CS 4824/ECE 4424: Autoencoder

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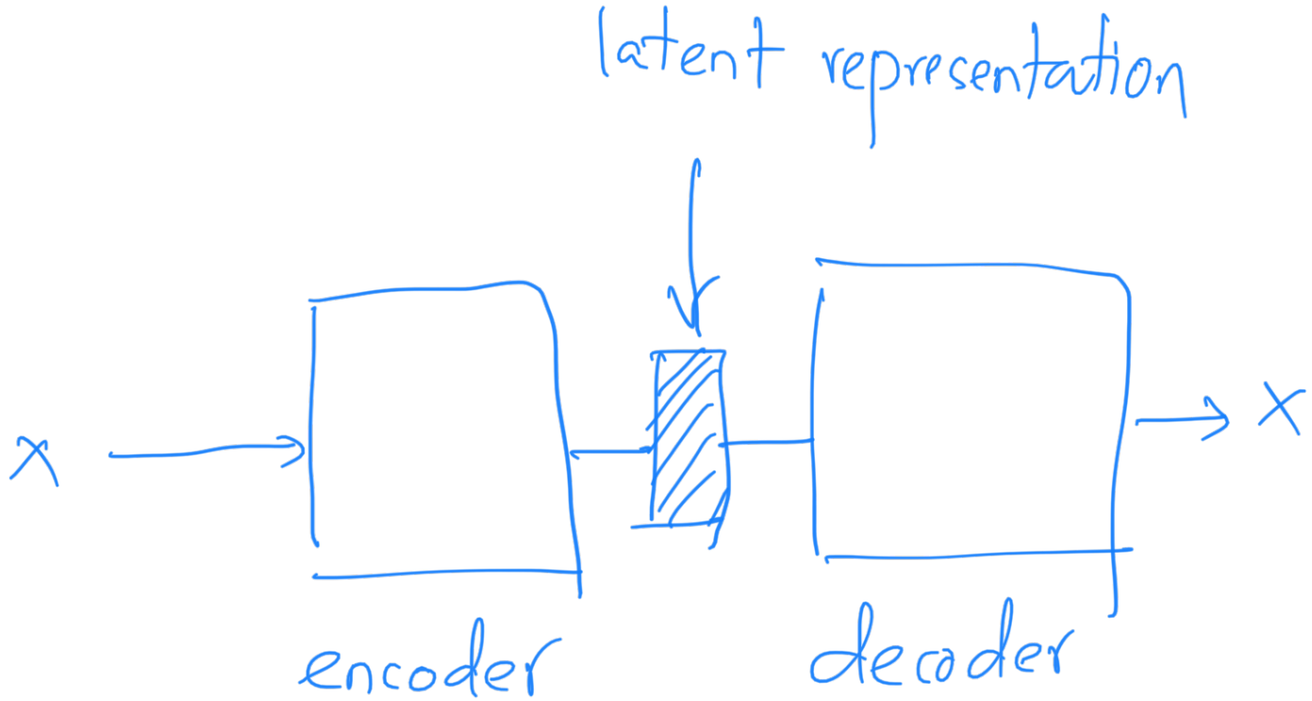
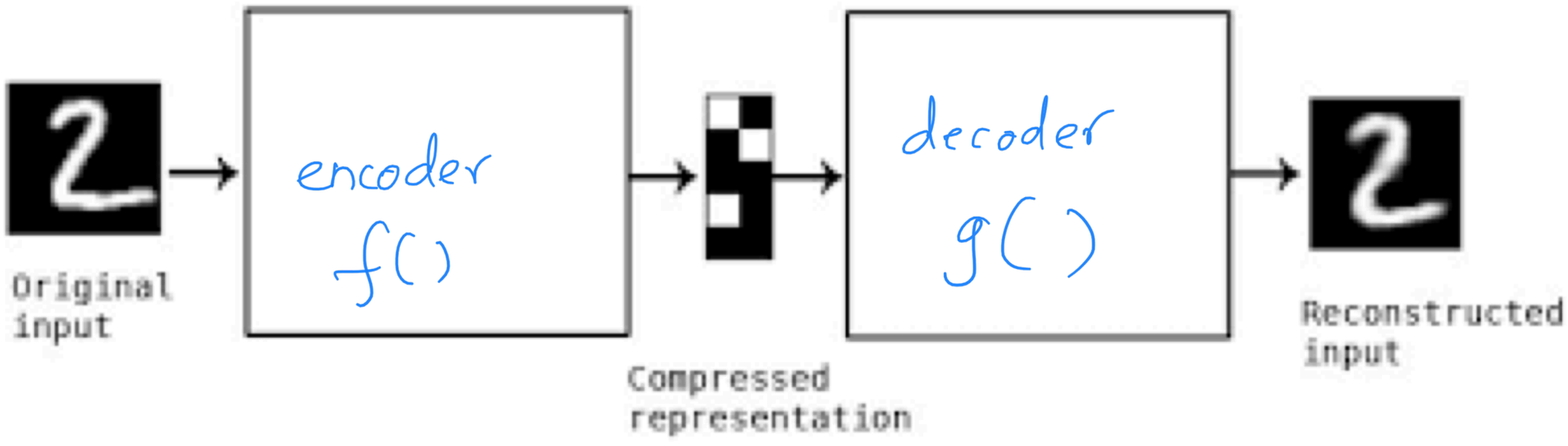
Autoencoder

