Energy-Aware Spectrum Sharing for Dynamic Spectrum Access via Monotonic Optimization

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***Abstract*—In this paper, we investigate the optimal spectrum leasing and sharing for dynamic spectrum access (DSA) network which actively shares its spectrum resource with a group of secondary users (SUs) for reaping revenue. The DSA aims at maximizing its total charge to the SUs based on their achieved throughput, while taking into account the additional power consumption of the primary user (PU) to overcome the SUs' interference such that quality of service (QoS) of the PU is guaranteed. The interference between the PU and the SUs as well as the interference within the SUs themselves make the spectrum leasing problem strictly nonconvex and thus difficult to solve in general. To tackle with this difficulty, we exploit the decomposition structure of the concerned spectrum leasing problem and propose a layered monotonic optimization approach to solve it efficiently. Numerical results are presented to validate our proposed approach and its computational efficiency.**

I. Introduction

With a rapid growth in mobile data service and the growing crowded spectrum space, dynamic spectrum access (DSA) has been considered a promising paradigm to improve the spectrum efficiency and provide better wireless local service as a supplementary to cellular networks [1]. The merit of DSA lies in its intelligent reuse of underutilized spectrum of the primary user (PU) such that the secondary users (SUs) could obtain an opportunistic usage of the PU's licensed spectrum for data transmission. An important approach for implementing the DSA is spectrum leasing, through which the DSA leases the PU's spectrum to the SUs for reaping additional benefits, e.g., the economic revenue and the improved transmission performance. Despite gaining these additional benefits, the spectrum leasing incurs interference between the PU and the SUs, which thus necessitates a careful design of the resource management such that the benefits of the spectrum leasing can be truly achieved without sacrificing the PU's performance.

There have been several related works about the spectrum leasing in DSA networks. [2] and [3] considered that the PU actively leased its spectrum to the SUs and charged the SUs' interference accordingly. Their objective was to maximize the PU's total charge to the SUs' interference while limiting the total interference from all the SUs below a given threshold. [5] and [6] adopted the auction models to investigate the spectrum leasing and sharing in DSA networks with the given interference thresholds for the PUs. [4] further considered the interference limit tunable and aimed at choosing an optimal interference limit to trade off between the PU's interference-based charge and its QoS degradation. Taking into account the difficulty in charging the interference in practice, our work [7] proposed charging the achieved throughput of the SUs and modeled the joint price and power optimization problem as a two-stage Stackelberg game between the PU and the SUs.

In this paper, we aim at optimizing the economic reward of the spectrum leasing from charging the SUs' throughput while taking into account the PU's additional power consumption to overcome the SUs' interference. Due to the interference between the PU and the SUs as well as the interference within the SUs themselves, the optimal spectrum leasing problem is strictly nonconvex and thus is difficult to solve. To tackle with this technical difficulty, we exploit *the decomposition structure of the optimal leasing problem* and propose *a layered monotonic optimization approach* to solve it efficiently.

II. System Model and Problem Formulation

A. System Model and the Throughput Model

We consider that a group of SUs, denoted by $\Omega = \{1, 2, ..., S\}$, coexist with one representative PU in a DSA network. The PU performs the uplink transmission to the base station (BS) of the network on its licensed spectrum. To gain the additional reward, the PU leases its spectrum to the SUs. We consider that each SU s is comprised of a transmitter (Tx) and a receiver (Rx). The spectrum leasing results in two types of interference, i.e., the interference between the PU and each SU s and the interference within the SUs themselves. Let p_0 and q_s denote the transmit-powers of the PU and SU s, respectively. The throughput of each SU s can be given by

$$R_s(q_s, p_0, \{q_j\}_{j\neq s, j\in\Omega}) = \log_2\left(1 + \frac{q_s g_{ss}}{n + p_0 g_{0s} + \sum_{j\neq s, j\in\Omega} q_j g_{js}}\right), \forall s \in \Omega. \quad (1)$$

Specifically, g_{ss}, g_{0s}, and g_{js} denote the channel power gains from the Tx of SU s to its Rx, from the PU to the Rx of SU s, and from the Tx of SU j to the Rx of SU s, respectively. Besides, n denotes the power of the background noise. Meanwhile, the throughput of the PU can be given by

$$R_0(p_0, \{q_s\}_{s\in\Omega}) = \log_2\left(1 + \frac{p_0 g_{0B}}{n + \sum_{s\in\Omega} q_s g_{sB}}\right), \quad (2)$$

where g_{0B} and g_{sB} denote the channel power gains from the PU to the BS and from the Tx of SU s to the BS, respectively.

