CS 4824/ECE 4424:
Deep Neural Networks I

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Deep Neural Networks

- **DNN**: neural network with many hidden layers
- **Advantage**: highly expressive
- **Challenges**:
  - How to effectively train a deep neural network?
  - How to avoid overfitting?

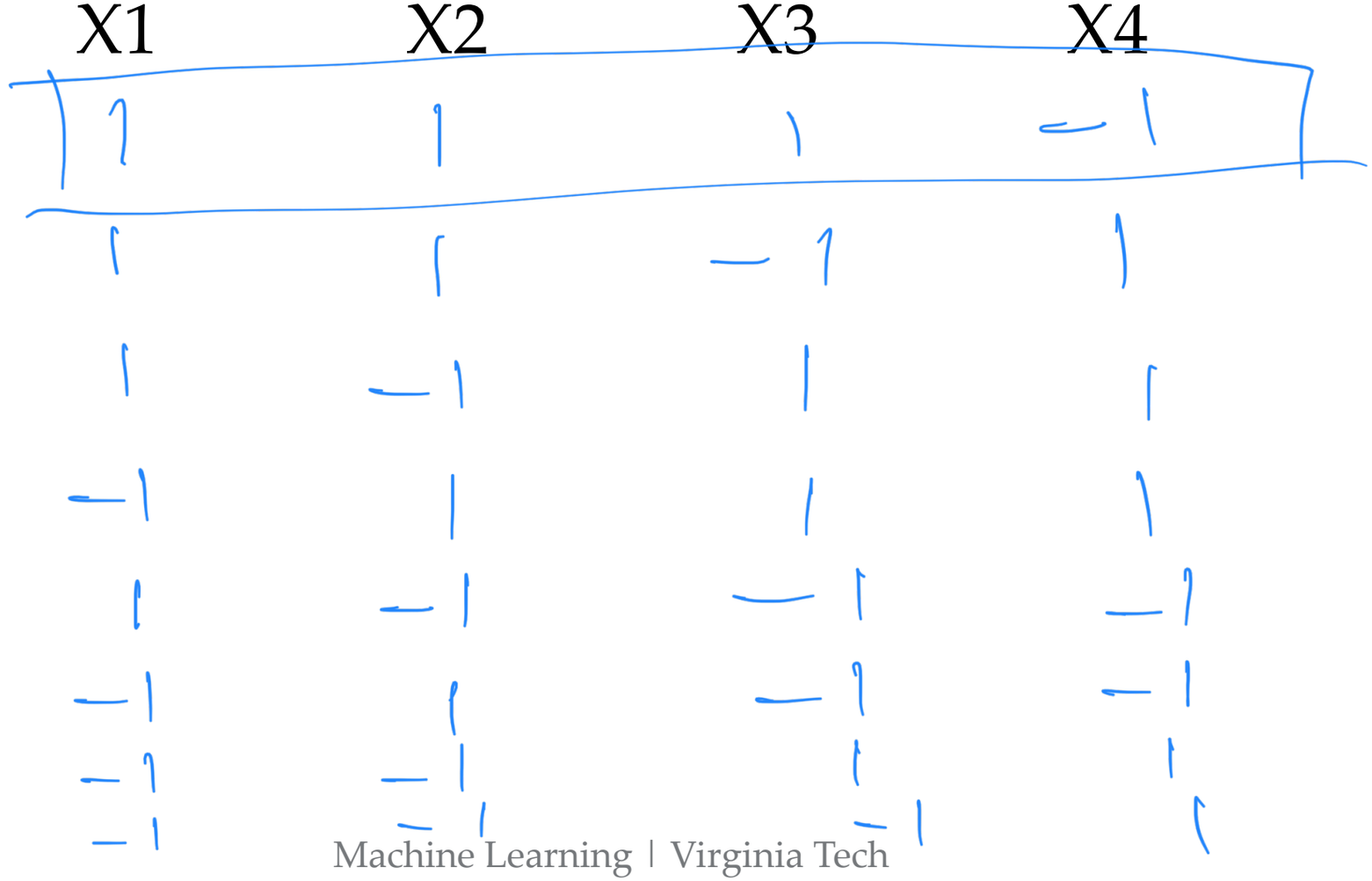
Expressiveness

- Neural networks with one hidden layer of sigmoid/tanh units can approximate arbitrarily closely neural networks with several layers of sigmoid/hyperbolic units
- However, as we increase the number of layers, the number of units needed may decrease exponentially (with the number of layers)

