# CS 4824/ECE 4424: Generative vs. Discriminative Classifiers

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# Generative vs. discriminative classifiers

- Training classifiers involve estimating $\boldsymbol{f}$: $\boldsymbol{X} \rightarrow \boldsymbol{Y}$ or $P(Y|X)$

- Generative classifiers (e.g., Naïve Bayes)
  - Assumes some functional form for P(X|Y), P(Y)
  - Estimates parameters of P(X|Y), P(Y) from training data
  - Use Bayes rule to calculate P(Y|X)
  - Y is boolean

- Discriminative classifiers (e.g., Logistic Regression)
  - Assumes some functional form for P(Y|X)
  - Estimates parameters of P(Y|X) directly from training data

- **NOTE**: Even through our derivation of the form of P(Y|X) made GNB-style assumptions, the *training procedure* for logistic regression does not!

# Use Naïve Bayes or Logistic Regression?

- Consider

  - Restrictiveness of modeling assumption
    - How well we can learn assuming we have infinite data?

  - Learning curve
    - Rate of convergence (in amount of training data) toward asymptotic (infinite data) hypothesis

# GaussianNaïve Bayes vs. Logistic Regression

$Y \in \{0,1\}$

- Consider boolean Y, continuous $X_i$'s $\quad X = \langle x_1 \ldots x_n \rangle$
- Number of parameters to estimate

  - GNB $\quad 4n+1$

    $\boxed{P(x_i \mid Y=y_k)} \sim \mathcal{N}(\mu_{ik}, \sigma_{ik})$

  - GNB2 $\quad 3n+1$

    $P(x_i \mid Y=y_k) \sim \mathcal{N}(\mu_{ik}, \sigma_i)$

  - LR $\quad \boxed{n+1}$

    $$P(Y=0 \mid X, W) = \frac{1}{1+\exp(w_0 + \sum_{i=1}^{n} w_i x_i)}$$

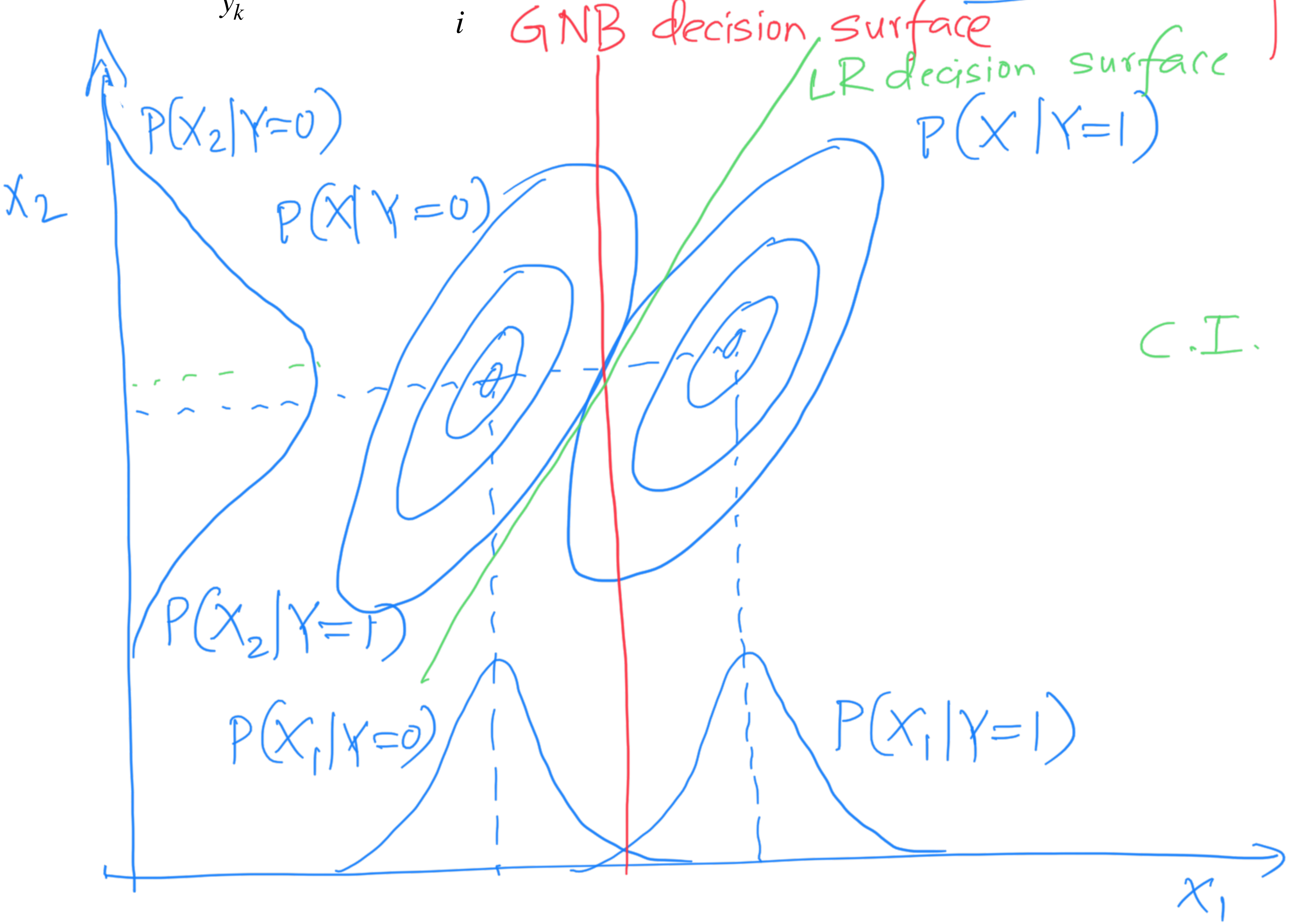
# Gaussian Naïve Bayes vs. Logistic Regression