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B. Revenue Optimization of the PU and Its Feasibility

The PU has a throughput requirement denoted by R_0^{req}, i.e., requiring $R_0(p_0,\{q_s\}_{s\in\Omega}) \geq R_0^{\text{req}}$. Thus, the PU's transmit-power, as a response to the SUs' interference, should meet

$$p_0 \geq \theta_0 \frac{n+\sum_{s\in\Omega} q_s g_{sB}}{g_{0B}}, \tag{3}$$

where parameter $\theta_0 = 2^{R_0^{\text{req}}}-1$. Let us consider a *benchmark case* in which no spectrum leasing is incurred. In this benchmark case, the minimum transmit-power of the PU is given by $p_0^{\min} = \theta_0 \frac{n}{g_{0B}}$. Thus, taking into account the SUs' interference, the PU consumes the additional power equal to $\left(p_0 - p_0^{\min}\right)$, when leasing its spectrum to the SUs. Despite this additional cost, the PU charges the SUs according to their achieved throughput. Therefore, the PU's optimal spectrum leasing problem can be formulated as follows:

$$\text{(P1)}: \max \sum_{s\in\Omega} \alpha_s R_s(q_s,p_0,\{q_j\}_{j\neq s, j\in\Omega}) - \beta\left(p_0 - p_0^{\min}\right)$$

$$\text{subject to: } p_0 \geq \theta_0 \frac{n+\sum_{s\in\Omega} q_s g_{sB}}{g_{0B}}, \tag{4}$$

$$R_s(q_s,p_0,\{q_j\}_{j\neq s, j\in\Omega}) \geq R_s^{\text{req}}, \forall s\in\Omega, \tag{5}$$

$$0 \leq p_0 \leq P_0^{\max}, \tag{6}$$

$$0 \leq q_s \leq Q_s^{\max}, \forall s\in\Omega, \tag{7}$$

decision variable: p_0 and $\{q_s\}_{s\in\Omega}$.

Here, α_s denotes the PU's marginal charge to the throughput of each SU s, and β denotes the PU's marginal power consumption cost[1]. Constraints (4) and (5) ensure that the PU and each SU s achieve their throughput requirements R_0^{req} and R_s^{req}, respectively. Constraints (6) and (7) limit the transmit-powers of the PU and each SU s by their respective maximum levels denoted by $P_0^{\max}$ and $Q_s^{\max}$. In particular, it is noticed that constraint (5) is equivalent to

$$\frac{q_s g_{ss}}{n + p_0 g_{0s} + \sum_{j\neq s, j\in\Omega} q_j g_{js}} \geq \theta_s, \forall s\in\Omega, \tag{8}$$

with $\theta_s = 2^{R_s^{\text{req}}}-1$. We will use (8) in the rest of this paper.

To analyze the feasibility of Problem (P1), we define an S-by-S matrix $\mathbf{M}$ with its each element $M_{ss}=0, \forall s\in\Omega$,

$$\text{and} \quad M_{sj} = \frac{\theta_s\theta_0 g_{0s} g_{jB} + \theta_s g_{js} g_{0B}}{g_{ss}g_{0B} - \theta_s\theta_0 g_{sB} g_{0s}}, \forall s,j\in\Omega, \text{ and } s\neq j.$$

Recall that S is the total number of the SUs. We also define an S-by-1 vector $\mathbf{u}$ with its each element $u_s = \frac{\theta_s n g_{0B} + \theta_s\theta_0 n g_{0s}}{g_{ss}g_{0B} - \theta_s\theta_0 g_{sB} g_{0s}}, \forall s\in\Omega$. Let $\hat{p}_0$ and $\hat{q}_s$ denote the respective *minimum transmit-powers required by the PU and each SU s such that constraints (4) and (8) (i.e., (5)) are satisfied exactly*. We then obtain the result regarding $\hat{p}_0$ and $\{\hat{q}_s\}_{s\in\Omega}$ as follows.

Lemma *1:* Suppose that Conditions (C1) and (C2) are met: **(C1)**: $g_{ss}g_{0B} - \theta_s\theta_0 g_{sB}g_{0s} > 0$ holds for each SU $s\in\Omega$.
(C2): the spectrum radius $\rho(\mathbf{M})$ of $\mathbf{M}$, defined as $\rho(\mathbf{M}) = \max\{|\lambda| | \lambda \text{ is an eigenvalue of } \mathbf{M}\}$, satisfies that $\rho(\mathbf{M})<1$. Then, $\hat{p}_0$ and $\{\hat{q}_s\}_{s\in\Omega}$ can be compactly given by

$$\hat{p}_0 = \theta_0 \frac{n+\sum_{s\in\Omega}\hat{q}_s g_{sB}}{g_{0B}}, \text{ and } \hat{q}_s = \left((\mathbf{I}-\mathbf{M})^{-1}\mathbf{u}\right)_s,$$

where $\mathbf{I}$ is an S-by-S identity matrix. $(\mathbf{x})_s$ denotes the s-th element of vector $\mathbf{x}$.