- Special type of feed forward network for
  - Compression ←
  - Denoising ←
  - Sparse representation ←
  - Data generation



Autoencoder

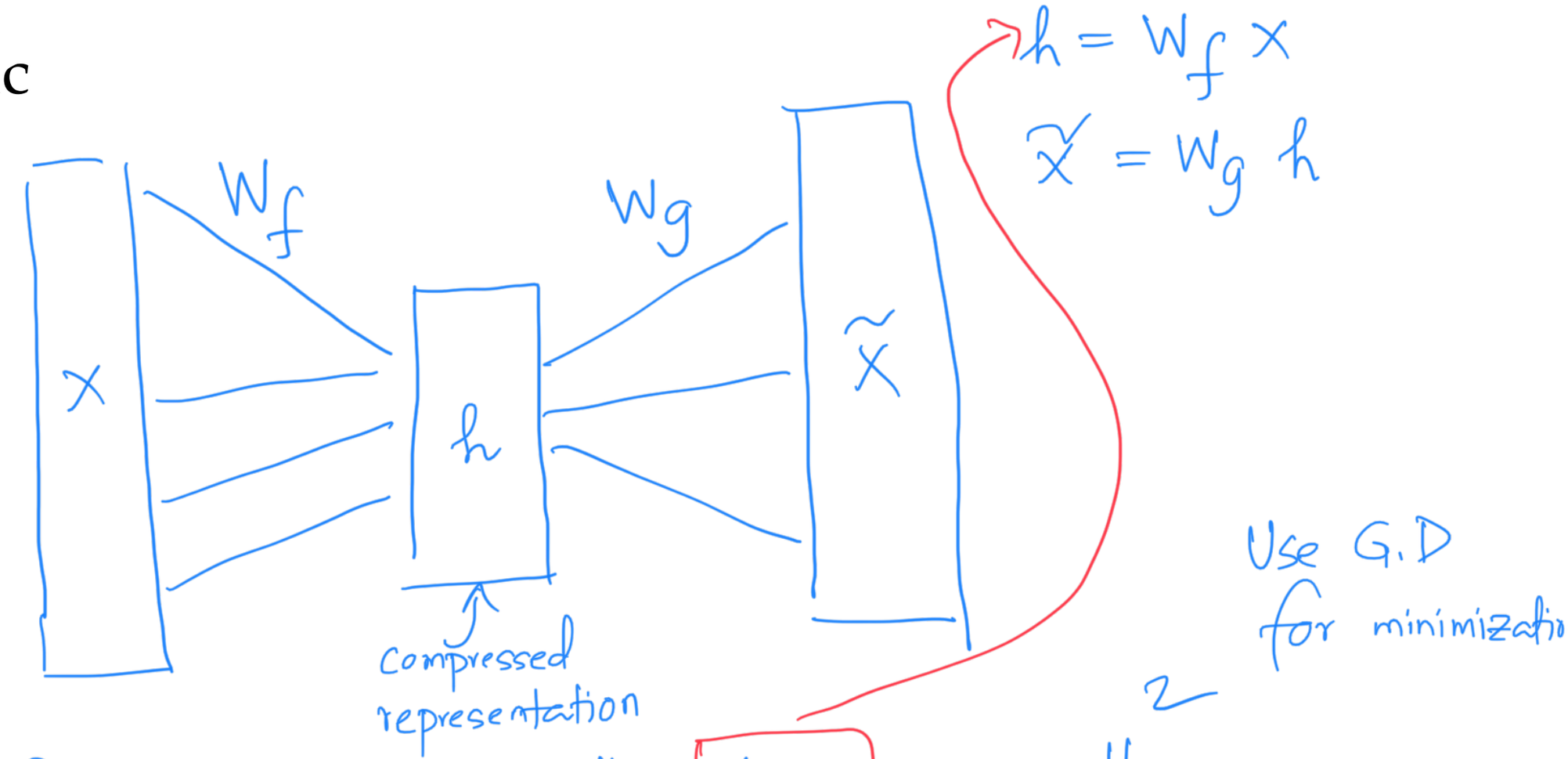
- Encoder: $f(\quad)$
- Decoder: $g(\quad)$
- Autoencoder: $g(f(\mathbf{x})) = \mathbf{x}$



