

# Classical Dataflow Analysis

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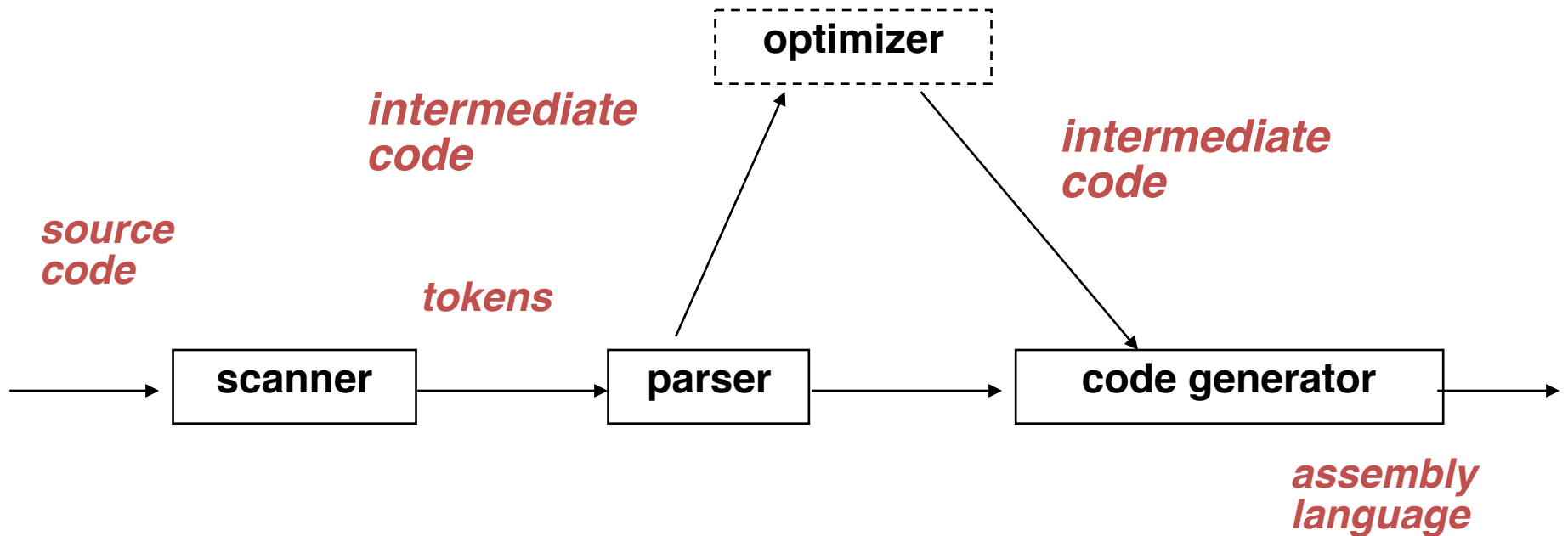
Cs 6304

Fall 2015

# Lecture 1 - Outline

- Classical Dataflow Analysis
  - Control flow graphs, Reaching definitions, Live uses of variables, Available Expressions
  - Dataflow equations (transfer functions)
  - References: optimization chapter of compiler textbooks

# Compilation Process



**Optimization is a semantics-preserving transformation**

# Static (compile-time) Analysis

- Semantic analysis of code to ensure correctness of machine independent optimization
  - Optimizing Fortran compiler - IBM Backus late 1960's
- Classical dataflow problems defined on Fortran serve as simple examples of defining and solving dataflow problems
- Assume knowledge of internal program representations of code
  - Rooted, digraphs: control flow graph (of a function), call graph (program calling structure)

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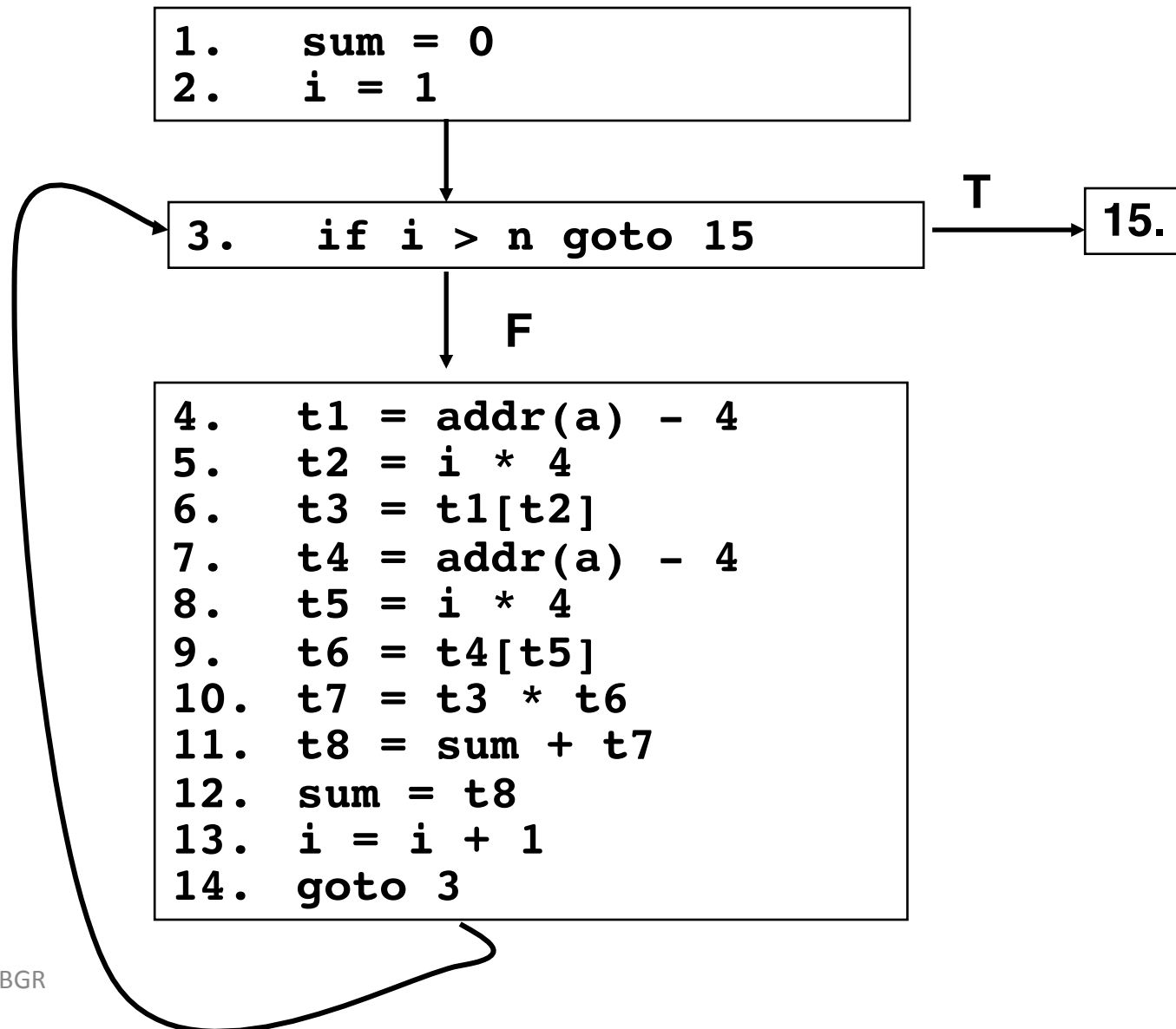
        sum = 0
        do 10 i = 1, n
10      sum = sum + a(i) * a(i)
        ...