- Consider boolean Y, continuous $X_i$'s
- Number of parameters to estimate
  - GNB: 4n+1
  - GNB2: 3n+1
  - LR: n+1
- Estimation method
  - NB parameter estimates are ~~un~~coupled de-
  - LR parameter estimates are coupled

# Case study

- Assume Y=PlayBasketball (boolean) X1=Height X2=Age
  - $Y^{New} \leftarrow \arg\max_{y_k} P(Y \mid y_k) \prod_i P(X_i^{New} \mid Y = y_k)$ ; assume P(Y=1) = 0.5

$\boxed{\sigma_{ik} = \sigma_{ij} \; \forall j, k}$




C.I. is violated

# Gaussian Naïve Bayes vs. Logistic Regression

- Recall the two assumptions while deriving the form of LR from GNB
  - 1. $X_i$ are conditionally independent of $X_k$ given Y . C.I. assumption
  - 2. $P(X_i \mid Y = y_k) \sim \mathcal{N}(\mu_{ik}, \boxed{\sigma_i})$; NOT $\mathcal{N}(\mu_{ik}, \boxed{\sigma_{ik}})$

- Consider three learning methods:
  - GNB (assumption 1 only) can be non-linear d.s.
  - GNB2 (assumption 1 and 2) linear d.s.
  - LR linear d.s.

- Which method works better if we have infinite training data and

  - Both (1) and (2) are satisfied GNB = GNB2 = LR LR ? GNB
  - Neither (1) nor (2) is satisfied LR > GNB2 GNB > GNB2
  - (1) is satisfied but not (2) GNB > LR LR > GNB2

# Gaussian Naïve Bayes vs. Logistic Regression

- Recall the two assumptions while deriving the form of LR from GNB
  - 1. $X_i$ are conditionally independent of $X_k$ given Y
  - 2. $P(X_i \mid Y = y_k) \sim \mathcal{N}(\mu_{ik}, \sigma_i)$; NOT $\mathcal{N}(\mu_{ik}, \sigma_{ik})$

- Consider three learning methods:
  - GNB (assumption 1 only)
  - GNB2 (assumption 1 and 2)
  - LR

- Which method works better if we have <u>infinite training data</u> and

  - Both (1) and (2) are satisfied LR = GNB2 = GNB

  - Neither (1) nor (2) is satisfied LR > GNB2, GNB > GNB2

  - (1) is satisfied but not (2) GNB > LR, LR > GNB2

# Gaussian Naïve Bayes vs. Logistic Regression

- What if we have finite training data?
- GNB and LR converge at different rates to asymptotic (∞ data) error
- Let $\boxed{\varepsilon_{A,n}}$ refer to expected error of learning algorithm A after n training examples
- Let d be the number of features $<X_1, X_2, \ldots, X_d>$
  - $\varepsilon_{LR,n} = \varepsilon_{LR,\infty} + \boxed{O(\sqrt{\frac{d}{n}})}$
  - $\varepsilon_{GNB,n} = \varepsilon_{GNB,\infty} + O(\sqrt{\frac{\log d}{n}})$
- So GNB requires $\boxed{d = O(\log d)}$ to converge, but LR requires $\boxed{d = O(d)}$

# Naïve Bayes vs. Logistic Regression

- The bottom line
  - GNB2 and LR both use linear decision surface, GNB need not
  - Given infinite training data, LR is better than GNB2 because the training is free from assumptions (although our derivation of the form of P(Y|X) did)
  - But GNB2 converges more quickly to perhaps less-accurate asymptotic error
  - And GNB is more biased (assumption 1) and less (assumption 2) than LR, so neither might beat each other.