Linear Autoencoder

- f and g are linear
  - Matrix representation: $\mathbf{W}_f$ and $\mathbf{W}_g$

- Schematic



Linear Autoencoder

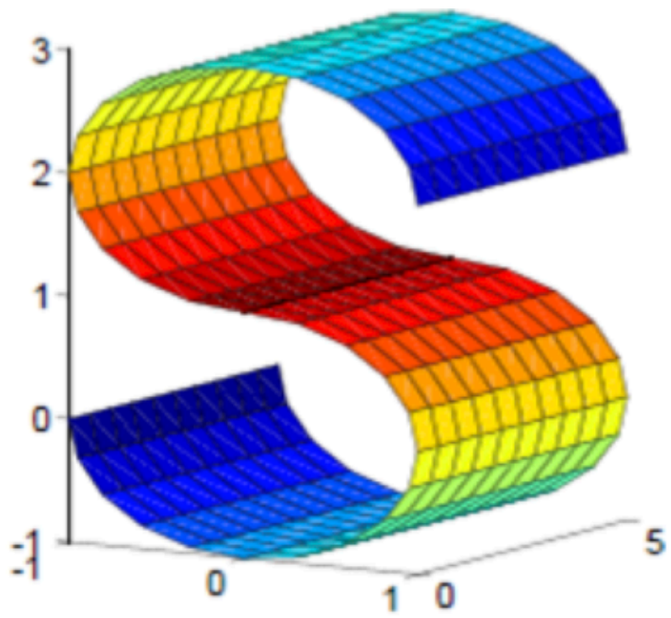
- **Objective**: find weights $\mathbf{W}_f$ and $\mathbf{W}_g$ that minimizes the reconstruction error
  - $$\arg\min_{\mathbf{W}} \frac{1}{2} \sum_n \|\mathbf{W}_g \mathbf{W}_f \mathbf{x}_n - \mathbf{x}_n\|_2^2$$
- Algorithm: Backpropagation
  - Gradient descent
- Hidden nodes: compressed representation