Example – Parity Function

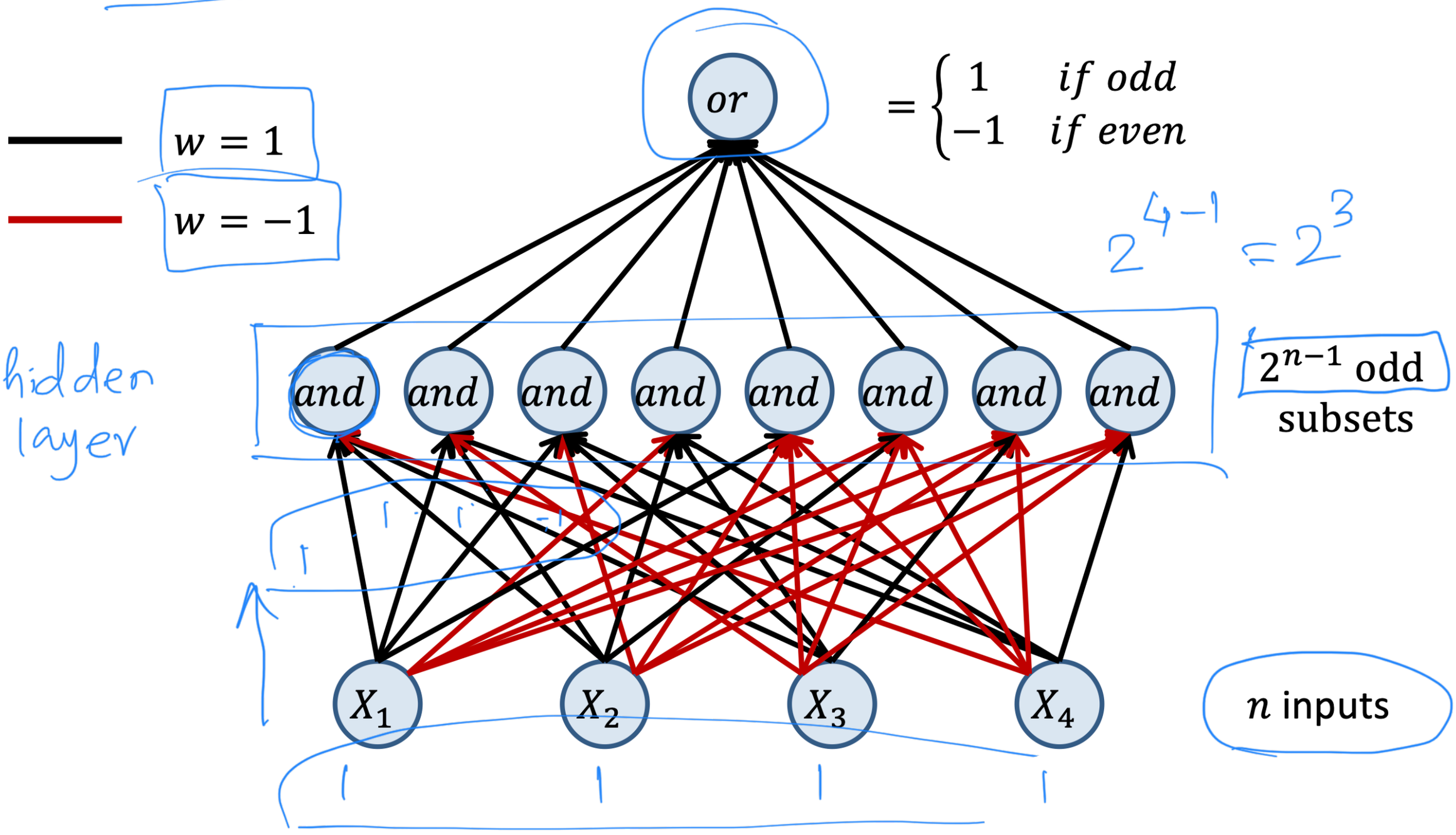
- Odd or even $\begin{cases} 1 & if\ odd \\ -1 & if\ even \end{cases}$