Proof: An important property of Problem (P1) is that *constraint (4) should be strictly binding for achieving the optimum* (otherwise, the PU can always reduce its transmit-power for increasing the objective function but without violating any constraint). By further setting (8) to be strictly binding and substituting p_0 with $\{q_s\}_{s\in\Omega}$ (i.e., via (4)), we obtain a set of linear equations, i.e., $(\mathbf{I}-\mathbf{M})\mathbf{q}=\mathbf{u}$, with vector $\mathbf{q}$ representing the vector form of $\{q_s\}_{s\in\Omega}$. Applying the Perron-Frobenius theorem [8] to these equations such that they have nonnegative solutions (i.e., $\hat{\mathbf{q}}\geq 0$), we obtain the results in Lemma 1. □

Remark 1: Condition (C1) takes into account the PU and each SU s, and it requires the interference channel gain between them to be weak enough. Condition (C2) takes into account the PU and all the SUs, and it essentially requires that the aggregate effect of the interference channel gain (including both those between the PU and each SU s, and those between different SUs s and j) is weak enough. Specifically, based on the property of spectrum radius $\rho(\mathbf{M})$ [8], a sufficient condition to guarantee $\rho(\mathbf{M})<1$ can be given as follows:

$$\theta_s\theta_0 g_{sB}g_{0s} + g_{0B}\theta_s \sum_{j\neq s, j\in\Omega} g_{js} + \theta_s\theta_0 g_{0s}\sum_{j\neq s, j\in\Omega} g_{jB} \leq g_{ss}g_{0B}, \forall s\in\Omega. \tag{9}$$

In (9), the 2nd part accounts for the interference between different SUs s and j, and the 3rd part accounts for the interference between SU j and the PU. Notice that (9) becomes Condition (C1) if there is only one SU in Ω.

Using $\hat{p}_0$ and $\{\hat{q}_s\}_{s\in\Omega}$ in Lemma 1, we can characterize the feasibility of Problem (P1) as follows. Problem (P1) is feasible, if Condition **(C3)** that $\hat{p}_0 \leq P_0^{\max}$ and $\hat{q}_s \leq Q_s^{\max}, \forall s\in\mathcal{S}$ holds. In this work, to focus on quantifying the PU's maximum benefit and designing an efficient algorithm to achieve this maximum benefit, we assume that Problem (P1) is feasible. Notice that Lemma 1 and Condition (C3) above can be used for *admission control* when a large number of SUs exist.

C. Layered Structure of Problem (P1)

Due to the interference between the PU and the SUs as well as the interference within the SUs, the objective function of Problem (P1) is strictly nonconvex. Thus, (P1) is a nonconvex optimization problem and is difficult to solve in general. To tackle with this difficulty, we exploit that (4) is binding at the optimum (as stated in the proof of Lemma 1) and vertically decompose (P1) into a *two-layered structure*. Specifically, *in the bottom-layer, given the PU's transmit-power p_0, we optimize the transmit-powers of the SUs* by solving

$$\text{(P1-Bottom)}: F(p_0) = \max_{\{q_s\}_{s\in\Omega}} \sum_{s\in\Omega} \alpha_s R_s(q_s,p_0,\{q_j\}_{j\neq s, j\in\Omega})$$

subject to: Constraints $(4), (7),$ and (8).

[1] In this work, we consider the PU's charge (or the pricing mechanism) as fixed and focus on optimizing the transmit-powers of the PU and the SUs by assuming that the SUs follow the decisions of the PU for achieving their respective throughput requirements (i.e., the PU and SUs are cooperative).

Different from Problem (P1), in (P1-Bottom), p_0 is given in advance in constraints (4), (7), and (8) (which represents constraint (5)). Notice that the maximum of the objective function of Problem (P1-Bottom) depends on the given p_0, and that is why we use function $F(p_0)$ to denote it.

By using $F(p_0)$ from the bottom-layer, we then solve the *top-layer problem that optimizes the PU's transmit-power* as:

$$\text{(P1-Top):} \max_{p_0^{\min} \leq p_0 \leq P_0^{\max}} F(p_0) - \beta(p_0 - p_0^{\min}).$$

Recall that $p_0^{\min} = \theta_0 \frac{n}{g_{0B}}$ is given before. In the next two sections, we focus on solving Problem (P1-Bottom) and Problem (P1-Top) in a way of backward induction.

III. Optimal Solution of Problem (P1-Bottom)

A. Hidden Monotonicity of Problem (P1-Bottom)

We solve Problem (P1-Bottom) in this section. It is observed that (P1-Bottom) is a nonconvex optimization problem and thus is difficult to solve. We first introduce the set of auxiliary variables $\{y_s\}_{s\in\Omega}$ based on the following continuous mapping:

$$y_s = \frac{q_s g_{ss}}{n + p_0 g_{0s} + \sum_{j\neq s, j\in\Omega} q_j g_{js}}, \forall s \in \Omega. \quad (10)$$

