname{Succ}(\mathbf{j})\)
```

where
pres( $\mathbf{m}$ ) is the set of uses preserved through node $\boldsymbol{m}$ (these will correspond to variables whose defs are preserved)
$\operatorname{ugen}(m)$ is the set of uses generated at node $m$ $\operatorname{succ}(\mathbf{j})$ is the set of immediate successors of node $j$

## Available Expressions

- An expression $X$ op $Y$ is available at program point $\boldsymbol{n}$ if EVERY path from program entry to $n$ evaluates $X$ op $Y$, and after every evaluation prior to reaching $n$, there are NO subsequent assignments to $X$ or $Y$.




## Available Expressions



## Available Expressions Equations

$$
\begin{aligned}
\operatorname{Avail}(\mathbf{j}) & =\bigcap\{\operatorname{Avail}(\mathrm{m}) \cap \operatorname{epres}(\mathrm{m}) \cup \operatorname{egen}(\mathrm{m})\} \\
\mathrm{m} & \in \operatorname{Pred}(\mathbf{j})
\end{aligned}
$$

where:
epres(m) is the set of expressions preserved through node $m$ egen $(\mathrm{m})$ is the set of (downwards exposed) expressions generated at node $m$
$\operatorname{pred}(\mathbf{j})$ is the set of immediate predecessors of node $\boldsymbol{j}$

## Very Busy Expressions

- An expression $X$ op $Y$ is very busy at program point $n$, if along EVERY path from $n$, we come to a computation of $X$ op $Y$ BEFORE any redefinition of $X$ or $Y$.



## Code Hoisting

- SAFETY: Assume X op $Y$ is in $\operatorname{VeryB(n)}$ and $n$ dominates all expression calculations that are hoisting candidates $p$.
- For every X op Y at program point $p$, trace backwards from $\boldsymbol{p}$ to $\boldsymbol{n}$ to ensure there is a path from $n-->p$ without any definitions of $X, Y, X$ op $Y$
- Hoist (Calculate $t=X$ op $Y$ ) at exit of node $n$; change candidate calculations from $s=X$ op $Y$ to $s=t$.
- PROFITABILITY: Check that copy propagation can eliminate all copies introduced in the previous step. If not, undo the hoist.


## Very Busy Expressions



## Very Busy Equations

$$
\begin{aligned}
\operatorname{VeryB}(\mathbf{j}) & =\cap\{\operatorname{VeryB}(\mathbf{m}) \cap \operatorname{epres}(\mathbf{m}) \cup \operatorname{vgen}(\mathbf{m})\} \\
\mathbf{m} & \in \operatorname{Succ}(\mathbf{j})
\end{aligned}
$$

where:
epres(m) is the set of expressions preserved through node $m$ $\operatorname{vgen}(m)$ is the set of (upwards exposed) expressions generated at node $m$
$\operatorname{succ}(\mathbf{j})$ is the set of immediate successors of node $\boldsymbol{j}$

| Dataflow Problems |  |  |
| :---: | :---: | :---: |
|  | May Problems | Must Problems |
| Forward <br> Problems | Reaching Defs | Available Exprs |
| Backward Problems | Live Uses of Variables | Very Busy Expressions |
|  |  |  |

