CS 4824/ECE 4424:
Deep Neural Networks II

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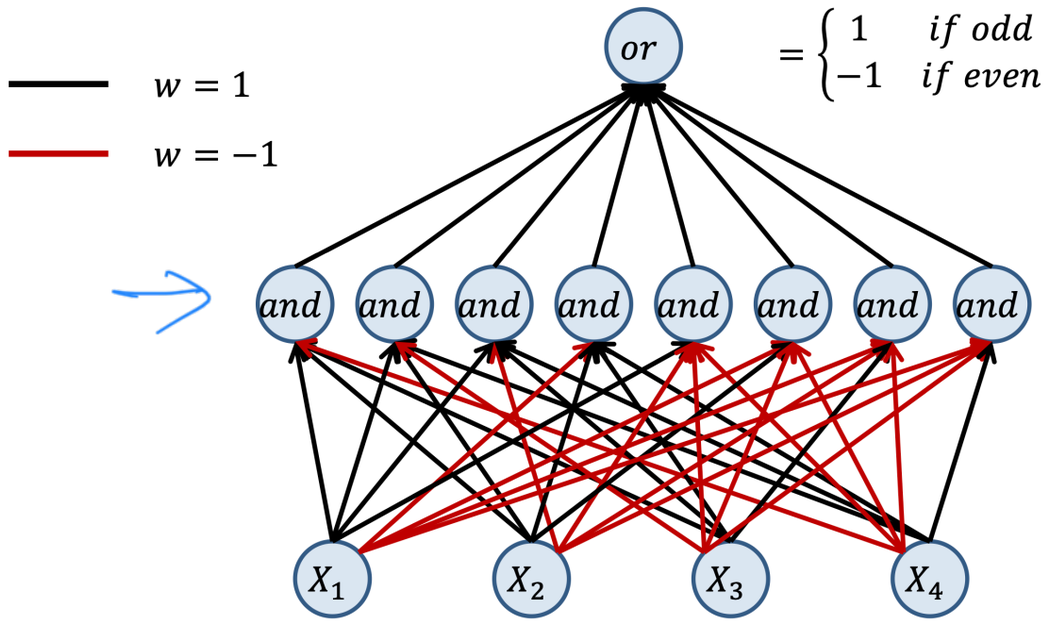
Parity Function — from shallow to deep neural network

- Deep neural networks of sigmoid and hyperbolic units often suffer from vanishing gradients



n inputs

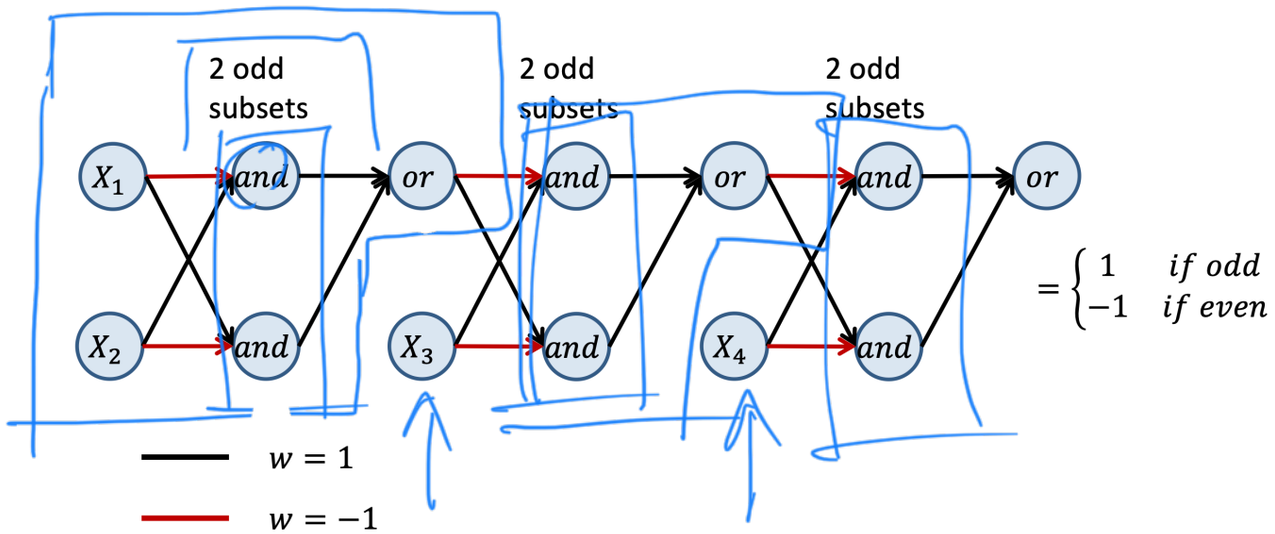
2^{n-1} odd subsets

single layer of hidden nodes
(shallow architecture)