Using $\{y_s\}_{s\in\Omega}$, we present the following Problem (P2):

$$\text{(P2):} \max_{\{y_s\}_{s\in\Omega}} \sum_{s\in\Omega} \alpha_s \log_2(1 + y_s), \text{ subject to: } \{y_s\} \in \mathcal{G} \cap \mathcal{H}.$$

where $\mathcal{G}$ is given by

$$\mathcal{G} = \Big\{\{y_s\}_{s\in\Omega} | 0 \leq y_s \leq \frac{q_s g_{ss}}{n + p_0 g_{0s} + \sum_{j\neq s, j\in\Omega} q_j g_{js}}, \forall s \in \Omega, \text{ and } 0 \leq q_s \leq Q_s^{\max}, \forall s \in \Omega, \text{ and } \sum_{s\in\Omega} q_s g_{sB} \leq \frac{p_0 g_{0B}}{\theta_0} - n\Big\},$$

and $\mathcal{H} = \{\{y_s\}_{s\in\Omega} | y_s \geq \theta_s, \forall s \in \Omega\}$. Particularly, according to Section 2.3 of [9], set $\mathcal{G}$ can be considered as the *normal hull* of the following set $\mathcal{D}$ as follows:

$$\mathcal{D} = \left\{\{q_s\}_{s\in\Omega} | 0 \leq q_s \leq Q_s^{\max}, \forall s \in \Omega, \sum_{s\in\Omega} q_s g_{sB} \leq \frac{p_0 g_{0B}}{\theta_0} - n\right\},$$

which is induced by the mapping given in (10). Thus, $\mathcal{G}$ is a normal set. Meanwhile, $\mathcal{H}$ is a reversed normal set. Notice that due to space limitation, the detailed definitions for the normal set and reverse normal set are skipped here. Interested readers please refer to [9] for the details.

In particular, we have the following property regarding (P2):

Proposition 1: Given the PU's transmit-power p_0, Problem (P2) is a monotonic optimization with respect to $\{y_s\}_{s\in\Omega}$.

Proof: The objective function of Problem (P2) is monotonically increasing, and the feasible set is the intersection of the normal set $\mathcal{G}$ and the reversed normal set $\mathcal{H}$. Thus, according to the canonical form of the monotonic optimization given in [9], Problem (P2) is a standard monotonic optimization. □

Proposition 2: Given p_0 that ensures that Problem (P1-Bottom) is feasible, solving Problem (P2) is equivalent to solving Problem (P1-Bottom). Specifically, let $\{y_s^*\}_{s\in\Omega}$ denote the set of the optimal solutions for Problem (P2). Then, the set of optimal solutions $\{q_s^*\}_{s\in\Omega}$ for Problem (P1-Bottom) can be directly obtained by solving the mapping given in (10).

Proof: The proof follows Proposition 2.3 in [9] directly, and we thus skip the details here due to space limitation. □

The monotonicity of Problem (P2) (i.e., Proposition 1) enables us to solve it by using the so-called *polyblock approximation (PA)* algorithm. The key idea of the PA algorithm is to construct a series of polyblocks to approximate the feasible set of (P2), and then to find the best vertex of the polyblocks that maximizes the objective function and meanwhile falls into the feasible set. However, due to space limitation, we skip the details about the PA, whose details can be referred to [9].

To solve (P1-Bottom), we propose Algorithm (A1) below that incorporates the PA algorithm in Step 1. The output of the PA, i.e., y_s^*, in fact is the optimal signal to interference plus noise ratio which each SU s can achieve. Using $\{y_s^*\}_{s\in\Omega}$, we then calculate the set of optimal transmit-powers $\{q_s^*\}_{s\in\Omega}$ for the SUs in Step 3, in which the S-by-S matrix $\mathbf{N}$ is defined as $N_{ss} = 0, \forall s \in \Omega$ and $N_{sj} = \frac{g_{js}\theta_s}{g_{ss}}, \forall s, j \in \Omega, j \neq s$.

Algorithm (A1): to solve Problem (P1-Bottom)

1: Given p_0, solve Problem (P2) by using the PA algorithm. Denote the set of optimal solutions of (P2) by $\{y_s^*\}_{s\in\Omega}$.
2: Set the S-by-1 vector $\mathbf{r}$ according to $r_s = y_s^* \frac{n+p_0 g_{0s}}{g_{ss}}, \forall s \in \Omega$.
3: Calculate $\mathbf{q}^* = (\mathbf{I} - \mathbf{N})^{-1}\mathbf{r}$, and output $\{q_s^*\}_{s\in\Omega}$ based on $\mathbf{q}^*$.

Notice that Algorithm (A1) presumes that Problem (P1-Bottom) is feasible. We will discuss about the choice of p_0 such that (P1-Bottom) is feasible in the next subsection.

B. Feasibility of Problem (P1-Bottom)

We investigate how the given p_0 influences the feasibility of Problem (P1-Bottom). Specifically, given p_0, the SUs' transmit-powers to meet their respective requirements $\{\theta_s\}_{s\in\Omega}$ (according to (8)) exactly can be obtained by solving

$$q_s - \sum_{j\neq s, j\in\Omega} \frac{g_{js}\theta_s}{g_{ss}} q_j = \theta_s \frac{n + p_0 g_{0s}}{g_{ss}}, \forall s \in \Omega. \quad (11)$$

Further define an S-by-1 vector $\mathbf{v}$ with its each element $v_s = \frac{\theta_s n}{g_{ss}}, s \in \Omega$ and an S-by-1 vector $\mathbf{w}$ with its each element $w_s = \frac{\theta_s g_{0s}}{g_{ss}}, s \in \Omega$. Then, the solution of eq. (11), i.e., the SUs' transmit-powers to meet $\{\theta_s\}_{s\in\Omega}$ exactly are given by

$$\hat{\mathbf{q}}(p_0) = (\mathbf{I} - \mathbf{N})^{-1}(\mathbf{v} + \mathbf{w}p_0). \quad (12)$$

Lemma *2:* $\hat{\mathbf{q}}(p_0)$ is nonnegative if $p_0 > 0$.