```

original Fortran

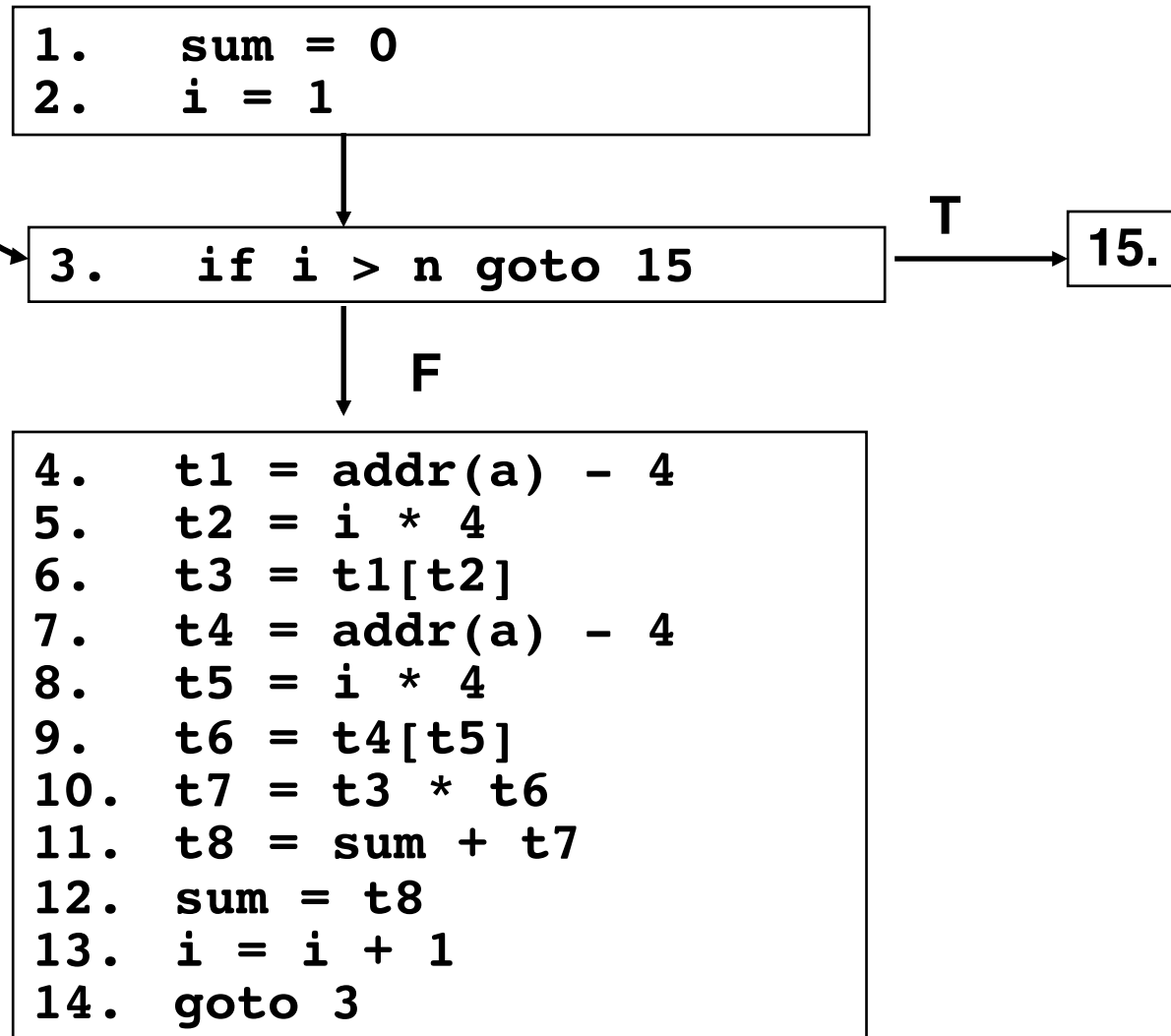
<pre> 1.  sum = 0 2.  i = 1 3.  if i &gt; n goto 15 4.  t1 = addr(a) - 4 5.  t2 = i * 4 6.  t3 = t1[t2] 7.  t4 = addr(a) - 4 8.  t5 = i * 4 9.  t6 = t4[t5] 10. t7 = t3 * t6 11. t8 = sum + t7 12. sum = t8 13. i = i + 1 14. goto 3 15. </pre>		<p>sum = 0; initialize loop counter loop test, check for limit</p> <p>a[i]</p> <p>a[i]</p> <p>a[i] * a[i] increment sum increment loop counter</p>
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# Control Flow Graph (CFG)



# Optimized Control Flow Graph (CFG)

Optimizations enabled by dataflow analysis extracting info about reads and writes - data dependences



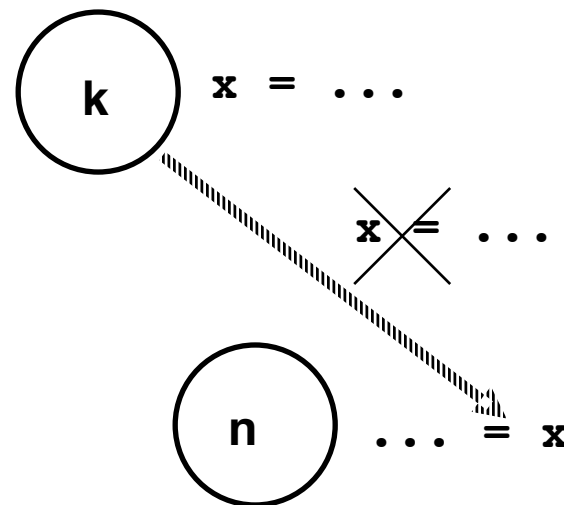
