Functional Programming

- Pure functional PLs
- S-expressions
  - *cons, car, cdr*
- Defining functions
- *read-eval-print* loop of Lisp interpreter
- Examples of recursive functions
  - Shallow, deep
- Equality testing

Pure Functional Languages

- *Referential transparency*
  - value of an expression is independent of context where the function application occurs
  - means that all variables in a function body must be local to that function; why?
- There is no concept of assignment
  - variables are bound to values only through parameter associations
  - no side effects

Pure Functional Languages

- Control flow accomplished through function application (and recursion)
  - a program is a set of function definitions and their application to arguments
- Implicit storage management
  - copy semantics, needs garbage collection
- Functions are 1st class values!
  - can be returned as value of an expression or function application
  - can be passed as an argument
  - can be put into a data structure and saved
- Unnamed functions exist as values

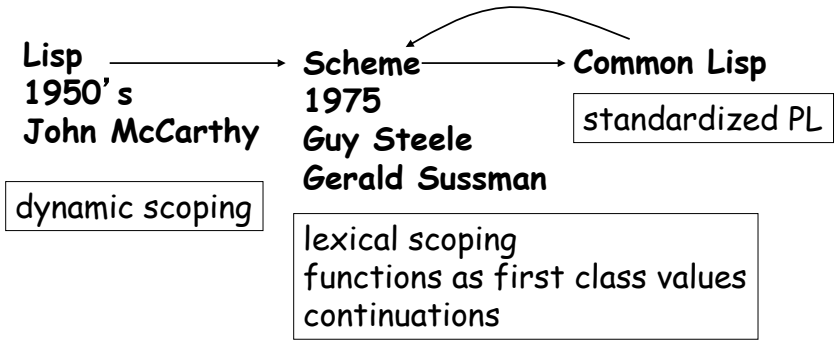
Pure Functional Languages

- Lisp designed for symbolic computing
  - simple syntax
  - data and programs have same syntactic form
    - S-expression
  - function application written in prefix form

    (e1 e2 e3 … ek) means
    - Evaluate e1 to a function value
    - Evaluate each of e2,…,ek to values
    - Apply the function to these values

    (+ 1 3) evaluates to 4

History



Racket is a PL based on Scheme;
Scheme has eager evaluation and dynamic typing

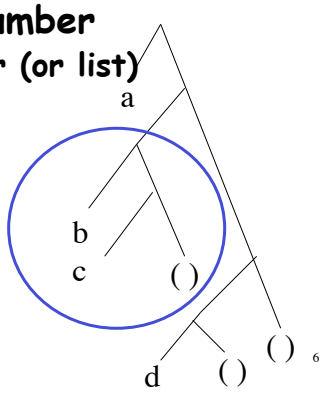
S-expressions

S-expr ::= Name | Number | ({ S-expr })

Name is a symbolic constant, some string of chars which starts off with anything that can't start a Number

Number is an integer or real number

- E.g., (a (b c) (d)) is an S-expr (or list)
- car selects the first element
  - car of this S-expr is a
- cdr selects the rest of the list
  - cdr of this S-expr is ((b c) (d))

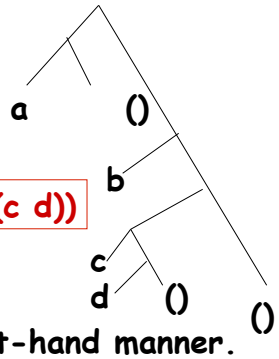


List Operators

- *Car* and *cdr*
  - Given a list, they decompose it into first element, rest of list portions
- *Cons*
  - Given an element and a list, cons builds a new list with the element as its car and the list as its cdr
- () means the empty list in Scheme

History:
the words car and cdr come from the first implementation of Lisp on the IBM 704.
car stands for Contents of the Address part of Register
cdr stands for Contents of the Decrement part of the Register
and the implementation represented lists so that the head was a car
and the tail was a cdr

Examples



((a) b (c d))

(car '(a b c)) is a
(car '((a) b (c d))) is (a)
(cdr '(a b c)) is (b c)
(cdr '((a) b (c d))) is (b (c d))

Can compose these operators in a short-hand manner.
Can reach any arbitrary list element by composition of car's and cdr's.

(car (cdr (cdr '((a) b (c d))))) = can also be written
(caddr '((a) b (c d)))

(car (cdr '(b (c d))) =
(car '((c d)) = (c d).

Examples

(cons 'a b c) '((a) b (c d))) is ((a b c) (a) b (c d))

(cons 'd '(e)) is (d e)

(cons '(a b) '(c d)) is ((a b) c d)

Useful predicates in Scheme. Note the quote prevents evaluation of the argument as an S-expr.

(symbol? 'sam) returns #true (symbol? 1) returns #f

(number? 'sam) returns #false (number? 1) returns #t

(list? '(a b)) returns #true (list? 'a) returns #f

(null? '()) returns #true (null? '(a b)) returns #f

(zero? 0) returns #true (zero? 1) returns #f

Can compose these.