Proof: According to the definitions of matrices $\mathbf{M}$ and $\mathbf{N}$ provided before, there always exits $N_{sj} \leq M_{sj}, \forall s, j$. Recall that $\rho(\mathbf{M}) < 1$ is assumed in Condition (C2) (in Lemma 1). Thus, based on Theorem 8.4.5 [10], there always exists $\rho(\mathbf{N}) \leq \rho(\mathbf{M}) < 1$. By further using the Perron-Frobenius theorem [8], we obtain the result stated in Lemma 3. □

Based on $\hat{\mathbf{q}}(p_0)$, we further obtain the following result.

Lemma *3:* Problem (P1-Bottom) is feasible, if the condition that $\underline{P} \leq p_0 \leq \overline{P}$ is satisfied, where

$$\underline{P} = \frac{n + \sum_{s\in\Omega} g_{sB}\left((\mathbf{I}-\mathbf{N})^{-1}\mathbf{v}\right)_s}{\frac{g_{0B}}{\theta_0} - \sum_{s\in\Omega} g_{sB}\left((\mathbf{I}-\mathbf{N})^{-1}\mathbf{w}\right)_s}, \quad (13)$$

$$\overline{P} = \min_{s\in\Omega} \frac{\left((\mathbf{I}-\mathbf{N})\mathbf{Q}^{\max} - \mathbf{v}\right)_s}{\mathbf{w}_s}, \quad (14)$$

and the S-by-1 vector $\mathbf{Q}^{\max} = \{Q_1^{\max}, Q_2^{\max}, ..., Q_S^{\max}\}$ (recall that $(\mathbf{x})_s$ denotes the s-th element of vector $\mathbf{x}$).

Proof: Recall that (4) and $\hat{q}_s(p_0) \leq Q_s^{\max}, \forall s \in \Omega$ are required to ensure that Problem (P1-Bottom) is feasible. By substituting (12) into (4) and performing some manipulations, we obtain the lower bound $\underline{P}$ for p_0 in (13). Meanwhile, by comparing (12) with $\mathbf{Q}^{\max}$, we obtain the upper bound $\overline{P}$ in (14). □

C. Critical Threshold to make Constraint (4) Binding

Notice that constraint (4) should be binding at the optimum of Problem (P1-Bottom), which enables us to decompose Problem (P1) into (P1-Top) and (P1-Bottom). To this end, we quantify a critical threshold (denoted by P_{th}) within $[\underline{P}, \overline{P}]$, such that (4) is binding at the optimum of Problem (P1-Bottom) when $\underline{P} \leq p_0 \leq P_{\text{th}}$. In particular, the smaller the p_0, the more likely that (4) will be binding at the optimum of (P1-Bottom). Therefore, we propose Algorithm (A2), which is based on the bisection search, to determine P_{th} efficiently.

Algorithm (A2): to determine the value of P_{th}

1: Set δ and ϵ. Set $p_{\text{lower}} = \underline{P}$ and $p_{\text{upper}} = \overline{P}$.
2: **while** $|p_{\text{lower}} - p_{\text{upper}}| > \epsilon$ **do**
3: Set $p_0 = \frac{p_{\text{lower}} + p_{\text{upper}}}{2}$.
4: Use Algorithm (A1) to solve Problem (P1-Bottom). Denote the set of optimal solutions by $\{q_s^o\}_{s\in\Omega}$.
5: Evaluate $\eta = |p_0 - \theta_0 \frac{n + \sum_{s\in\Omega} q_s^o g_{sB}}{g_{0B}}|$.
6: If $\eta < \delta$, set $p_{\text{lower}} = p_0$. Otherwise, set $p_{\text{upper}} = p_0$.
7: **end while**
8: Output $P_{\text{th}} = p_0$.

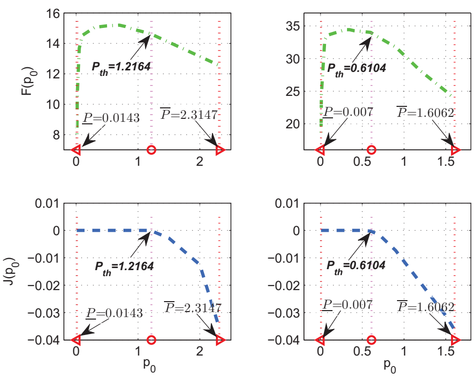

Fig. 1. Illustration of $\underline{P}$, $\overline{P}$, and P_{th}. The left two subfigures are with $R_0^{\text{req}} = 6$ b/s/Hz, $\theta_s = 3, \forall s$, and $\alpha_s = 1, \forall s$. The right two subfigures are with $R_0^{\text{req}} = 5$ b/s/Hz, $\theta_s = 4, \forall s$, and $\alpha_s = 2, \forall s$.