# Some Classical Data Flow Problems

- Reaching definitions, Live uses of variables, Available expressions, used historically for low-level code optimizations
- Def-use and use-def chains, built from Reach and Live provide semantic basis for data dependence analysis
- Available expressions enable common subexpression elimination



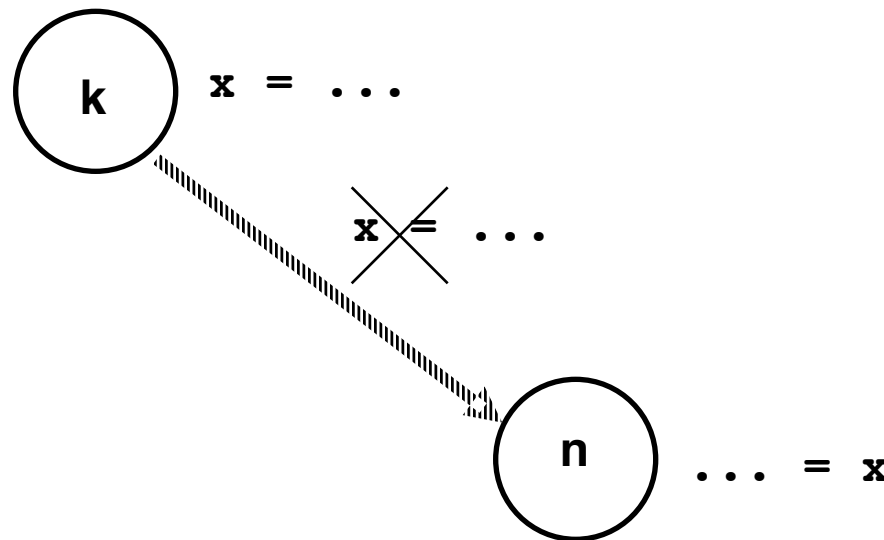
# Reaching Definitions

- *Definition* A statement which may change the value of a variable
- A definition of a variable  $x$  at node  $k$  *reaches* node  $n$  if there is a definition-clear path from  $k$  to  $n$ .



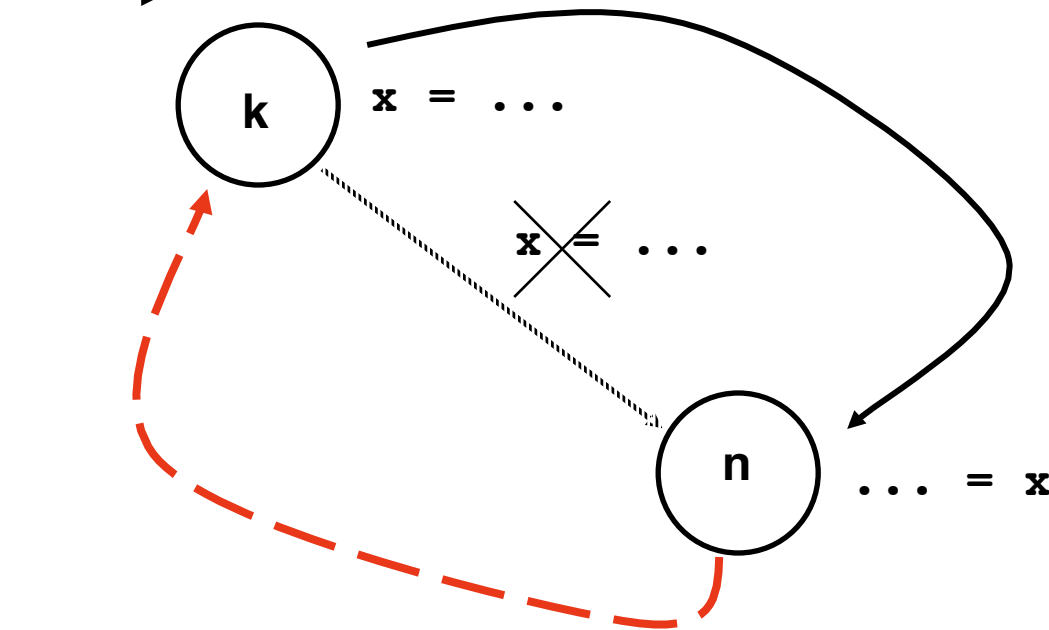
# Live Uses of Variables

- Use Appearance of a variable as an operand of a 3 address statement
- A use of a variable  $x$  at node  $n$  is *live on exit* from node  $k$  if there is a definition-clear path for  $x$  from  $k$  to  $n$ .