odd

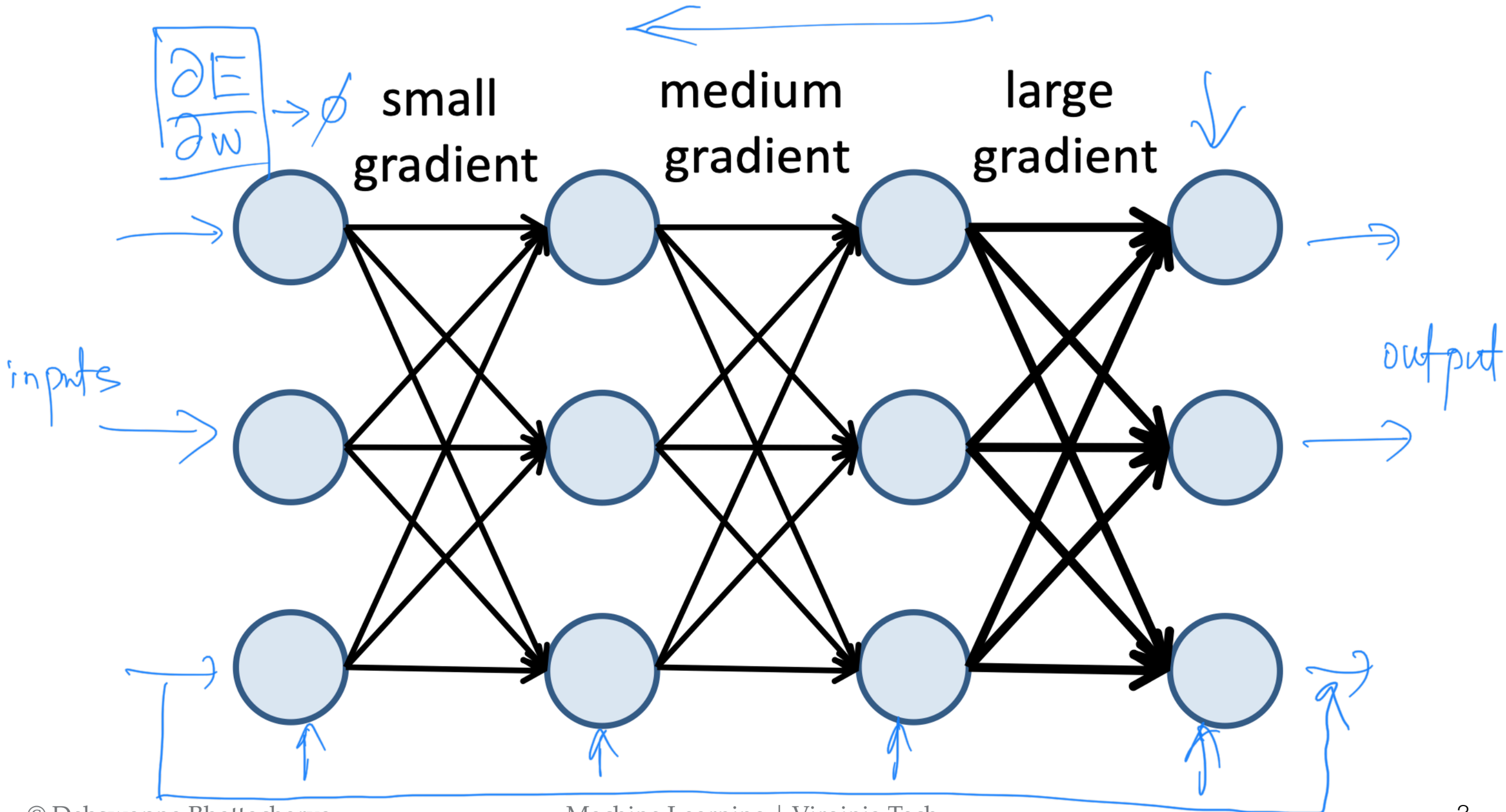
x_1	x_2	x_3	x_4
1	1	1	−1
1	1	−1	1
1	−1	1	1
−1	1	1	1

2n − 2 layers of hidden nodes
(deep architecture)