Figure 1 illustrates $\underline{P}$, $\overline{P}$, and P_{th} discussed above by enumerating p_0. In each subfigure, we mark out $\underline{P}$ and $\overline{P}$, which are derived according to (13) and (14), respectively. Besides, we also mark out P_{th}, which is obtained by using Algorithm (A2). To validate (A2), in the two subfigures at the bottom, we plot the value of $J(p_0) = \theta_0 \frac{n + \sum_{s\in\Omega} q_s g_{sB}}{g_{0B}} - p_0$. The results show that when $p_0 \leq P_{\text{th}}$, $J(p_0)$ keeps at zero, meaning that constraint (4) is binding. In comparison, when $p_0 > P_{\text{th}}$, $J(p_0)$ becomes negative, meaning that (4) becomes inactive. We emphasize that finding P_{th} is important, since it helps reduce the search space of p_0 (based on the rationale of our proposed decomposition structure). As shown in Figure 1, the threshold P_{th} is usually much smaller than the upper bound $\overline{P}$, especially when $\{Q_s^{\max}\}$ are large.

IV. Optimal Solution of Problem (P1-Top)

A. Property of Problem (P1-Top)

After solving (P1-Bottom), we next solve Problem (P1-Top) in this section. Specifically, based on $\underline{P}$ and P_{th} obtained before, the top-problem (P1-Top) can be reexpressed as follows

$$\text{(P1-Top): } \max F(p_0) - \beta\left(p_0 - p_0^{\min}\right)$$
$$\text{subject to: } \max\left\{p_0^{\min}, \underline{P}\right\} \leq p_0 \leq \min\{P_{\text{th}}, \overline{P}\}.$$

The key difficulty in solving Problem (P1-Top) lies in that we cannot obtain $F(p_0)$ in closed form. In other words, Problem (P1-Top) is an optimization problem, in which the objective function cannot be given analytically. Hence, Problem (P1-Top) is difficult to solve in general.

Fortunately, an important observation from Fig. 1 is that $F(p_0)$ is always unimodal, i.e., there exists a special threshold Γ such that $F(p_0)$ is increasing when $\max\{\underline{P}, p_0^{\min}\} \leq p_0 \leq \Gamma$, and is decreasing when $\Gamma \leq p_0 \leq \min\{\overline{P}, P_{\text{th}}\}$. Despite that we cannot analytically prove this unimodal property, our extensive numerical examples (e.g., as shown in Fig. 1 and Fig. 2 below) always show that this property holds for $F(p_0)$. Based on the unimodal property, we propose Algorithm (A3) to find Γ efficiently. Notice that Algorithm (A3) relies on approximating the gradient of $F(p_0)$, denoted by v, in Steps 5 and 6. Based on v, Algorithm (A3) updates p_0 by using the bisection method in Steps 3 and 7.

Figure 2 illustrates Γ discussed above by enumerating p_0. For each tested case, we mark out Γ (denoted by the red star), which is obtained by using Algorithm (A3). The results verify that Γ achieves the maximum of $F(p_0)$ for $\underline{P} \leq p_0 \leq P_{\text{th}}$.

B. Monotonicity of Problem (P1-Top) and Its Optimal Solution

Since $\beta\left(p_0 - p_0^{\min}\right)$ (i.e., the second part of the objective function) is increasing in p_0, the optimal solution of Problem (P1-Top) can only happen when $\max\left\{p_0^{\min}, \underline{P}\right\} \leq p_0 \leq \Gamma$. In other words, Problem (P1-Top) becomes equivalent to

$$\text{(P1-Top-E): } \max F(p_0) - \beta\left(p_0 - p_0^{\min}\right)$$
$$\text{subject to: } \max\left\{p_0^{\min}, \underline{P}\right\} \leq p_0 \leq \Gamma. \quad (15)$$

An important property of Problem (P1-Top-E) is that its objective function is structured by the difference of two increasing functions. Using the property, we have the following

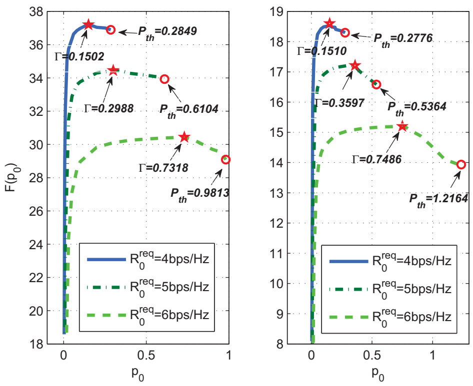

Fig. 2. Illustration of Γ. Left subfigures: $\theta_s = 4, \forall s$, and $\alpha_s = 2, \forall s$. Right subfigures: $\theta_s = 3, \forall s$, and $\alpha_s = 1, \forall s$.