- Possible odd combinations



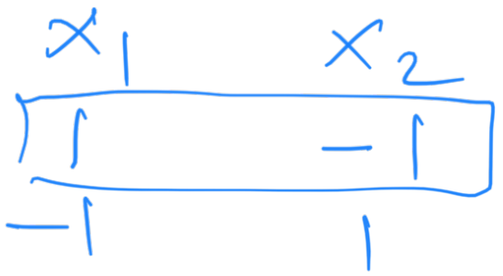
Example – Parity Function

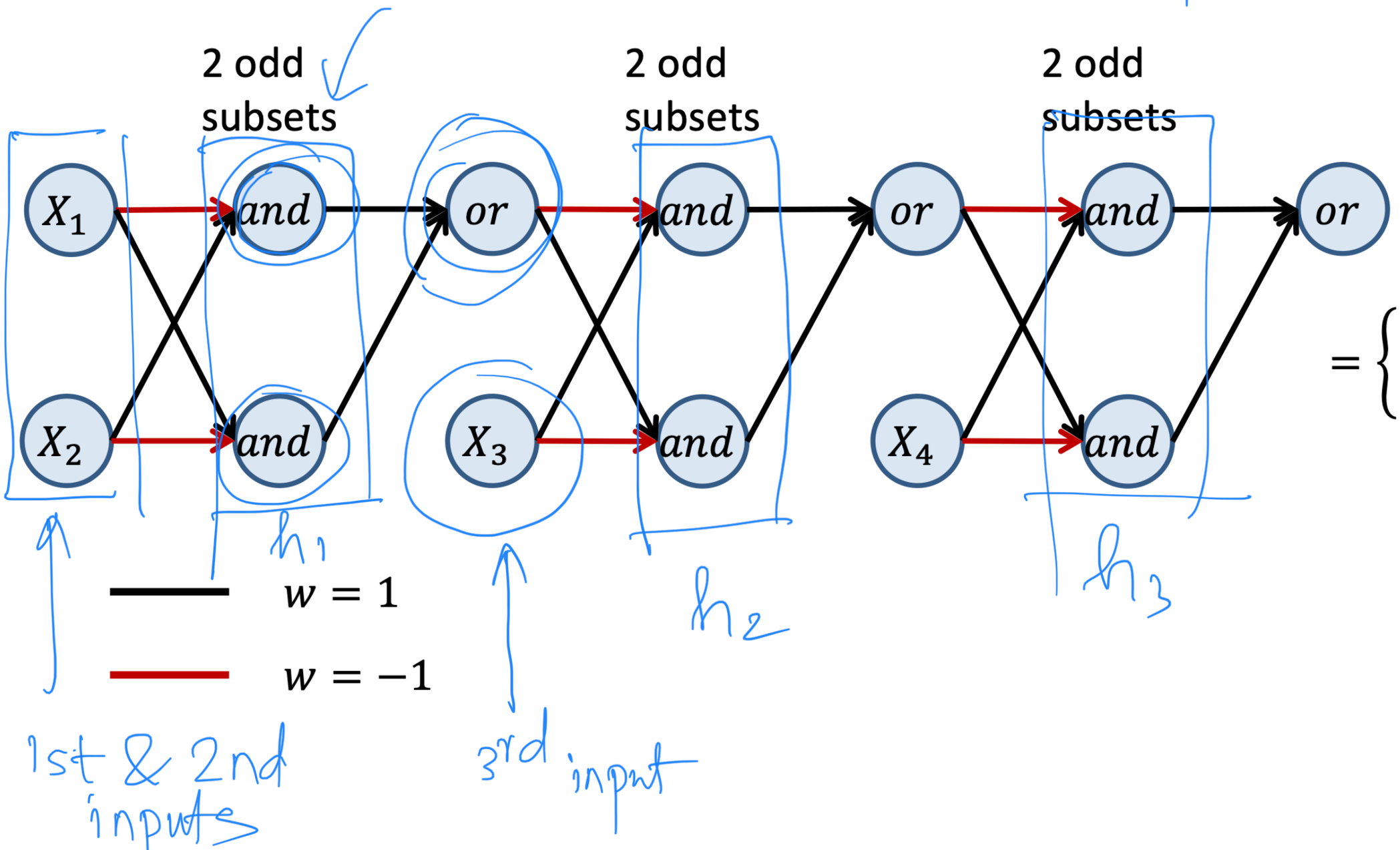
- Single layer of hidden nodes