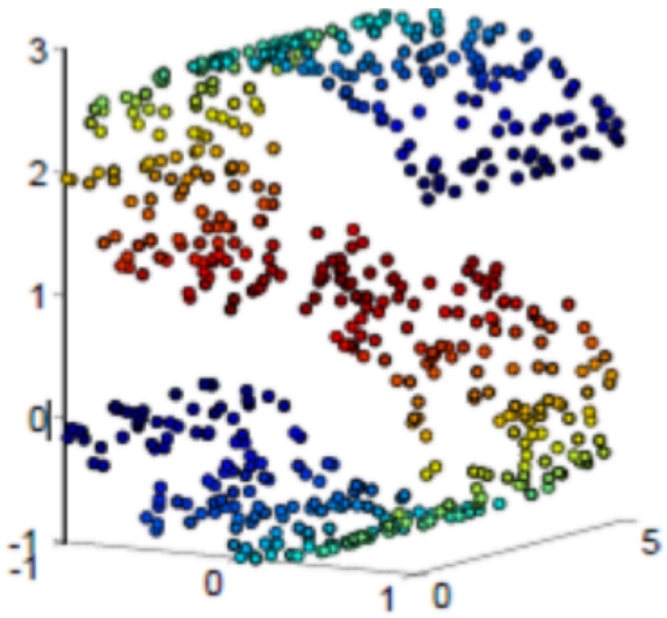
Nonlinear Autoencoder

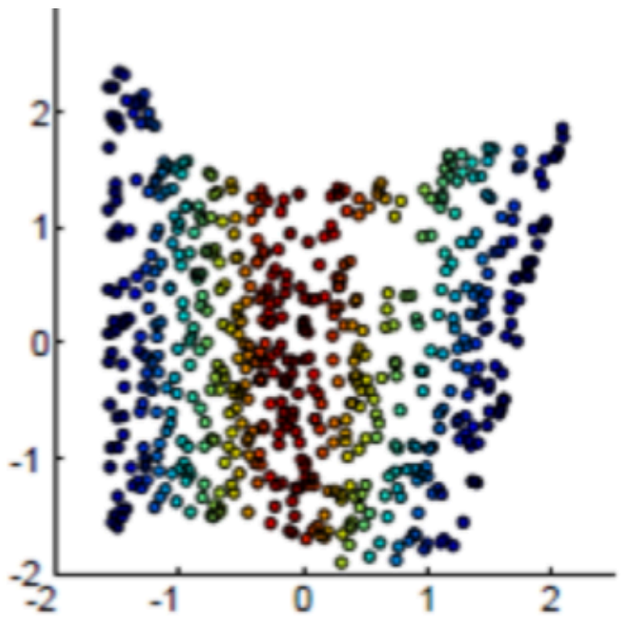
- f and g are nonlinear functions
  - $$\arg\min_{\mathbf{W}} \frac{1}{2} \sum_n \| g(f(\mathbf{x}_n; \mathbf{W}_f); \mathbf{W}_g) - \mathbf{x}_n \|_2^2$$