Vanishing Gradients

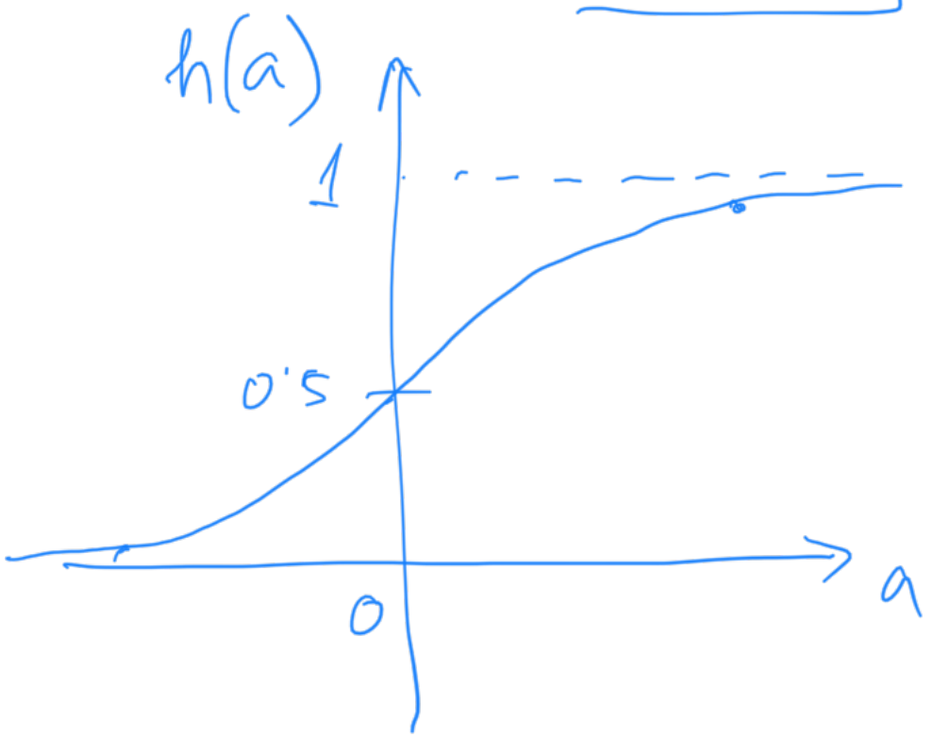
- Deep neural networks often suffer from vanishing gradients



Common activation functions

Sigmoid

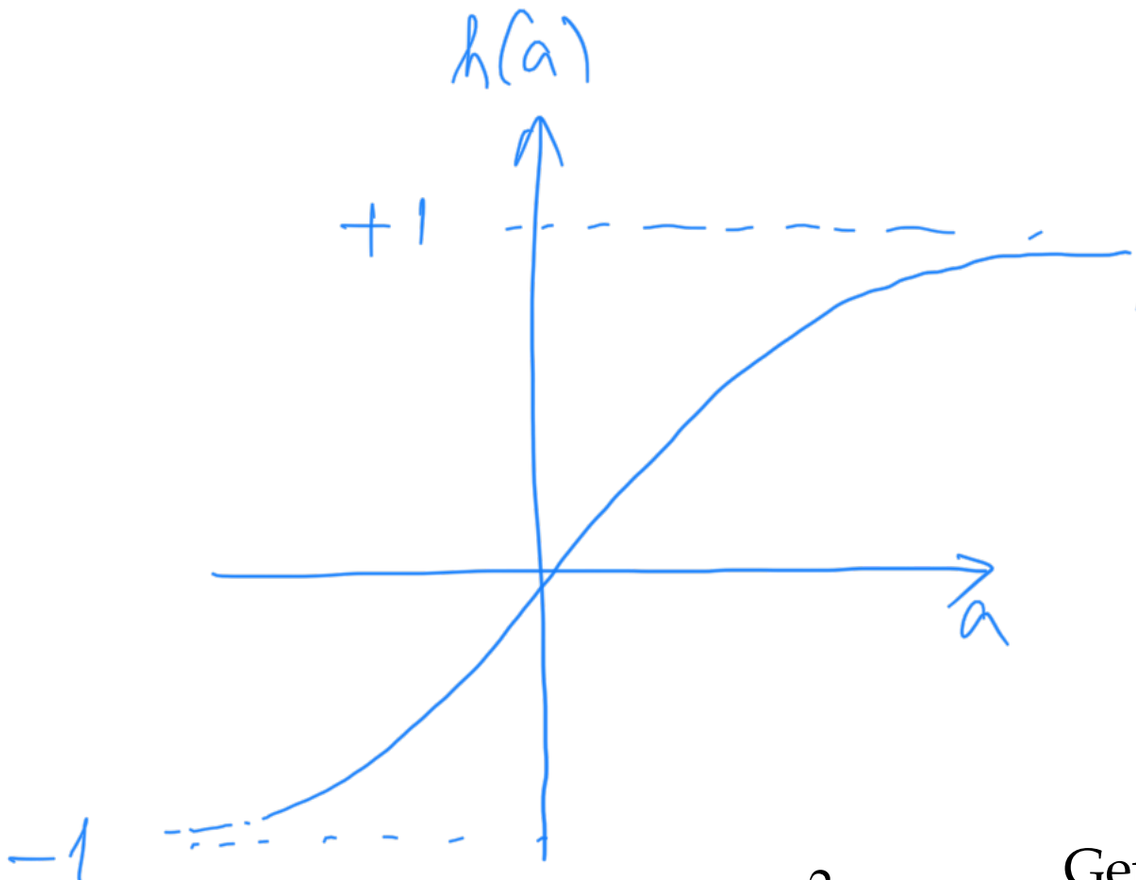
$$h(a) = \sigma(a) = \frac{1}{1 + e^{-a}}$$



