CS 4824/ECE 4424: Neural Networks

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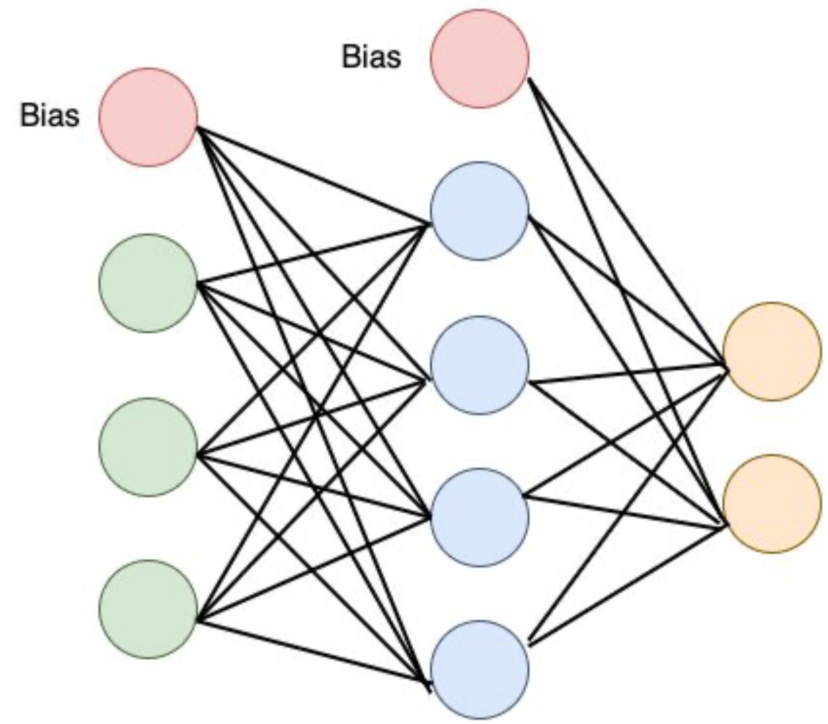
Two-layer Feed-forward Network

- Architecture



- Hidden nodes: $z_j = h_1\ (\boldsymbol{w_j}^{(1)T}\ \boldsymbol{x})$
- Output nodes: $y_k = h_2\ (\boldsymbol{w_k}^{(2)T}\ \boldsymbol{z})$
- Overall: $y_k = h_2(\sum_j w_{kj}^{(2)}\ h_1(\sum_i w_{ji}^{(1)}\ x_i))$

Common Activation Functions h

- Identity $h(a) = a$
- Threshold $h(a) = \begin{cases} 1 & \mathit{if}\ a \geq 0 \\ 0 & \mathit{if}\ a < 0 \end{cases}$
- Sigmoid $h(a) = \sigma(a) = \dfrac{1}{1 + e^{-a}}$
- Gaussian $h(a) = e^{-\frac{1}{2}\left(\frac{a-\mu}{\sigma}\right)^2}$
- Tanh $h(a) = tanh(a) = \dfrac{e^a - e^{-a}}{e^a + e^{-a}}$