Example – Parity Function

- 2n – 2 layers of hidden nodes

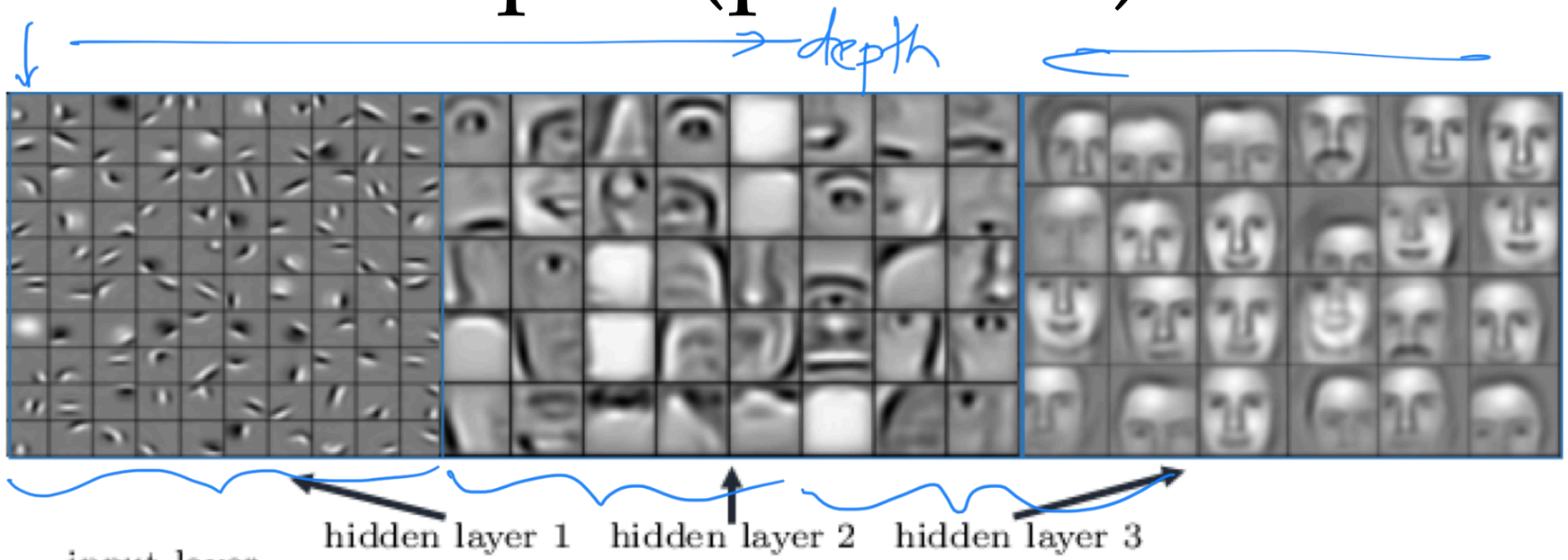


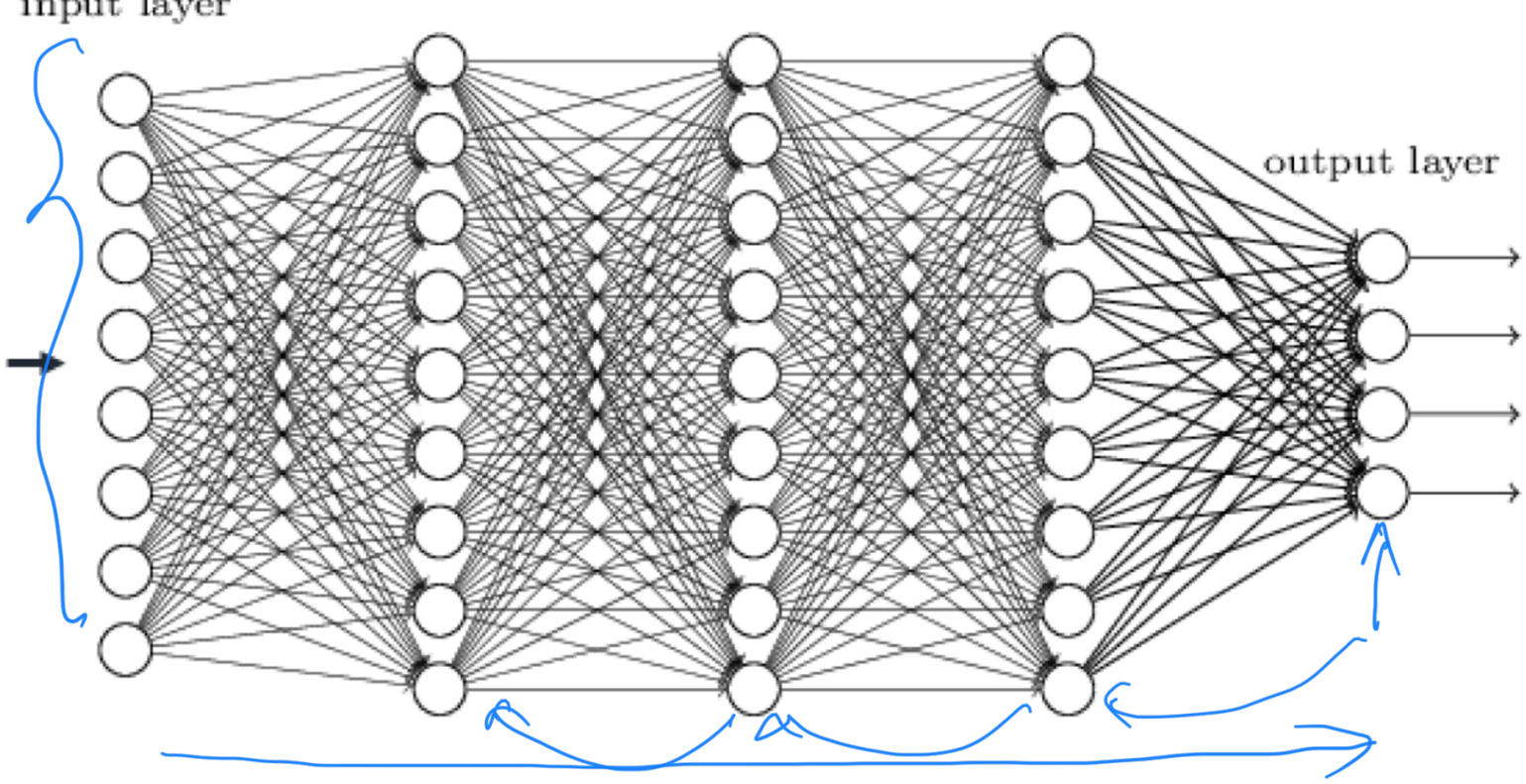


$$= \begin{cases} 1 & if\ odd \\ -1 & if\ even \end{cases}$$

The power of depth (practice)

Deep neural networks learn hierarchical feature representations





- Challenge: how to train deepNNs?