$$\sigma'(a) = \sigma(a)(1 - (\sigma(a))$$

Tanh

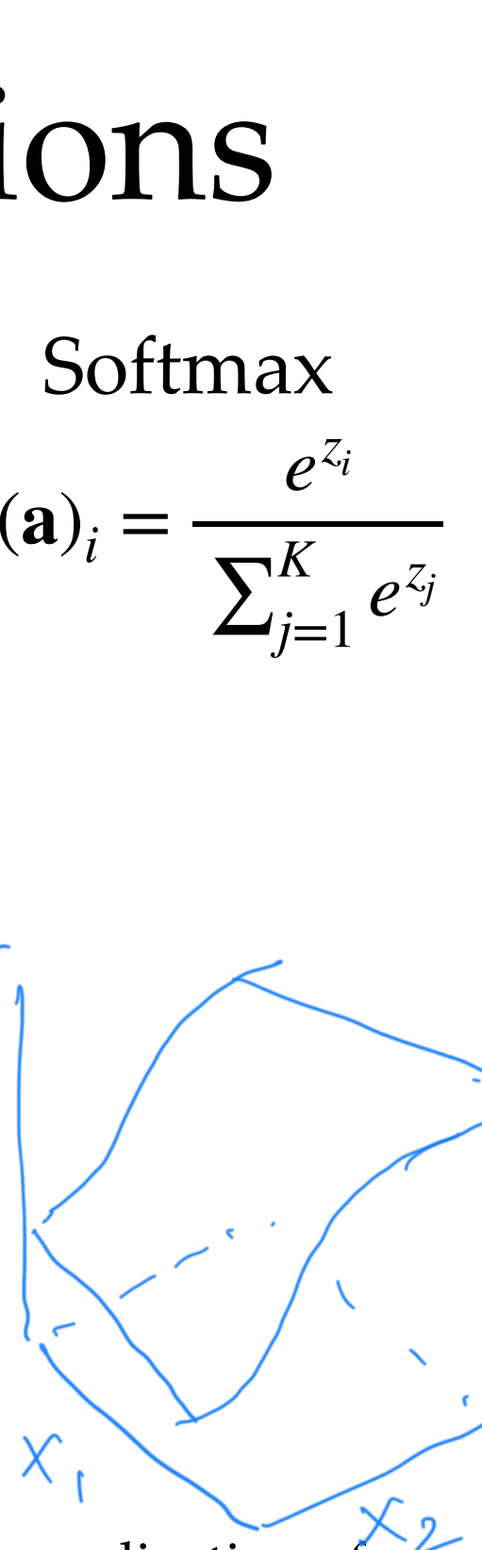
$$h(a) = tanh(a) = \frac{e^{a} - e^{-a}}{e^{a} + e^{-a}}$$



$$tanh'(a) = (1 - (tanh(a))^2$$

Softmax

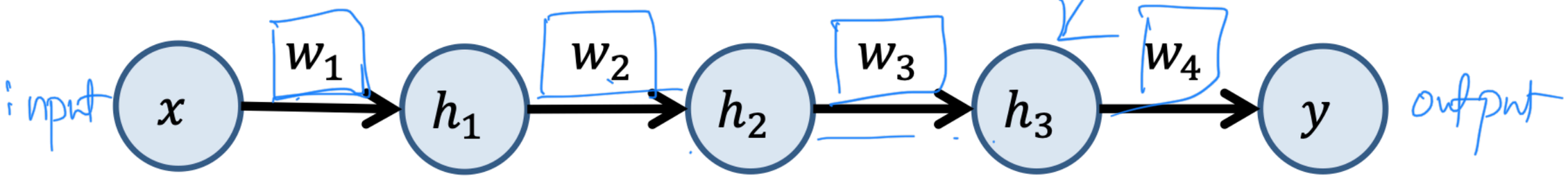
$$h(\mathbf{a}) = \sigma(\mathbf{a})_i = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}$$



Generalization of sigmoid/logistic fn to multiple dimension

Simple Example

$$y = \sigma\left(w_4\,\sigma\left(w_3\,\sigma(w_2\,\sigma(w_1\,x))\right)\right)$$



- Common weight initialization in (0,1) or in (-1, 1)
- Sigmoid function and its derivative always less than 1
- This leads to vanishing gradients:

$$\frac{\partial y}{\partial w_4} = \sigma'(a_4)\sigma(a_3)$$

$$\frac{\partial y}{\partial w_3} = \sigma'(a_4)w_4\sigma'(a_3)\sigma(a_2)$$

$$\frac{\partial y}{\partial w_2} = \sigma'(a_4)w_4\sigma'(a_3)w_3\sigma'(a_2)\sigma(a_1)$$

$$\frac{\partial y}{\partial w_1} = \sigma'(a_4)w_4\sigma'(a_3)w_3\sigma'(a_2)w_2\sigma'(a_1)x$$

As products of factors less than 1 gets longer, gradient vanishes

Avoiding Vanishing Gradients

- Several popular solutions:
  - Pre-training
  - **Rectified linear units and maxout units**
  - Skip connections
  - Batch normalization