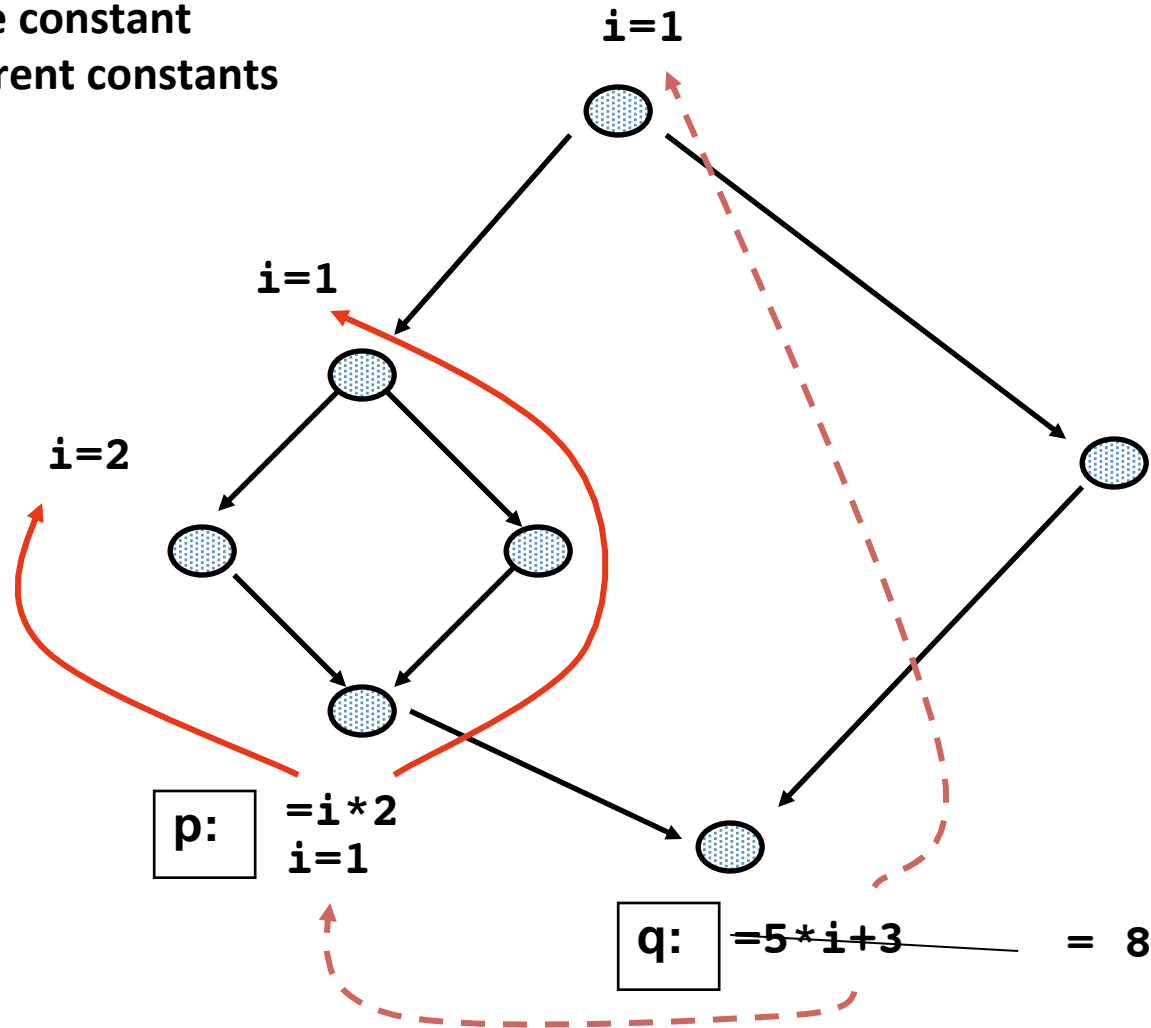
# Def-use Relations

- Use-def chain links an use to a definition that reaches that use  $\dashrightarrow$
- Def-use chain links a definition to an use that it reaches  $\longrightarrow$



# Constant Propagation

---> same constant  
-> different constants



# Reaching Definitions Equations

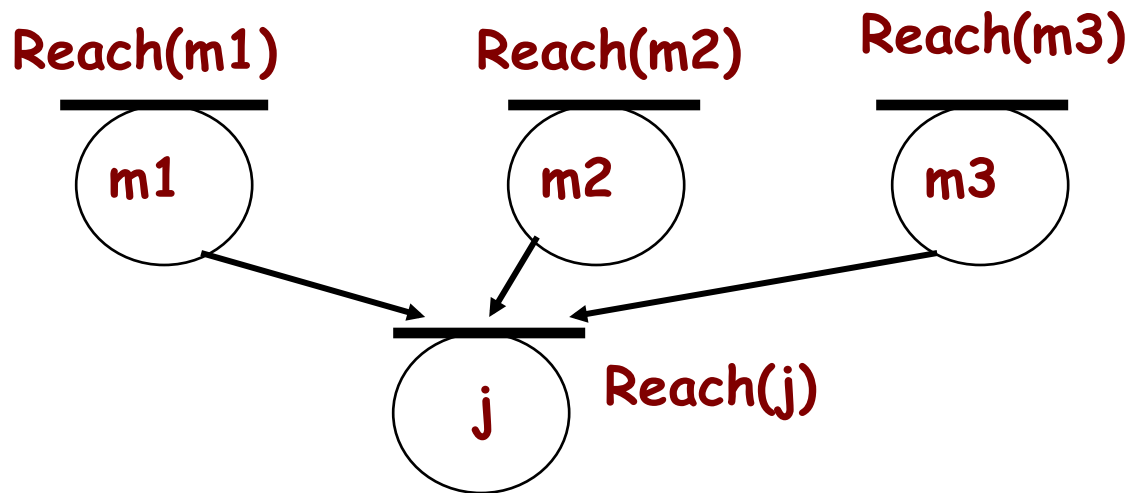
$$\text{Reach}(j) = \bigcup_{m \in \text{Pred}(j)} \{ \text{Reach}(m) \cap \text{pres}(m) \cup \text{dgen}(m) \}$$

where:

$\text{pres}(m)$  is the set of defs preserved through node  $m$

$\text{dgen}(m)$  is the set of defs generated at node  $m$

$\text{Pred}(j)$  is the set of immediate predecessors of node  $j$



# Live Uses Equations

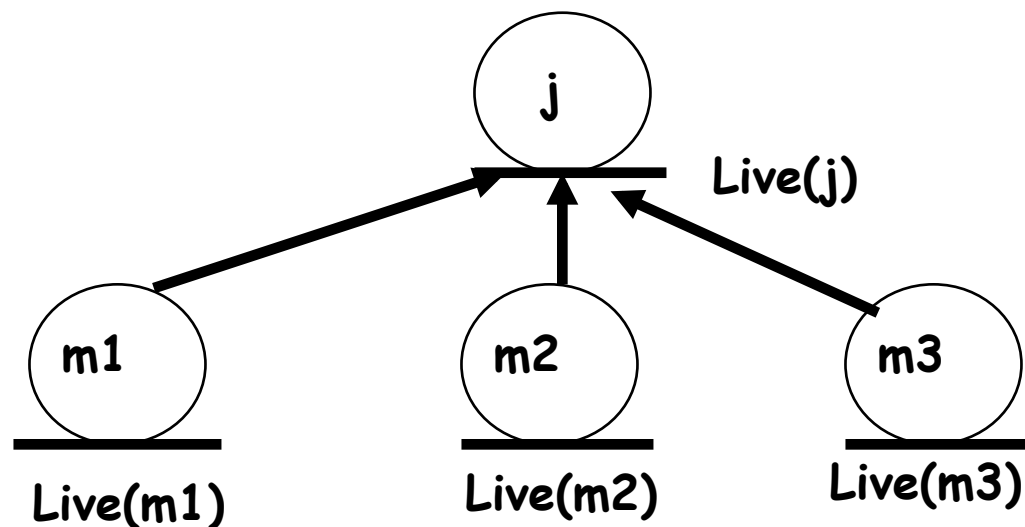