(zero? (- 3 3)) returns #true *--note that since this language is fully parenthesized, there are no precedence problems in the expressions!*

Scheme

Fcn-def ::= (define (Fcn-name Param {Param}) S-expr)

Fcn-name should be a new name for a fcn.

Param should be variable(s) that appear in the S-expr which is the function body.

Fcn-def ::= (define Fcn-name Fcn-value)

Fcn-value::= (lambda (Param {Param}) S-expr)

where Param variables are expected to appear in the S-expr; called a *lambda expression.*

Scheme Examples

```
( define (zerocheck? x)
  (if (= x 0) #true #false) )
```

If-expr ::= (if S-expr0 S-expr1 S-expr2)

where S-expr0 must evaluate to a boolean value; if that value is true, then the If-expr returns the value of S-expr1, else the value of S-expr2.

(zerocheck? 1) returns #false, (zerocheck? (* 1 0)) returns #true

```
(define (notNumber x) (not (number? x)))
```

Where *number?* returns #true if its argument is a number and #false otherwise; note *not* is a logical operator

Scheme Examples

```
(define square (lambda (n) (*  n  n)) )
```

- This associates the Fcn-name square with the function value (lambda (n) (* n n)))
- *Lambda calculus* is a formal system for defining recursive functions and their properties
  - Set of functions definable using lambda calculus (Church 1941) is same as set of functions computable as Turning Machines (Turing 1930's)

Read-eval-print loop

- How does a Scheme interpreter work?
  - *Read* input from user
    - A function definition or abstraction
    - A function evaluation
  - *Evaluate* input
    - Store function definition
    - (e1 e2 e3 … ek)
      - Evaluate e1 to obtain a function
      - Evaluate e2, … , ek to values
      - Execute function body using values from previous step as formal parameter values
      - Return value of function
  - *Print* return value

Conditional Execution

(if e1 e2 e3)

(cond (e1 h1) (e2 h2)…(en-1 hn-1) (else hn))

- Cond is like a nested if-then-elseif construct

```
(define (zerocheck? x)
      (cond ((= x 0) #t) (else #f)))
```

OR

```
(define (zchk? x)
  (cond ((number? x) (zero? x))
        (else #f)) )
```

Recursive Functions

```
(define (len x)  (cond ((null?  x)  0)  (else (+ 1 (len (cdr  x))))))
```

(len '(1 2)) should yield 2.

len is a shallow recursive function

Trace: (len '(1 2)) --top level call
x = (1 2)
(len '(2)) --recursive call 1
x = (2)
(len '( )) -- recursive call 2
x = ( )
returns 0 --return for call 2
returns (+ 1 0) =1 --return for call 1
returns (+ 1 1) = 2 --return for top level call

(len '((a) b (c d))) returns 3

List Append

```
(define (app  x  y)
  (cond  ((null?  x)  y)
         ((null?  y)  x)
         (else  (cons (car x)  (app (cdr x)  y)))))
```

(app '( ) '( )) yields ( )
(app '( ) '(1 4 5)) yields (1 4 5)
(app '(5 9) '(a (4) 6)) yields (5 9 a (4) 6)
another shallow recursive function

- **Can we write a function that counts the number of atoms in a list? (this will have to be a *deep recursive function*)**

Atomcount Function

atomcount is a deep recursive function

```
(define (atomcount x)
  (cond ((null? x) 0)
        ((atom? x) 1)
        (else (+ (atomcount (car x)) (atomcount (cdr x))))))
```

(atomcount '(1)) yields 1
(atomcount '(1 (2 (3)) (5))) yields 4
Trace: (atomcount '(1 (2 (3)))
1> (+ (atomcount 1) (atomcount '((2 (3)))))
2> (+ (atomcount '(2 (3))) (atomcount '( )))
3> (+ (atomcount 2) (atomcount '((3))) etc.
4> (+ (atomcount '(3)) (atomcount '( )))
1 ← 5> (+ (atomcount 3) (atomcount '( )))

Equality Testing

eq?
- predicate that can check atoms for equal values
- doesn't work on lists

eql?
- comparison function for lists

```
(define (eql? x y)
  (or (and (null? x) (null? y)) // null? '() is #true
      (and (atom? x) (atom? y) (eq? x y))
                  //atom? '() is false; checking atoms
      (and (not (atom? x)) (not (atom? y))
           (not (null? x)) (not (null? y))//error checks
           (eql? (car x) (car y))
           (eql? (cdr x) (cdr y)))))
```

Examples

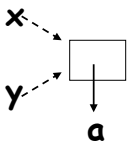
(eql? ‘(a) ‘(a)) yields #t
(eql? ‘a ‘b) yields #f
(eql? ‘b ‘b) yields #t
(eql? ‘((a)) ‘(a)) yields #f

(eq? ‘a ‘a) yields #t
(eq? ‘(a) ‘(a)) yields #f

How does eq? work?

(define (f x y) (list x y))
so (f ‘a ‘a) yields (a a).
How does Scheme implement this?
It binds both x and y to the same atom a.
eq? checks that x and y both point to the same place

Scheme uses a reference model for variables



Say we called (f ‘(a) ‘(a)). then x and y don’t point to the same list at all!