$g(f(x)) - x$

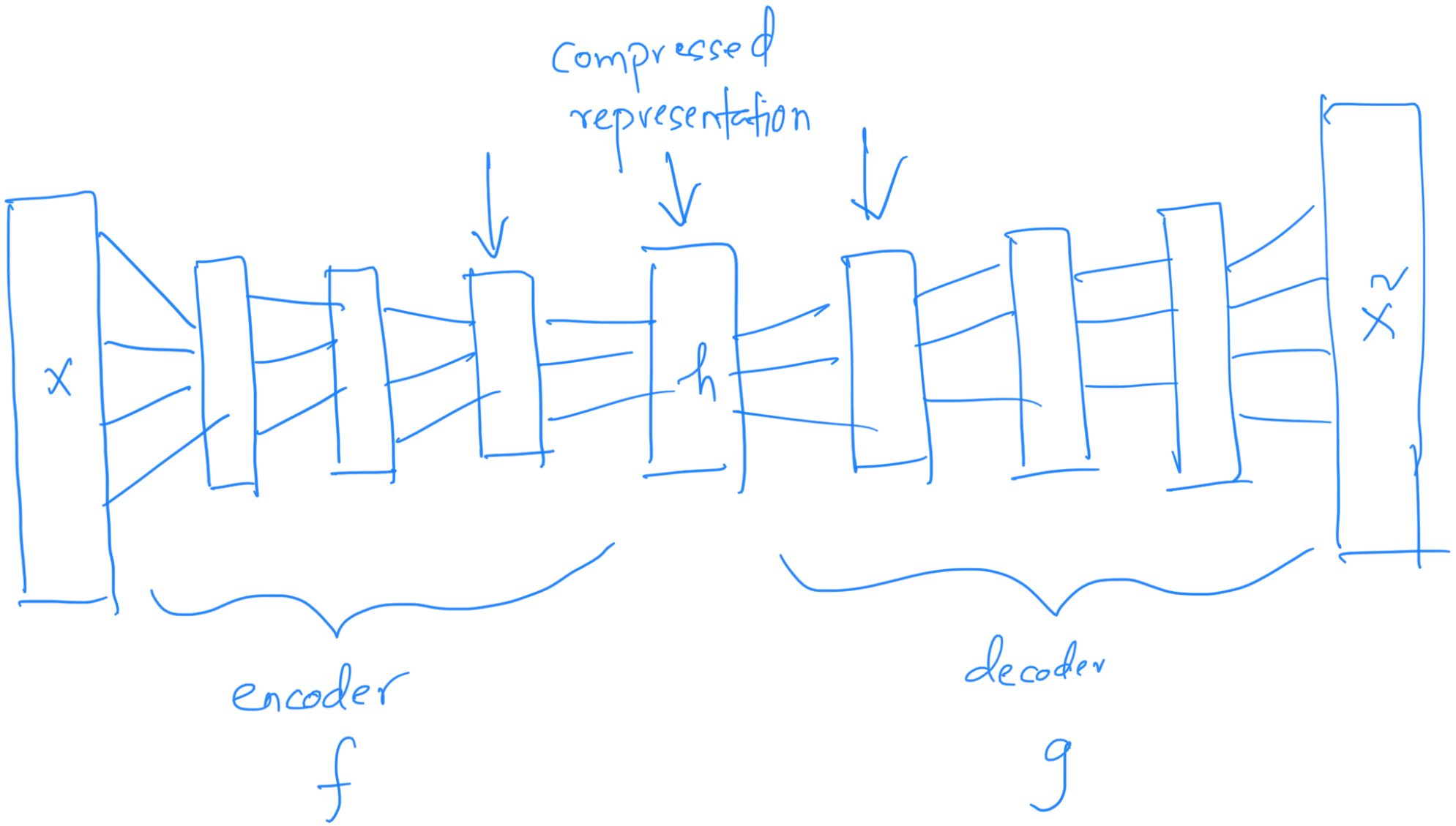
- Hidden nodes: nonlinear manifold







Deep Autoencoder



Deep Autoencoder

- f and g often consist of multiple layers
- In theory, one hidden layer in f and g is sufficient to represent any possible compression
- Multiple hidden layers in f and g is often better

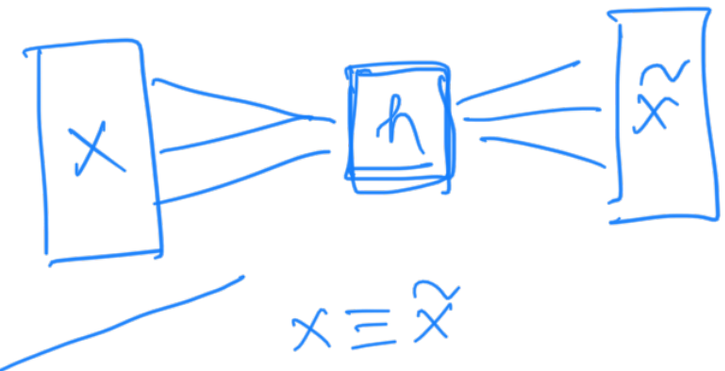
Sparse Representations

- When more hidden nodes than inputs, use regularization to constrain autoencoder