$$\text{Live}(j) = \bigcup_{m \in \text{Succ}(j)} \{ \text{Live}(m) \cap \text{pres}(m) \cup \text{ugen}(m) \}$$

where  $m \in \text{Succ}(j)$

$\text{pres}(m)$  is the set of uses preserved through node  $m$   
(these will correspond to variables whose defs are preserved)

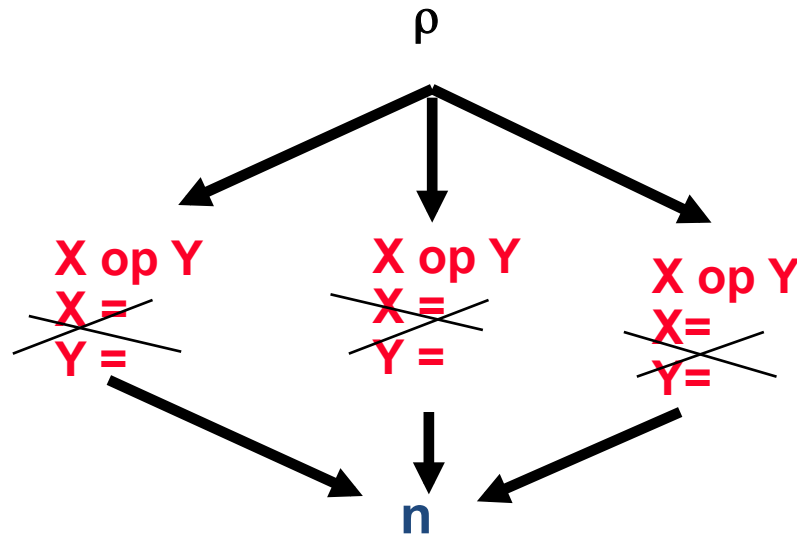
$\text{ugen}(m)$  is the set of uses generated at node  $m$

$\text{succ}(j)$  is the set of immediate successors of node  $j$

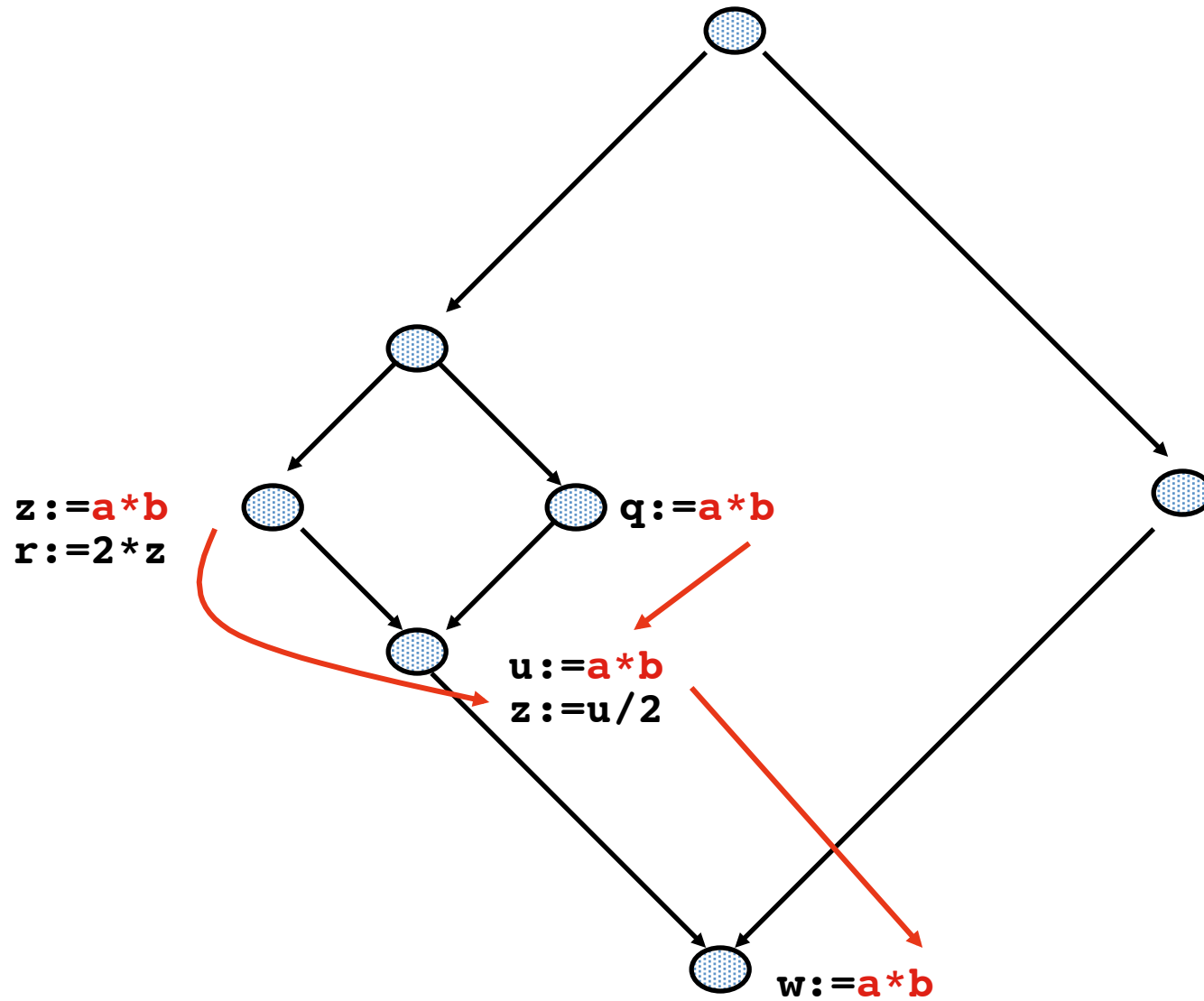


# Available Expressions

- An expression  $X \text{ op } Y$  is *available* at program point  $n$  if EVERY path from program entry to  $n$  evaluates  $X \text{ op } Y$ , and after every evaluation prior to reaching  $n$ , there are NO subsequent assignments to  $X$  or  $Y$ .