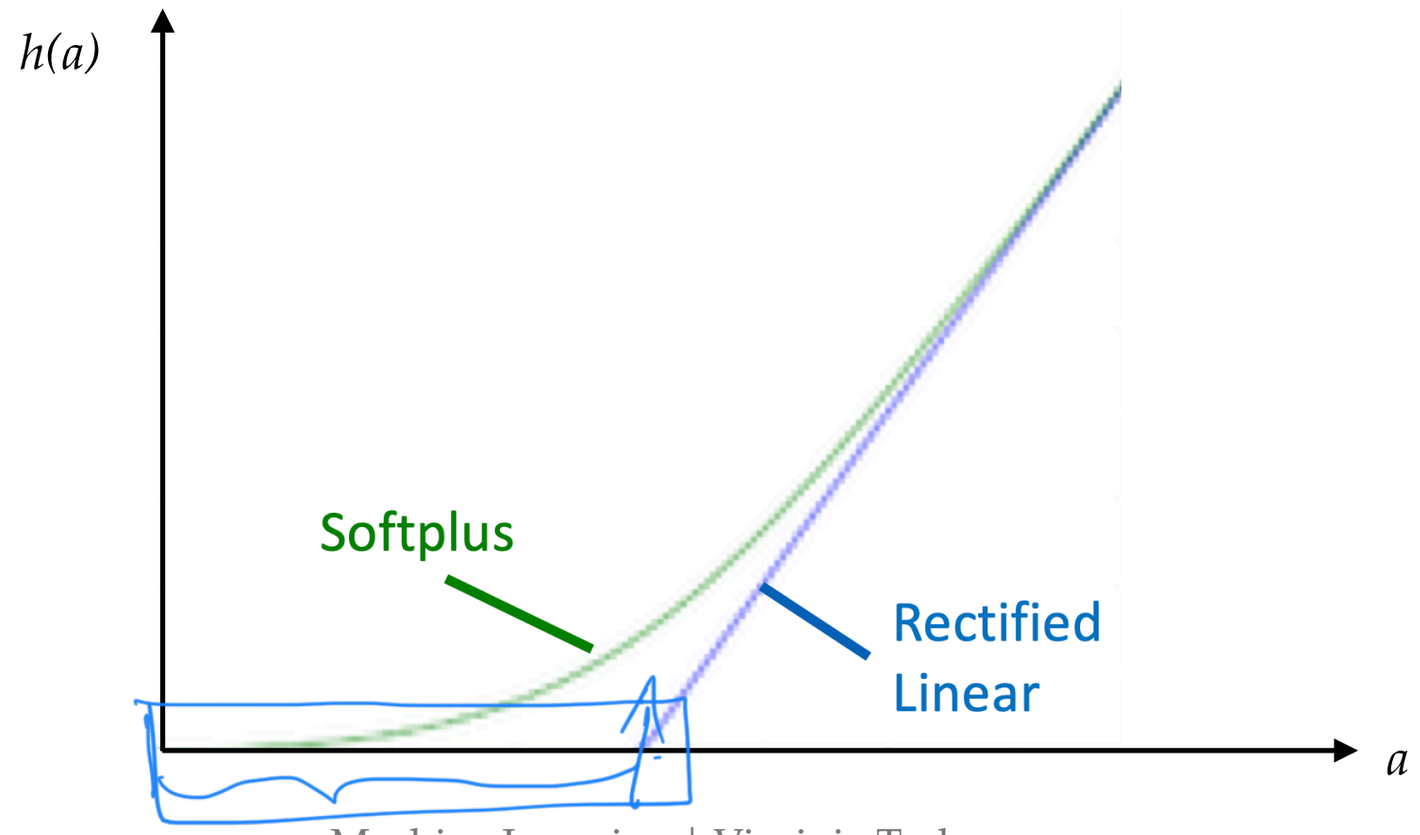
Rectified Linear Units (ReLU)

- **Rectified linear**: $h(a) = \max(0, a)$
  - Gradient is 0 or 1 w.r.t. a
  - Sparse computation

- **Soft version ("softplus")**: $h(a) = \log(1 + e^a)$
  - But softplus does not prevent gradient vanishing (gradient < 1)
  - Making rectified linear unit smooth does not help!