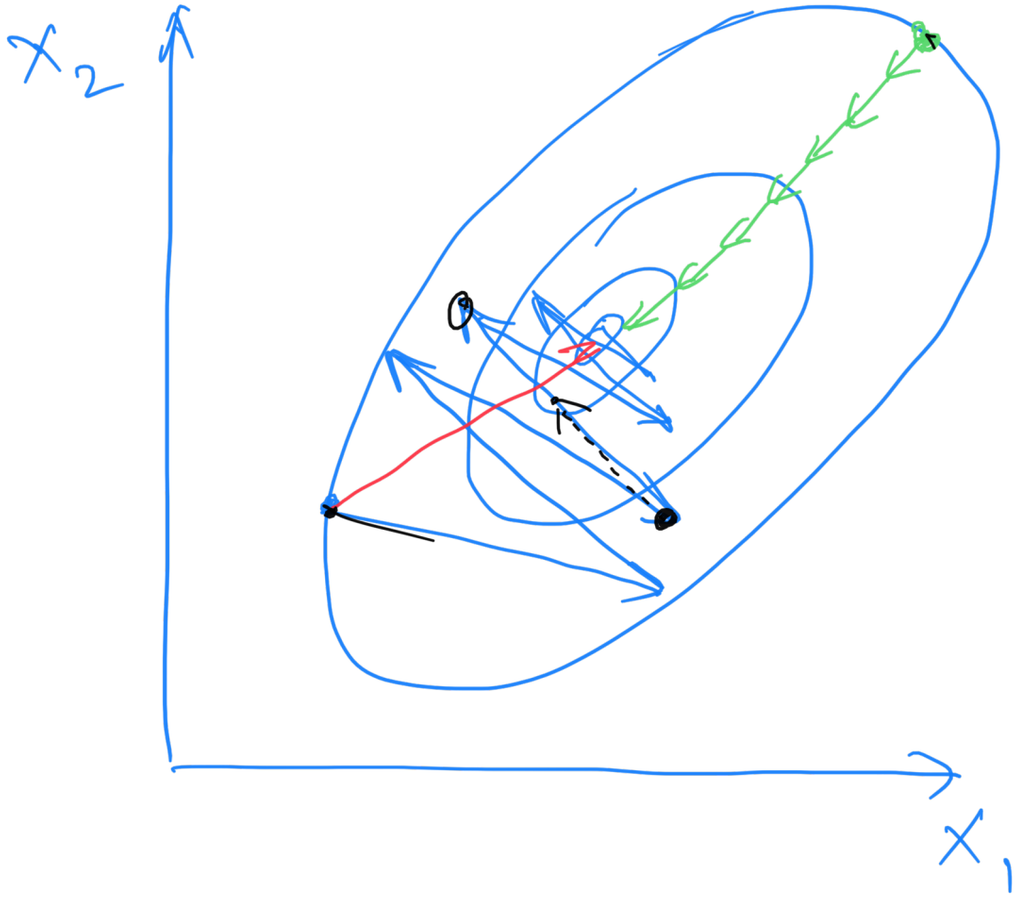
Gradient-based training

- Efficient gradient computation: linear in number of weights

- Convergence:
  - Slow convergence (linear rate)
  - May get trapped in local optima

Slow Convergence

- **Issue**: gradient is not always ideal
- Illustration:



Adaptive Gradients

- **Idea**: adjust the learning rate of each dimension separately
- AdaGrad:
  - $r_t \leftarrow r_{t-1} + \left(\frac{\partial E_n}{\partial w_{ji}}\right)^2$ (sum of squares of partial derivative)
  - $w_{ji} \leftarrow w_{ji} - \frac{\eta}{\sqrt{r_t}} \frac{\partial E_n}{\partial w_{ji}}$ (update rule)
- **Problem**: learning rate $\frac{\eta}{\sqrt{r_t}}$ decays too quickly

$$w'_{ji} \leftarrow w_{ji} - \eta \frac{\partial E_n}{\partial w_{ji}}$$

RMSprop

- **Idea**: divide by root mean square (RMS) (instead of square root of the sum) of partial derivatives
- **RMSprop**

$$r_t \leftarrow \alpha r_{t-1} + (1-\alpha)\left(\frac{\partial E_n}{\partial w_{ji}}\right)^2 \quad (0 \le \alpha \le 1)$$

- $$w_{ji} \leftarrow w_{ji} - \frac{\eta}{\sqrt{r_t}} \frac{\partial E_n}{\partial w_{ji}} \quad \text{(update rule)}$$

- **Problem**: gradient lacks momentum

Adaptive Moment Estimation

- **Idea**: replace gradient by its moving average to induce momentum

$r_t \leftarrow \phi$; $s_t \leftarrow \phi$ before invoking G.D.

- **Adam**:

$$r_t \leftarrow \alpha r_{t-1} + (1-\alpha)\left(\frac{\partial E_n}{\partial w_{ji}}\right)^2 \quad (0 \le \alpha \le 1)$$

$$s_t \leftarrow \beta s_{t-1} + (1-\beta)\left(\frac{\partial E_n}{\partial w_{ji}}\right) \quad (0 \le \beta \le 1)$$

RMSprop update rule

- $$w_{ji} \leftarrow w_{ji} - \frac{\eta}{\sqrt{r_t}} s_t \qquad \text{(update rule)}$$

$$w_{ji} \leftarrow w_{ji} - \frac{\eta}{\sqrt{r_t}}\left(\frac{\partial E_n}{\partial w_{ji}}\right)$$

Challenges in Deep Neural Networks

- Deep neural networks often suffer from vanishing gradients
- High expressivity of deep neural networks increases the risk of overfitting