# Global Common Subexpressions





# Available Expressions Equations

$$Avail(j) = \bigcap_{m \in Pred(j)} \{ Avail(m) \cap epres(m) \cup egen(m) \}$$

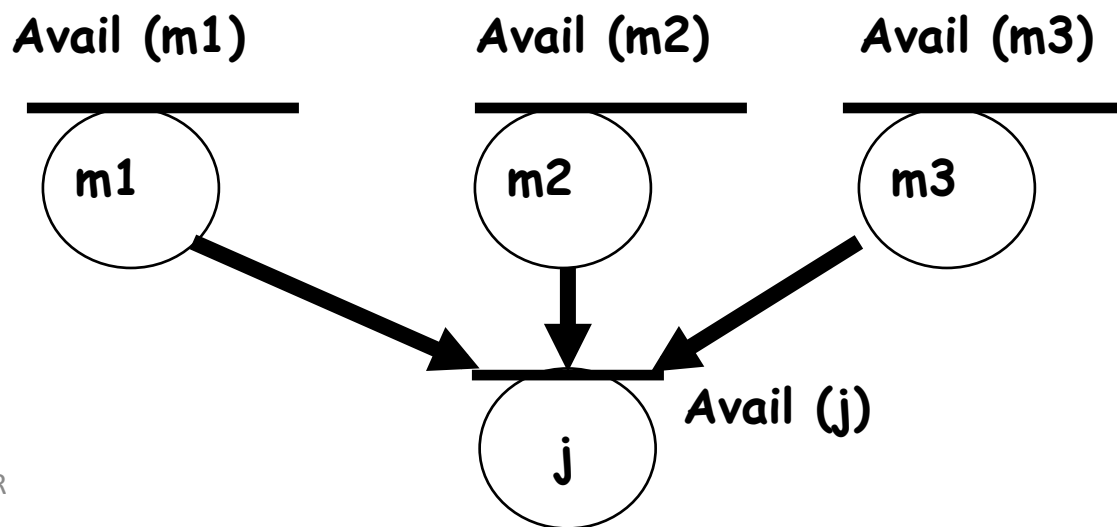
$m \in Pred(j)$

where:

$epres(m)$  is the set of expressions preserved through node  $m$

$egen(m)$  is the set of (downwards exposed) expressions generated at node  $m$

$pred(j)$  is the set of immediate predecessors of node  $j$



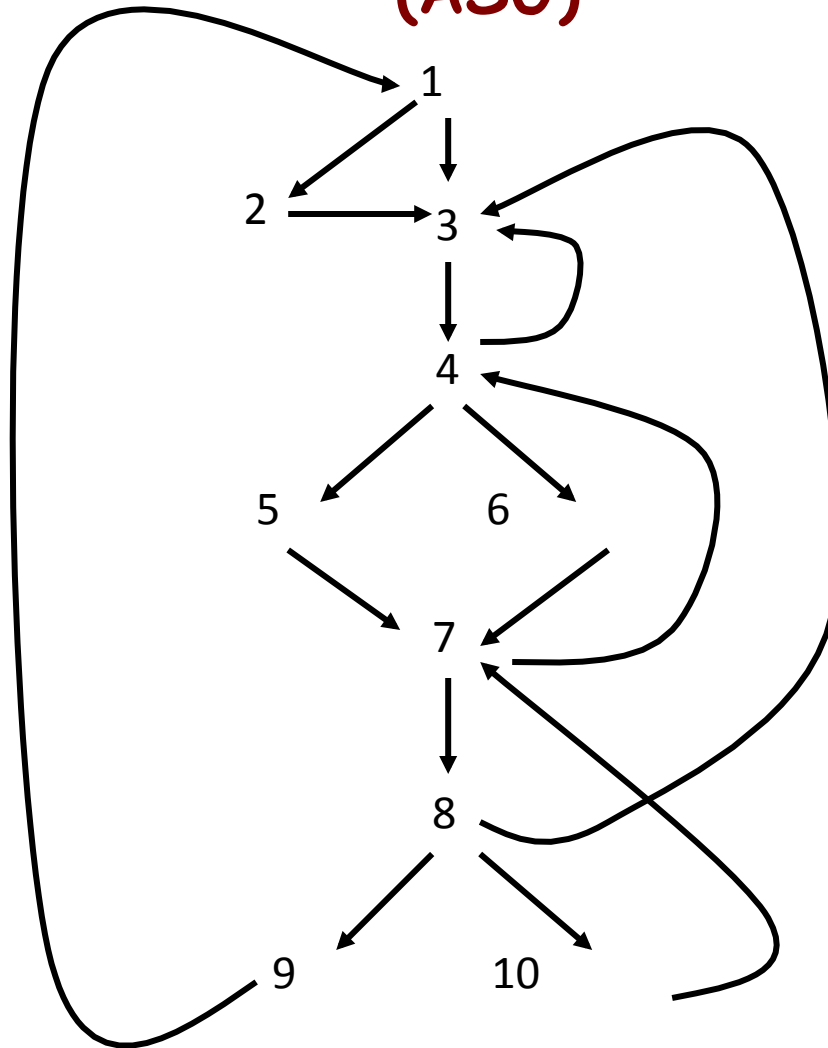
# Classical Dataflow Problems

	<u>May</u> Problems	<u>Must</u> Problems
Forward Problems	Reaching Defs	Available Exprs
Backward Problems	Live Uses of Variables	Very Busy Expressions

# Dominators and Natural Loops

- A **dominator** of a node  $x$  in a rooted digraph is a node  $y$  such that all paths from the root to  $x$  must pass through  $y$
- A node  $x$  can have many dominators. There is one dominator  $y$  such that there are no other dominators on a path from  $y$  to  $x$ . Then  $y$  is  $x$ 's immediate dominators.
- Dominators and spanning trees can define natural loops on a rooted digraph.