$h(a) = \log(1 + N^a)$

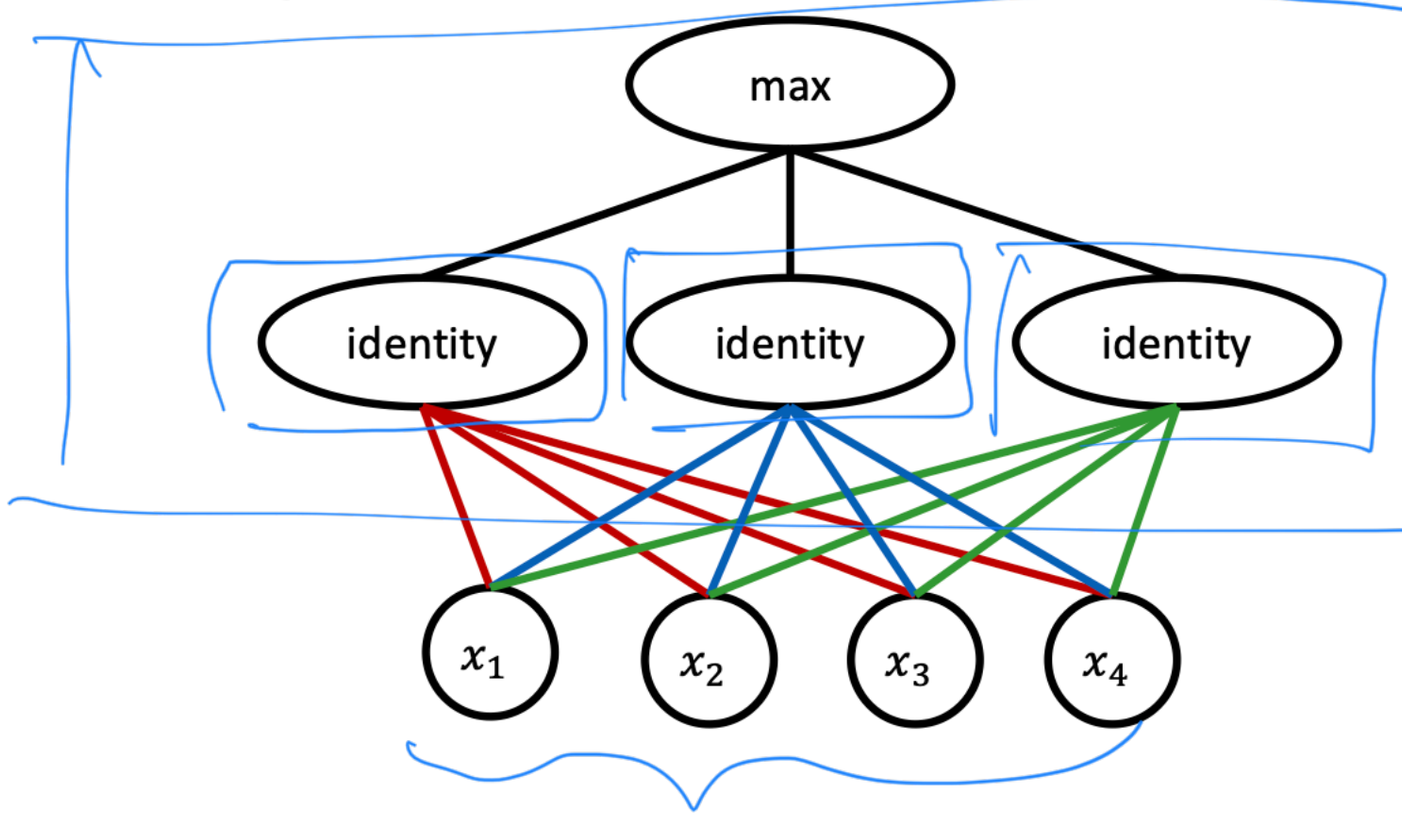
$N = 2000$

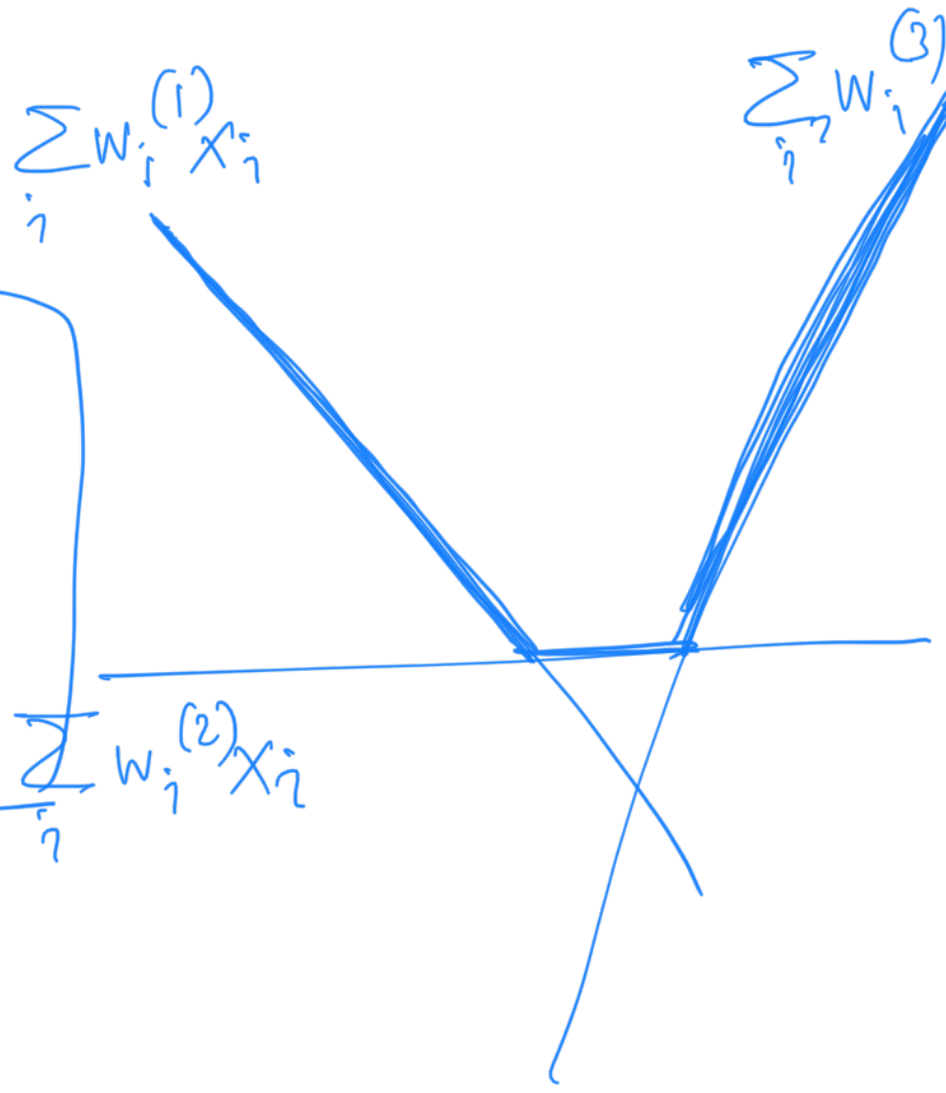


Maxout Units

- Generalization of rectified linear units

$$\max\left\{\sum_i w_i^{(1)} x_i, \sum_i