Ensemble Kalman Filter, lecture 2

Asynchronous data assimilation

Pavel Sakov

Nansen Environmental and Remote Sensing Center, Norway

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Summer data assimilation school, Sibiu
27 July - 7 August 2009

Outline

Synchronous and asynchronous assimilation

Synchronous, or "3D" assimilation = observations are assumed to be taken at the assimilation time

Asynchronous, or "4D" assimilation = observations can be taken at time different than the assimilation time

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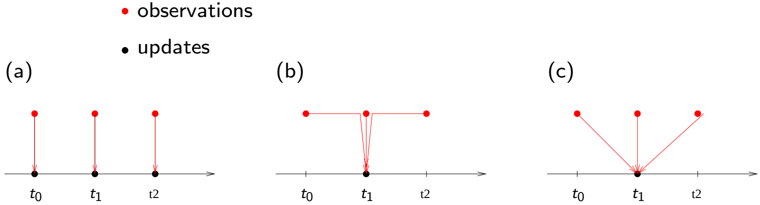
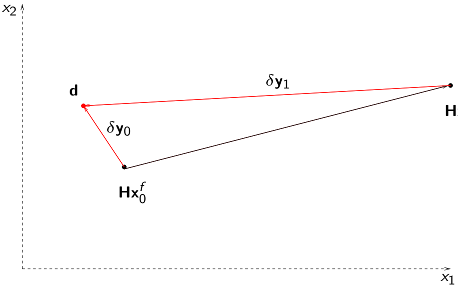


(a) Synchronous assimilation at each observation time
(b) Synchronous assimilation; asynchronous observations are assumed to be synchronous
(c) Asynchronous assimilation

Reasoning

0-order correction:



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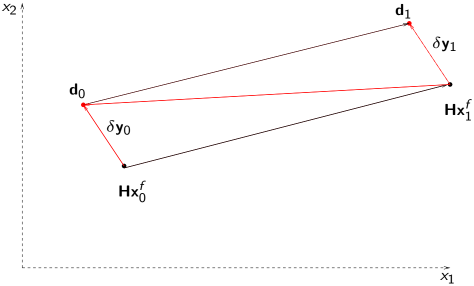

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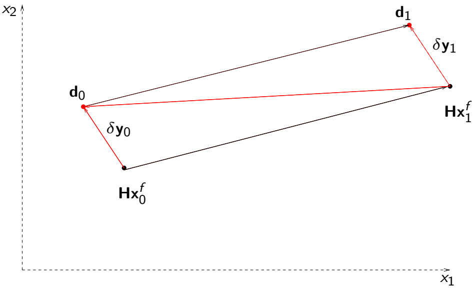

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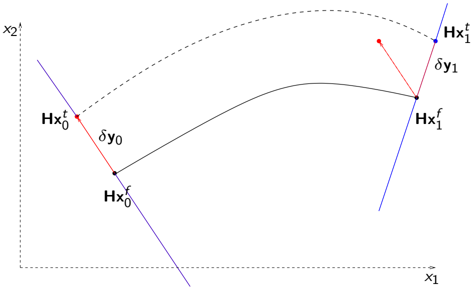

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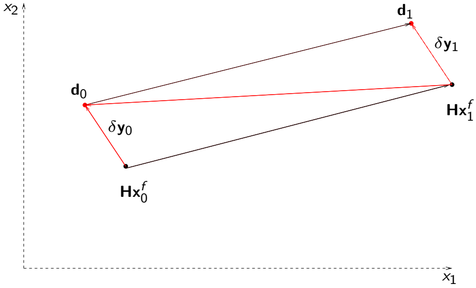

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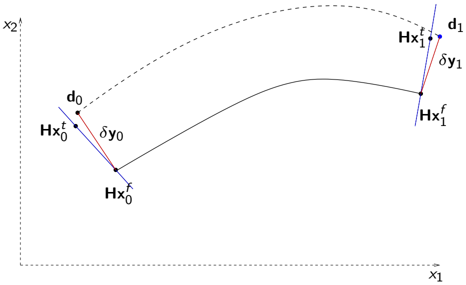

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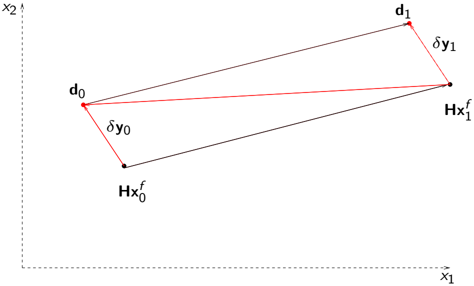

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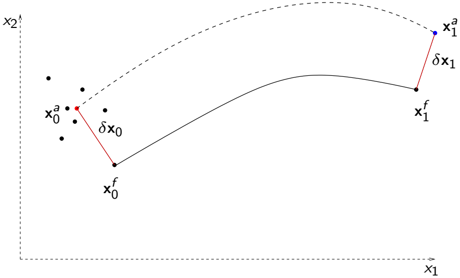

Historical overview

- Ensemble smoother (ES) (van Leeuwen and Evensen, 1996)
- Ensemble Kalman smoother (EnKS) (Evensen and van Leeuwen, 2000)
- 4DEnKF (Hunt et al., 2004)
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EnKF update and ensemble observations

The linear update equations:

$$\mathbf{x}^a - \mathbf{x}^f \equiv \delta\mathbf{x} = \mathbf{A}^f \mathbf{G}\,\mathbf{s},$$

$$\mathbf{A}^a - \mathbf{A}^f \equiv \delta\mathbf{A} = \mathbf{A}^f \mathbf{T}$$

where $\quad \mathbf{s} = \mathbf{R}^{-1/2}(\mathbf{y} - \mathbf{H}\mathbf{x}^f)/\sqrt{m-1}$