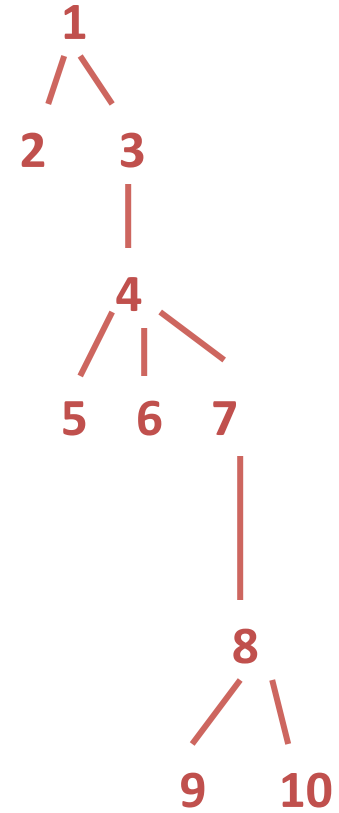
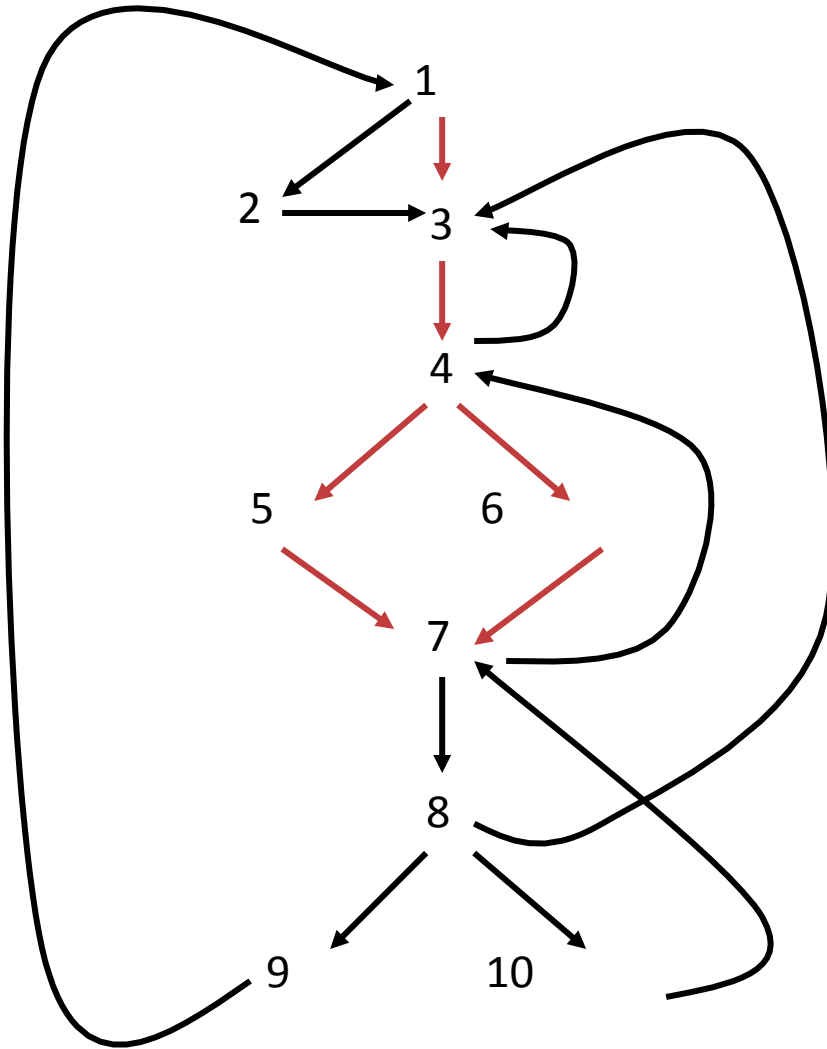
# Loops Example from Aho, Sethi, Ullman (ASU)



How to find the loops  
on this graph?

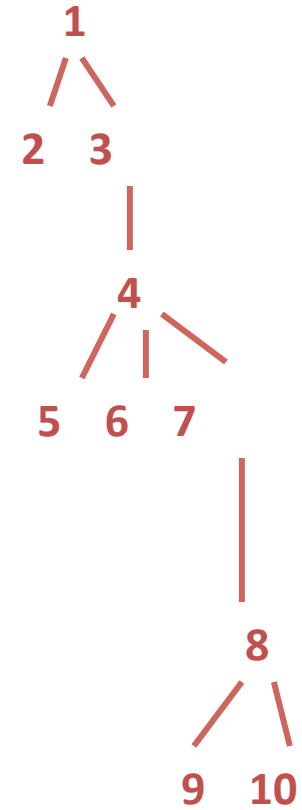
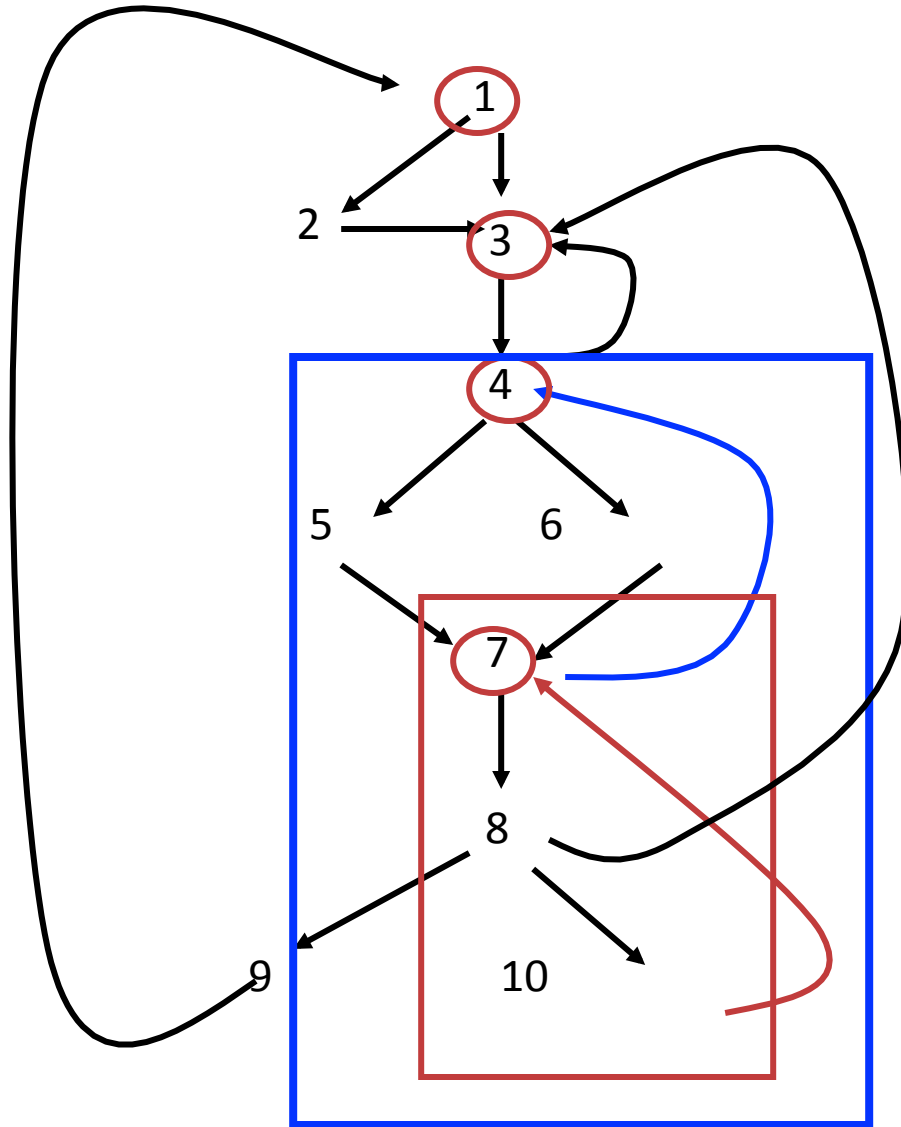
node 1 dominates node 7

# Example



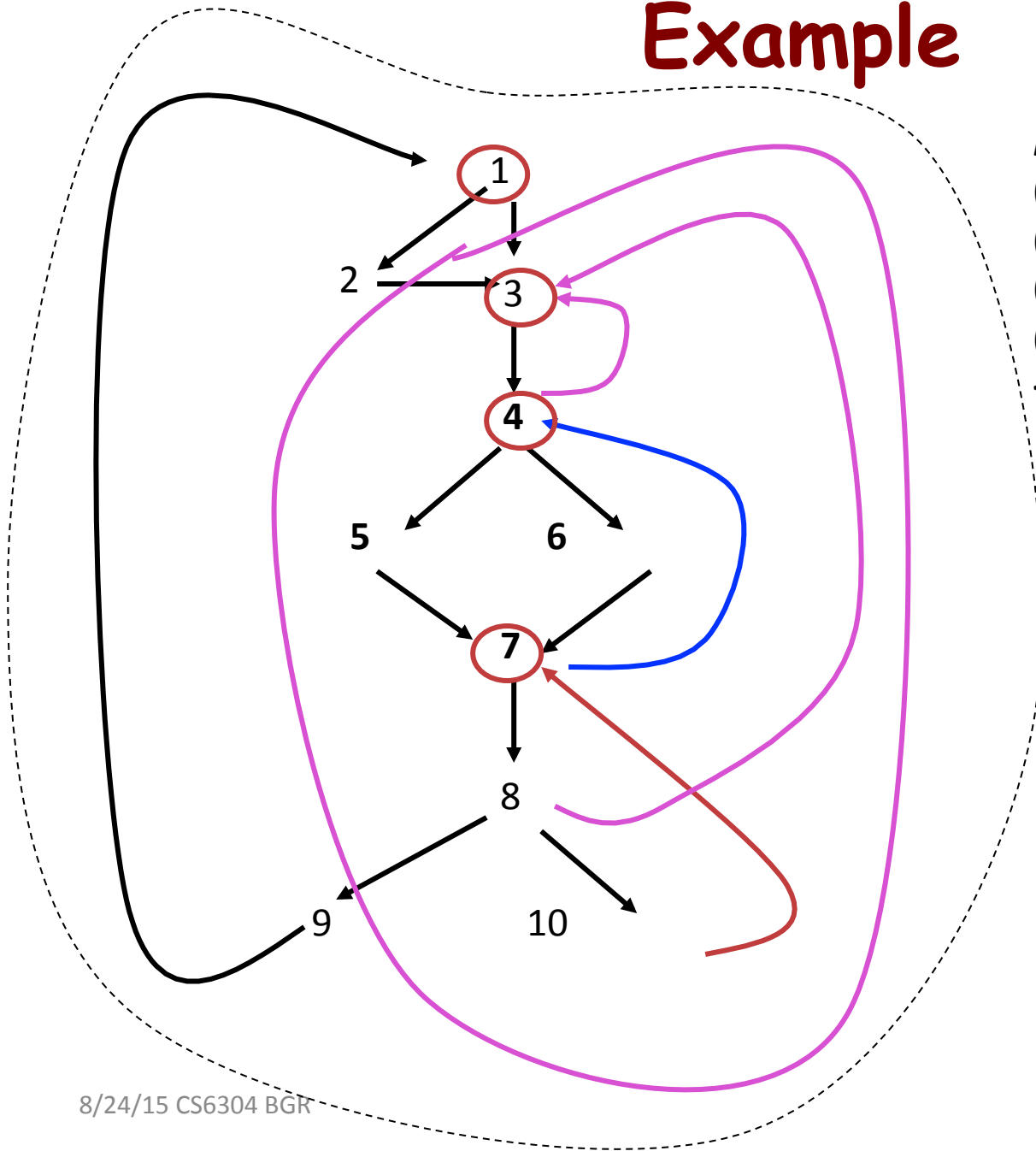
Dominator Tree

# Example



Dominator Tree

# Example



## Loops

(10,7): {7,8,10}

(7,4): {4,5,6,7,8,10}

(4,3)(8,3): {3,4,5,6,7,8,10}

(9,1):

{1,2,3,4,5,6,7,8,9,10}

# Dominators, ASU

- How to find dominators of CFG,  $G=(N,E,\rho)$ ? Use fixed point iteration (justification later)

$D(\rho) = \{\rho\}$

for  $n \in N - \{\rho\}$  do

{  $D(n) = N;$

while changes to any  $D(n)$  occur do

{ for  $n \in N - \{\rho\}$  do

$D(n) = \{n\} \cup \bigcap_{p \in \text{pred}(n)} D(p)$

}

$p \in \text{pred}(n)$



# Dominators

- Algorithm terminates since at every step some set  $D(k)$  becomes smaller; this cannot occur indefinitely, so loop terminates
- Invariant: Node  $k$  is parent of node  $n$  in the dominator tree, if node  $k$  is the *immediate dominator* of  $n$