w_i^{(2)} x_i, \sum_i w_i^{(3)} x_i, \ldots\right\}$$





Overfitting

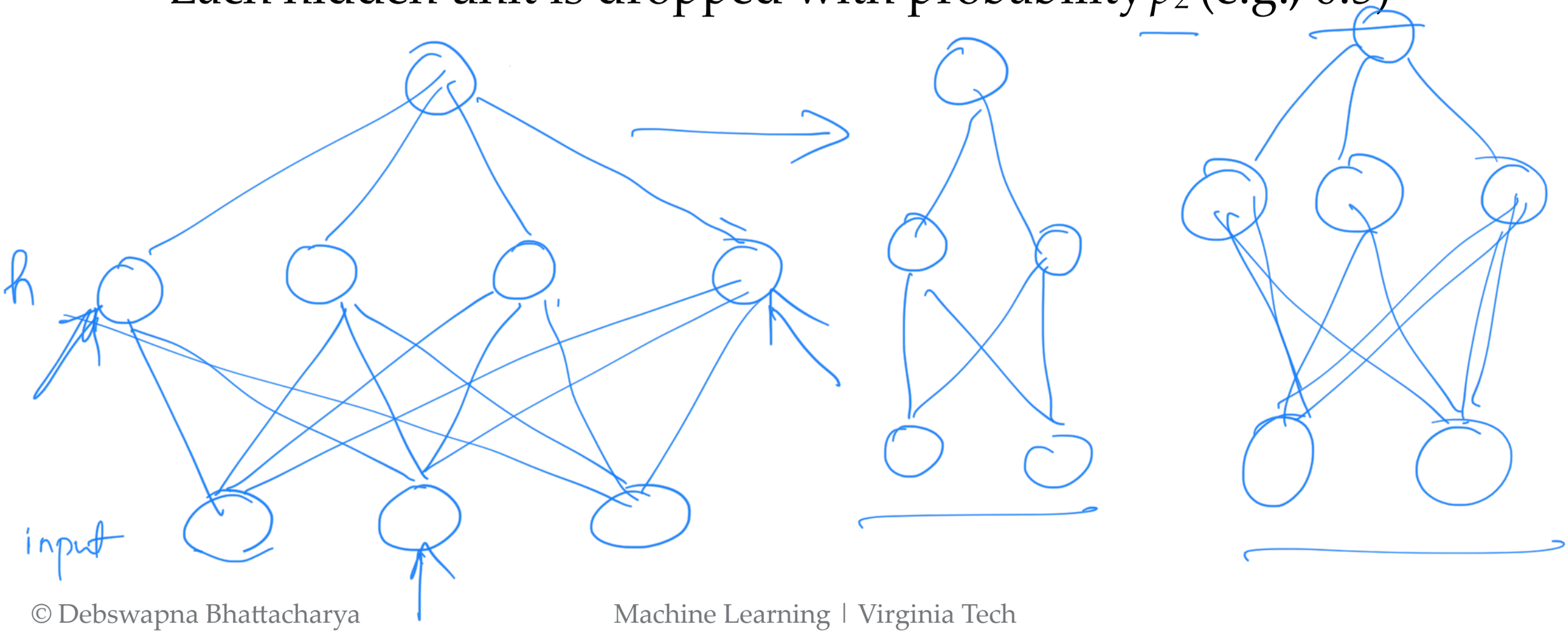
- High expressivity increases the risk of overfitting
  - # of parameters is often larger than the amount of data