Two-layer Feed-forward Network

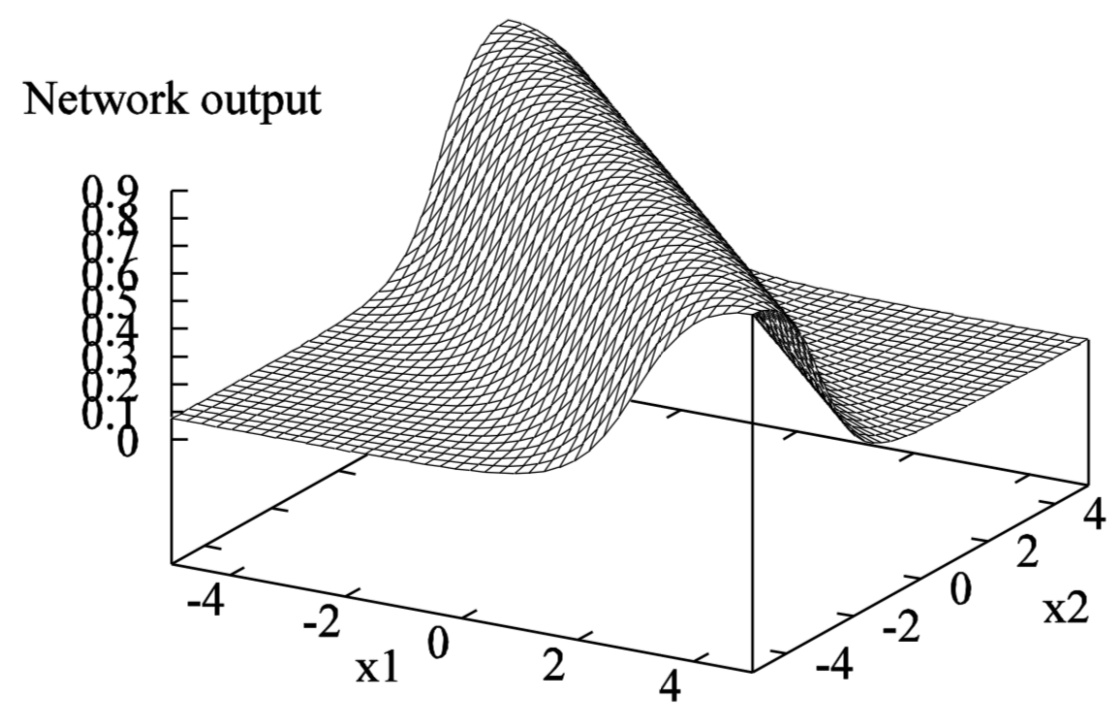
- Regression

- Classification

Combining Activation Functions

- Adding two sigmoid nodes with parallel but opposite "cliffs" produces a **ridge**