The gain writes as

$$\mathbf{G} = (\mathbf{I} + \mathbf{S}^{\mathrm{T}}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{T}}$$

$$= \mathbf{S}^{\mathrm{T}}(\mathbf{I} + \mathbf{S}\mathbf{S}^{\mathrm{T}})^{-1}$$

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EnKF: $\quad \mathbf{T} = \mathbf{G}(\mathbf{D} - \mathbf{S})$

ETKF: $\quad \mathbf{T} = (\mathbf{I} + \mathbf{S}^{\mathrm{T}}\mathbf{S})^{-1/2} - \mathbf{I}$

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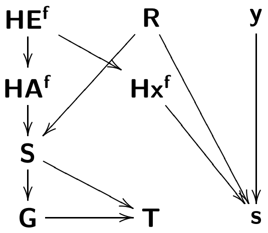

Evolution of corrections

Let us assimilate observations at t_0

$$\delta \mathbf{x}_0 = \mathbf{A}_0^f \boxed{\mathbf{G}_0 \, \mathbf{s}_0}$$

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Let M_{01} be the tangent linear propagator along the forecast system trajectory between t_0 and t_1:

$$\delta \mathbf{x}_1 = \mathbf{M}_{01} \, \delta \mathbf{x}_0 + O\left(\|\delta \mathbf{x}_0\|^2\right)$$

At t_1 the corrections become:

$$\delta \mathbf{x}_1 \sim \mathbf{M}_{01} \, \delta \mathbf{x}_0 = \mathbf{M}_{01} \mathbf{A}_0^f \mathbf{G}_0 \, \mathbf{s}_0 \sim \mathbf{A}_1^f \boxed{\mathbf{G}_0 \, \mathbf{s}_0}$$

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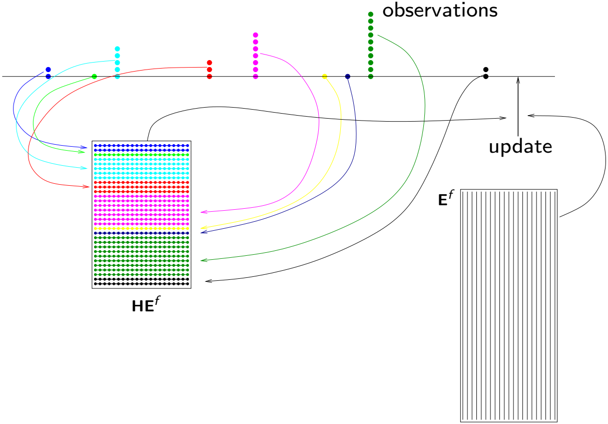
Parallel assimilation of asynchronous data with the EnKF

$$\mathbf{s} = [\mathbf{s}_1^{\mathrm{T}} \ldots \mathbf{s}_k^{\mathrm{T}}]^{\mathrm{T}}$$

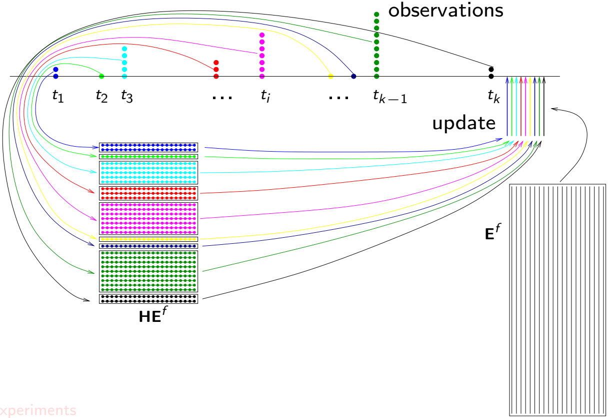
$$\mathbf{S} = [\mathbf{S}_1^{\mathrm{T}} \ldots \mathbf{S}_k^{\mathrm{T}}]^{\mathrm{T}}$$



EnKS

$$\mathbf{E}^a = \mathbf{E}^f \prod_{i=1}^{k} \mathbf{X}_5(t_i),$$

$$\mathbf{X}_5 = \frac{1}{m}\mathbf{1}\mathbf{1}^{\mathrm{T}} + (\mathbf{I} - \frac{1}{m}\mathbf{1}\mathbf{1}^{\mathrm{T}})(\mathbf{G}\,\delta\mathbf{y}\mathbf{1}^{\mathrm{T}} + \mathbf{T})$$



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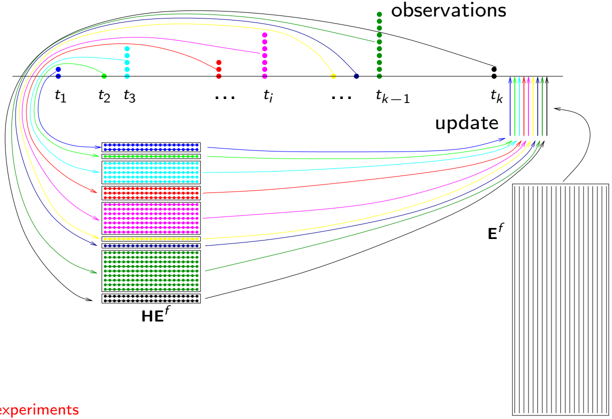

experiments

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Asynchronous assimilation: resume

- Recipe: use ensemble observations stored at observation times, as in "4D-LETKF" description (Hunt et al., 2007)
- This method is scheme-independent
- It is essentially equivalent to the EnKS solution (Evensen and van Leeuwen, 2000)
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References

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