Algorithm (A3): to determine Γ

Set δ and ϵ. Set $p_{\text{lower}} = \max\{p_0^{\min}, \underline{P}\}$ and $p_{\text{upper}} = P_{\text{th}}$. Set $v = \epsilon + 1$.
while $|v| > \epsilon$ **do**
  Set $p_0 = \frac{p_{\text{lower}} + p_{\text{upper}}}{2}$.
  Set $f_1 = F(p_0)$ by using Algorithm (A1).
  Set δ as a very small number. Set $f_2 = F(p_0 + \delta)$ and $f_3 = F(p_0 - \delta)$ by using Algorithm (A1).
  Set $v = \frac{1}{2}\left(\frac{f_2 - f_1}{\delta} + \frac{f_1 - f_3}{\delta}\right)$.
  If $v < -\epsilon$, set $p_{\text{upper}} = p_0$. If $v > \epsilon$, set $p_{\text{lower}} = p_0$.
end while
Output $\Gamma = p_0$.

result regarding the optimal solution (which is denoted by p_0^*) of Problem (P1-Top-E).

Proposition 3: The optimal solution p_0^* of Problem (P1-Top-E) can be given by $p_0^* = z^* + \max\left\{p_0^{\min}, \underline{P}\right\}$, where z^* is the optimal solution of the following Problem (P3):

$$\text{(P3)}: \{z^*, t^*\} = \max_{z,t} F\left(z + \max\{p_0^{\min}, \underline{P}\}\right) + t - \beta(\Gamma - p_0^{\min})$$

$$\text{subject to}: 0 \le z \le \Gamma - \max\{p_0^{\min}, \underline{P}\}, \quad (16)$$

$$0 \le t \le \beta\left(\Gamma - \max\{p_0^{\min}, \underline{P}\}\right), \quad (17)$$

$$\beta z + t \le \beta\left(\Gamma - \max\{p_0^{\min}, \underline{P}\}\right). \quad (18)$$

Proof: The increasing property of $\beta(p_0 - p_0^{\min})$ means that we can introduce the auxiliary variable t that meets

$$\beta(p_0 - p_0^{\min}) + t = \beta(\Gamma - p_0^{\min}), \quad (19)$$

with $0 \le t \le \beta\left(\Gamma - \max\{p_0^{\min}, \underline{P}\}\right)$ because of (15). By using (19) and putting t into the objective function of Problem (P1-Top-E), we obtain its equivalent form as follows:

$$\begin{aligned} \max_{p_0, t} \quad & F(p_0) + t - \beta\left(\Gamma - p_0^{\min}\right) \\ \text{subject to:} \quad & \beta p_0 + t \le \beta\Gamma, \\ & 0 \le t \le \beta\left(\Gamma - \max\{p_0^{\min}, \underline{P}\}\right), \\ & \text{and constraint (15).} \end{aligned}$$

By further using $p_0 = z + \max\{p_0^{\min}, \underline{P}\}$, we obtain the form of (P3), in which constraint (16) is from constraint (15). □

Problem (P3) has an important property as follows.

Proposition 4: Problem (P3) is a monotonic optimization with respect to the decision variables (z, t).

Proof: It is easy to see that the objective function of Problem (P3) is increasing in (z, t). Moreover, the feasible set given by (16), (17), and (18) can be considered as the intersection of the normal set $\left\{(z,t)|z \ge 0, t \ge 0, \text{ and } \beta z + t \le \beta\left(\Gamma - \max\{p_0^{\min}, \underline{P}\}\right)\right\}$ and the reversed normal set $\left\{(z,t)|0 \le z \le \Gamma - \max\{p_0^{\min}, \underline{P}\}, \text{ and } 0 \le t \le \beta\left(\Gamma - \max\{p_0^{\min}, \underline{P}\}\right)\right\}$. As a result, Problem (P3) is a monotonic optimization with respect to the decision variables (z, t), according to [9]. □

Based on Proposition 4, we can again use the PA algorithm to solve Problem (P3) efficiently, and thus obtain the corresponding optimal solutions (z^*, t^*). Consequently, the PU's optimal transmit-power is obtained as $p_0^* = z^* + \max\left\{p_0^{\min}, \underline{P}\right\}$ according to Proposition 3. We thus finish solving Problem (P1-Top) completely.

V. Numerical Result

We present the numerical results validate our analysis and the proposed algorithms. We consider a geographic area of 20-meters long and 20-meters wide. The BS is located at the center of this area, and there are one PU and four different SUs independently and uniformly distributed in this area. The channel power gain is set according to the path loss model. For instance, the channel power gain from the PU to the Rx of SU s is set as $g_{0s} = \frac{\xi_{0s}}{l_{0s}^{\kappa}}$, where l_{0s} denotes the distance between the PU and the Rx of SU s, and κ is the power-scaling factor for path loss (we use $\kappa = 2$). We also consider the effect small-scale channel fading, and this is captured by the parameter ξ_{0s} which is uniformly distributed within $[0, 1]$.