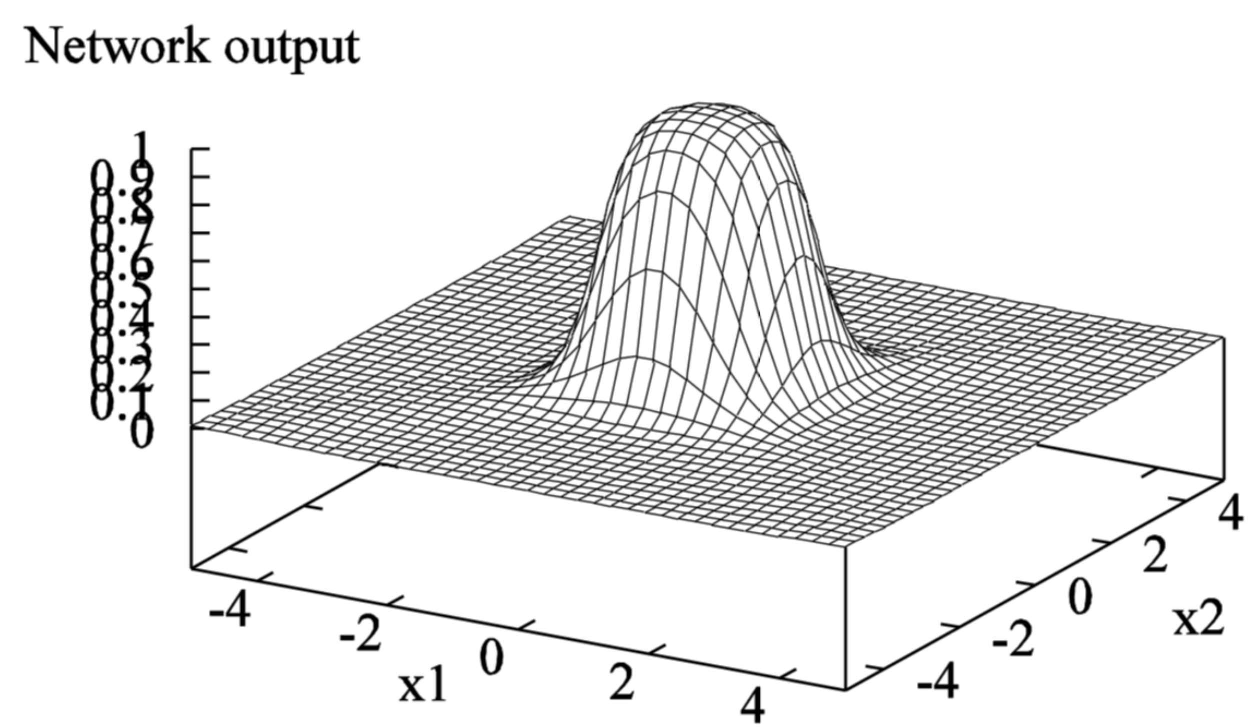
- Schematic



Combining Activation Functions

- Adding two intersecting ridges (and thresholding) produces a **bump**

- Schematic



Combining Activation Functions

- A bump can classify linearly non-separable data points

- By tiling bumps of various heights together, we can approximate any function

Combining Activation Functions

- Combining activation functions in a neural network enables us to approximate any function, hence millions of applications
  - Machine translation
  - Computer vision
  - Speech recognition
  - Word embedding
  - …

Optimizing the Weights

- Parameters: <$\boldsymbol{W}^{(1)}$, $\boldsymbol{W}^{(2)}$, …>
- Objective:
  - **Error minimization**
    - **Backpropagation (aka backprop)**

Backpropagation Algorithm

- Two phases:
  - Forward phase: compute output z_j for each node j
  - Backward phase: compute output δ_j for each node j

Forward Phase

- Propagate inputs forward through the network to compute the output of each node
- Output z_j at node j
  - $z_j = h(a_j)$ where $a_j = \sum_i w_{ji} z_i$

Backward Phase

- Use chain rule to recursively compute gradient
  - For each weight w_{ji}

$$\delta_{ji} = \frac{\partial E_n}{\partial w_{ji}} = \frac{\partial E_n}{\partial a_j}\frac{\partial a_j}{\partial w_{ji}}$$

  - Let $\delta_j = \frac{\partial E_n}{\partial a_j}$
  - Then $$\delta_j = \begin{cases} h'(a_j)(z_j - y_j) & \textit{base case}: \ j \in \textit{output nodes} \\ h'(a_j)\sum_k w_{kj}\delta_k & \textit{recursion}: \ j \in \textit{hidden nodes} \end{cases}$$
  - Since

$$a_j = \sum_k w_{ji} z_i \ \textit{then} \ \frac{\partial a_j}{\partial w_{ji}} = z_i$$

  - Therefore, $\delta_{ji} = \frac{\partial E_n}{\partial w_{ji}} = \frac{\partial E_n}{\partial a_j}\frac{\partial a_j}{\partial w_{ji}} = \delta_j z_i$

An Example

- A simple two-layer network:
  - Hidden nodes (Tanh): $h(a) = tanh(a) = \dfrac{e^a - e^{-a}}{e^a + e^{-a}}$
    - Note: $tanh'(a) = (1 - (tanh'(a))^2$
  - Output nodes (Identity): $h(a) = a$
- Objective function: squared error

The derivation

- Forward phase
  - Hidden nodes: $a_j =$ $z_j =$
  - Output nodes: $a_k =$ $z_k =$
- Backward phase
  - Output nodes: $\delta_k =$
  - Hidden nodes: $\delta_j =$
- Gradients
  - Hidden layer: $\frac{\partial E_n}{\partial w_{ji}} =$
  - Output layer: $\frac{\partial E_n}{\partial w_{kj}} =$