- Some solutions:
  - Regularization
  - **Dropout**
  - Data augmentation

Dropout — Training

- **Idea**: randomly "drop" some units from the network when training

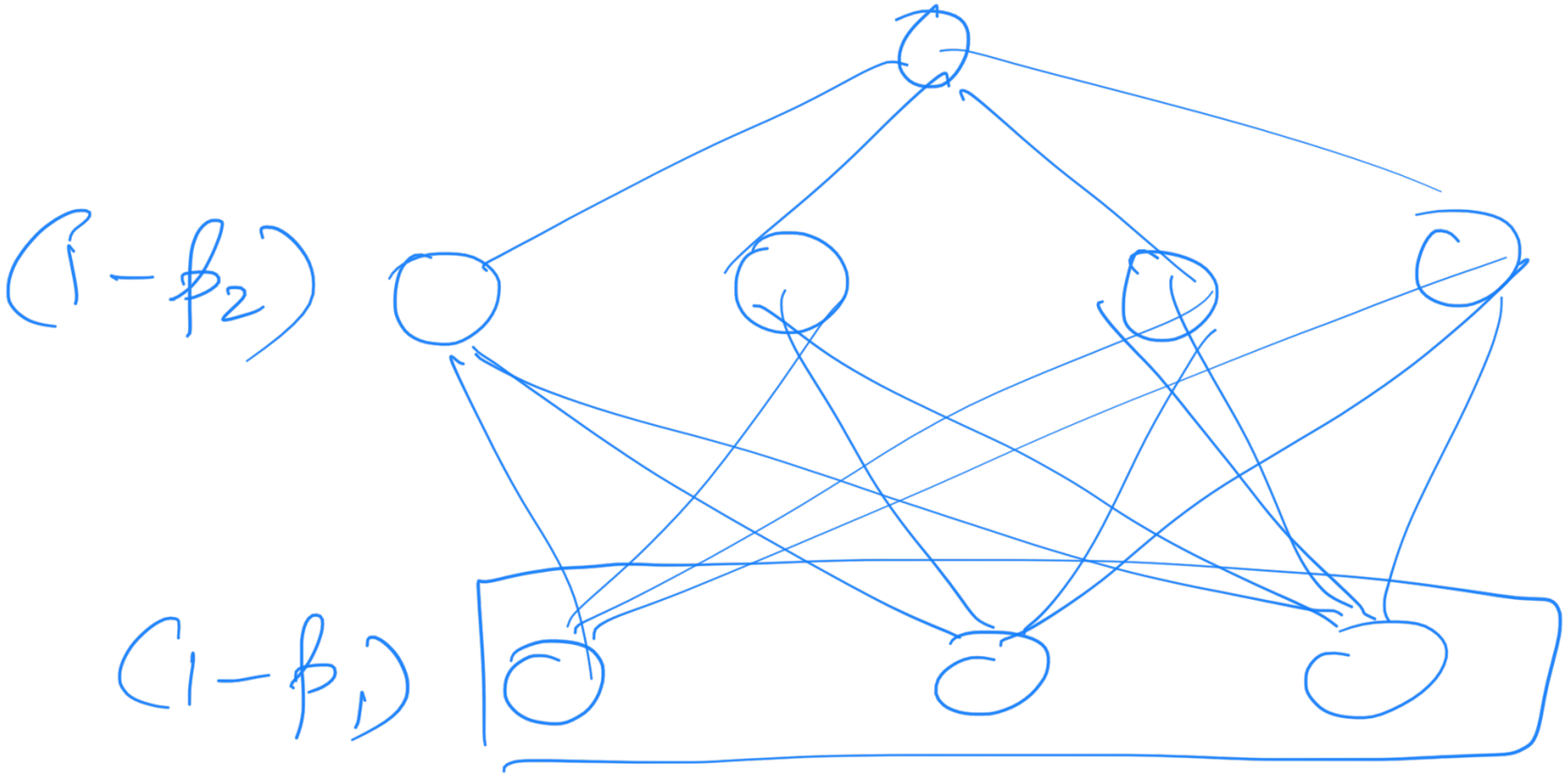
- Training: at each iteration of gradient descent
  - Each input unit is dropped with probability p_1 (e.g., 0.2)
  - Each hidden unit is dropped with probability p_2 (e.g., 0.5)



Dropout — Prediction

- **Idea**: during prediction, probabilistically account for the effect of randomly "dropped" units from the network during training

- Prediction(testing):
  - Multiply each input unit by $1 - p_1$
  - Multiply each hidden unit 1 - p_2



Dropout — Intuition

- Dropout can be viewed as an approximate form of ensemble learning
- In each training iteration, a different subnetwork is trained
- At test time, these subnetworks are “merged” by averaging their weights

Speech

- **2006** (Hinton and coworkers): first effective algorithm for deep NN
  - layer-wise training of Stacked Restricted Boltzmann Machines (SRBM)s
- **2009**: Breakthrough in acoustic modeling
  - replace Gaussian Mixture Models by SRBMs
  - Improved speech recognition at Google, Microsoft, IBM
- **2013**: recurrent neural nets (LSTM)
  - Google error rate: **23**% (2013) to **8**% (2015)
  - Microsoft error rate: **5.9**% (Oct 17, 2016) same as human performance
- **…**

Image Classification

- ImageNet Large Scale Visual Recognition Challenge