Figure 3 shows the performance of Algorithm (A1) to solve Problem (P1-Bottom) in comparison with the exhaustive search method. In Fig. 3, we use one set of channel power gains which are randomly generated as described before and vary p_0 within $[\underline{P}, \Gamma]$. The top subfigure shows that Algorithm (A1) achieves the results very close to the global optimum achieved by the exhaustive search method, thus validating the effectiveness of Algorithm (A1). Meanwhile, the bottom subfigure shows that Algorithm (A1) requires a computational time which is significantly less than that required by the exhaustive search method. Furthermore, Figure 4 shows the performance of Algorithm (A1) under 30 different sets of channel power gains which are randomly generated, while fixing $p_0 = 0.3$. Again, the top subfigure shows that Algorithm (A1) achieves the results very close to the global optimum achieved by the exhaustive search method, and the bottom subfigure shows that (A1) consumes a significantly less computational time than the exhaustive search. This advantage stems from exploiting the hidden monotonicity of (P1-Bottom) and solves it through the polyblock approximation.

Figure 5 shows the performance of our proposed layered monotonic optimization (denoted by "Layered MO") approach

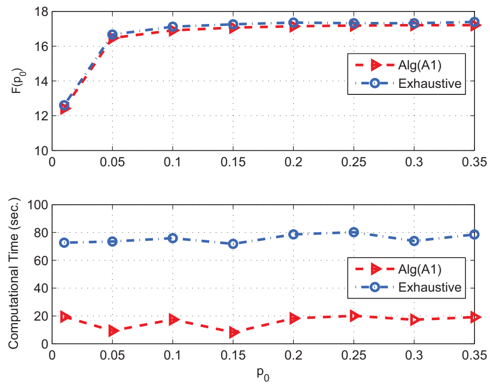

Fig. 3. Performance of Algorithm (A1) with different p_0 within $[\underline{P}, \Gamma]$. We use one set of channel power gains which are randomly generated.

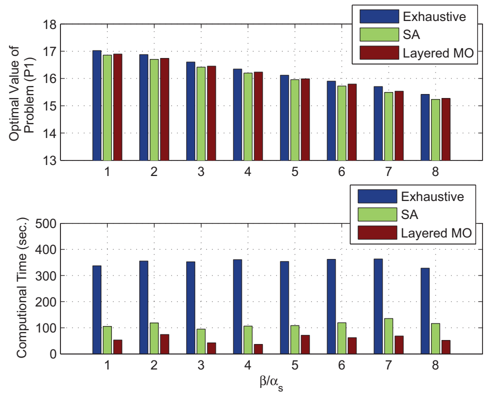

Fig. 5. Performance of the layered monotonic optimization to solve (P1).

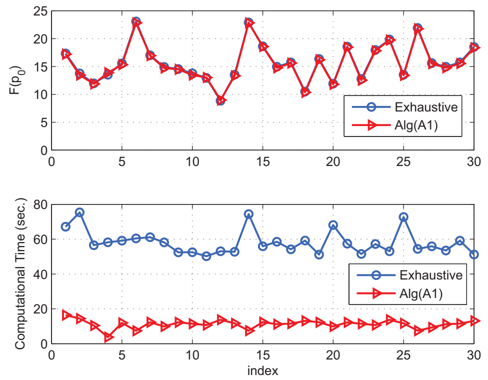

Fig. 4. Performance of Algorithm (A1) with different channel power gains.

to solve Problem (P1) completely (i.e., including solving both Problems (P1-Bottom) and (P1-Top)). In Fig. 5, we use one set of randomly generated channel power gains and enumerate different β/α_s. We compare the results obtained by our layered monotonic approach and those by the exhaustive search method and by the simulated annealing (SA) algorithm (which is a meta-heuristic optimization algorithm based on randomized search). The top subfigure shows that our layered monotonic approach can achieve the maximum revenue that is very close to the revenue obtained by the exhaustive search method despite acceptable relative errors. The results also show that our layered monotonic approach performs better than the SA algorithm. Besides, the bottom subfigure shows that our layered monotonic optimization approach can save the computational time significantly, thus validating its efficiency.

VI. Conclusion

We study the optimal spectrum leasing and sharing of DSA network which aims to maximize its total charge to the SUs based on their achieved throughput, while taking into account the additional power consumption of the PU to overcome the SUs' interference as well as the throughput requirements of the PU and the SUs. Despite of the non-convexity of the formulated spectrum leasing problem, we exploit its intrinsic decomposition structure and propose a layered monotonic optimization approach to obtain the optimal transmit-powers of the PU and the SUs. Numerical results are provided to validate the proposed approach and its computational efficiency.

In this paper, we assume that the PU and the SUs are cooperative and formulate a centralized model in which the PU, performing as a controller, aims at maximizing its revenue by determining its transmit-power as well as the SUs', and the SUs follow the PU's decisions yet achieve their throughput requirements. The result represents the most positive case which the PU can expect and thus can be considered a performance benchmark for evaluating other models (e.g., the game-theoretic formulations that capture the strategic interactions between the PU and the SUs). The future works are to take into account the willingness of the SUs to follow the PU's decisions and to optimize the PU's charge scheme accordingly.

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