- Example: force hidden nodes to be sparse
  - $$\arg\min_{\mathbf{W}} \frac{1}{2} \sum_n \|g(f(\mathbf{x}_n; \mathbf{W}_f); \mathbf{W}_g) - \mathbf{x}_n\|_2^2 + cnnz(f(\mathbf{x}_n; \mathbf{W}_f))$$
    - Where $cnnz(f(\mathbf{x}_n; \mathbf{W}_f))$ is the number of non-zero entries produced by f

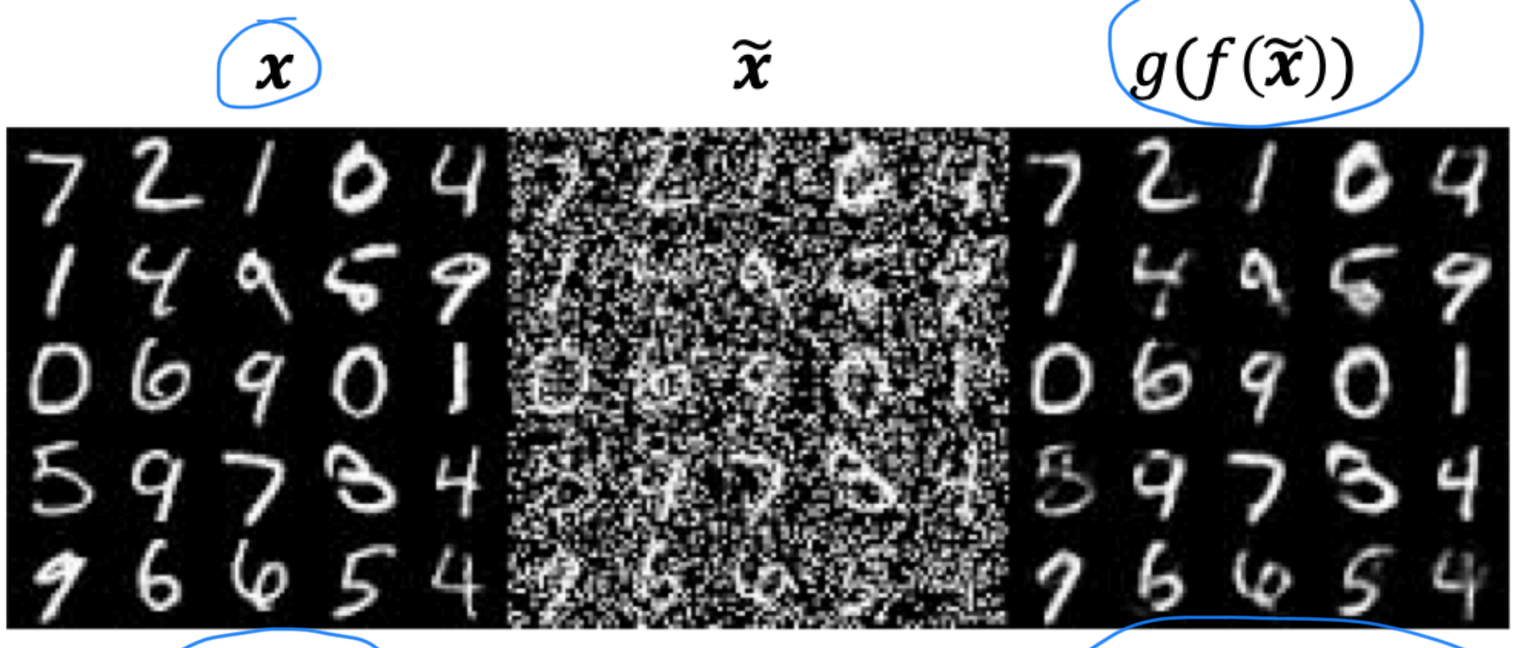
- Approximate objective: L1 regularization
  - $$\arg\min_{\mathbf{W}} \frac{1}{2} \sum_n \|g(f(\mathbf{x}_n; \mathbf{W}_f); \mathbf{W}_g) - \mathbf{x}_n\|_2^2 + c\|f(\mathbf{x}_n; \mathbf{W}_f)\|_1$$