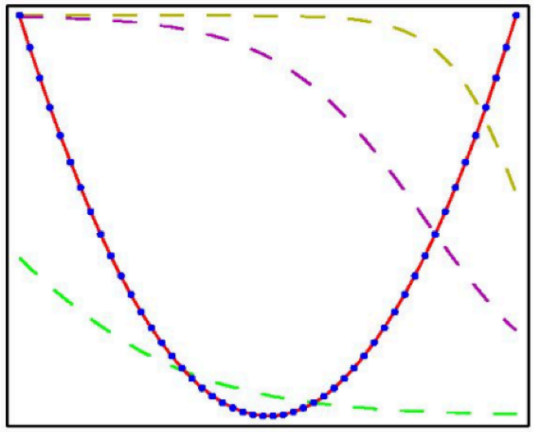
The derivation

- Forward phase
  - Hidden nodes: $a_j = \sum_i w_{ji} x_i$ $\quad z_j = tanh(a_j)$
  - Output nodes: $a_k = \sum_j w_{kj} z_j$ $\quad z_k = a_k$
- Backward phase
  - Output nodes: $\delta_k = z_k - y_k$
  - Hidden nodes: $\delta_j = [1 - (z_j)^2] \sum_k w_{kj} \delta_k$
- Gradients
  - Hidden layer: $\frac{\partial E_n}{\partial w_{ji}} = \delta_j x_i = [1 - (z_j)^2] \sum_k w_{kj} \delta_k x_i$
  - Output layer: $\frac{\partial E_n}{\partial w_{kj}} = \delta_k z_j = (z_k - y_k) z_j$

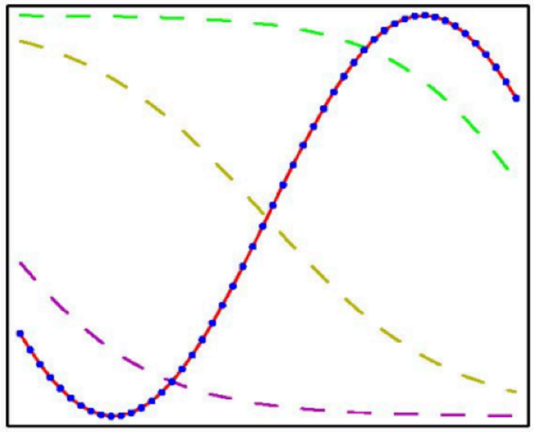
Non-linear regression examples

- Two layer network:
  - 3 tanh hidden units and 1 identity output unit

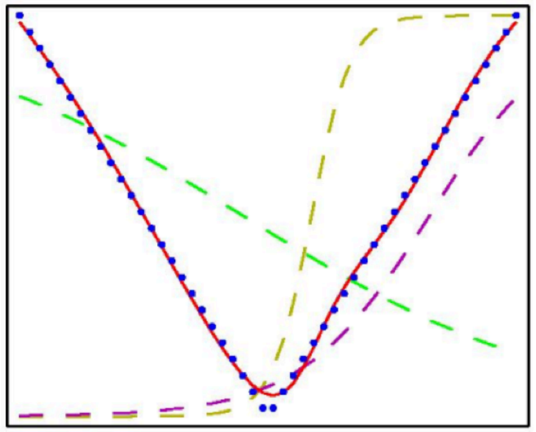
$y = x^2$



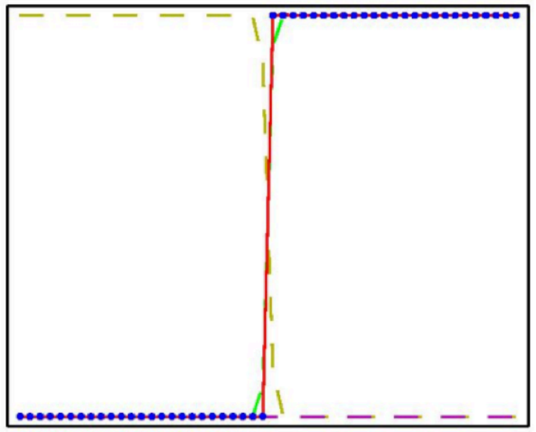
$y = \sin x$



$y = |x|$



$y = \int_{-\infty}^{x} \delta(t)dt$



Demo Time 🙂

https://playground.tensorflow.org/