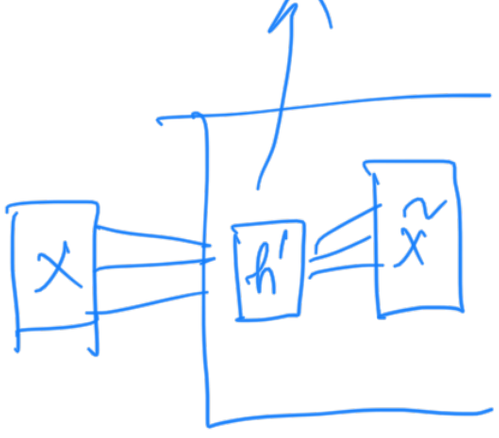
Denoising Autoencoder



- Consider noisy version $\tilde{\mathbf{x}}$ of the input $\mathbf{x}$
- Data denoising:
  - $$\arg\min_{\mathbf{W}} \frac{1}{2} \sum_{n} \| g(f(\tilde{\mathbf{x}}_n; \mathbf{W}_f); \mathbf{W}_g) - \mathbf{x}_n \|_2^2 + c \| f(\tilde{\mathbf{x}}_n; \mathbf{W}_f) \|_1$$

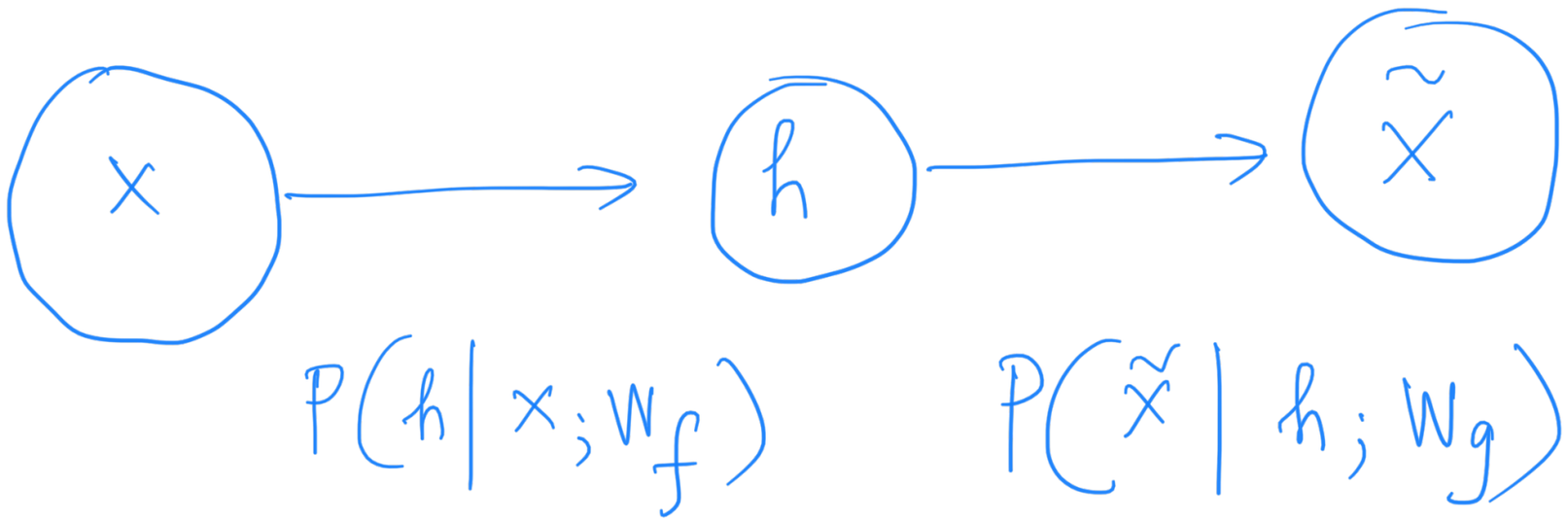
$x \equiv \tilde{x}$



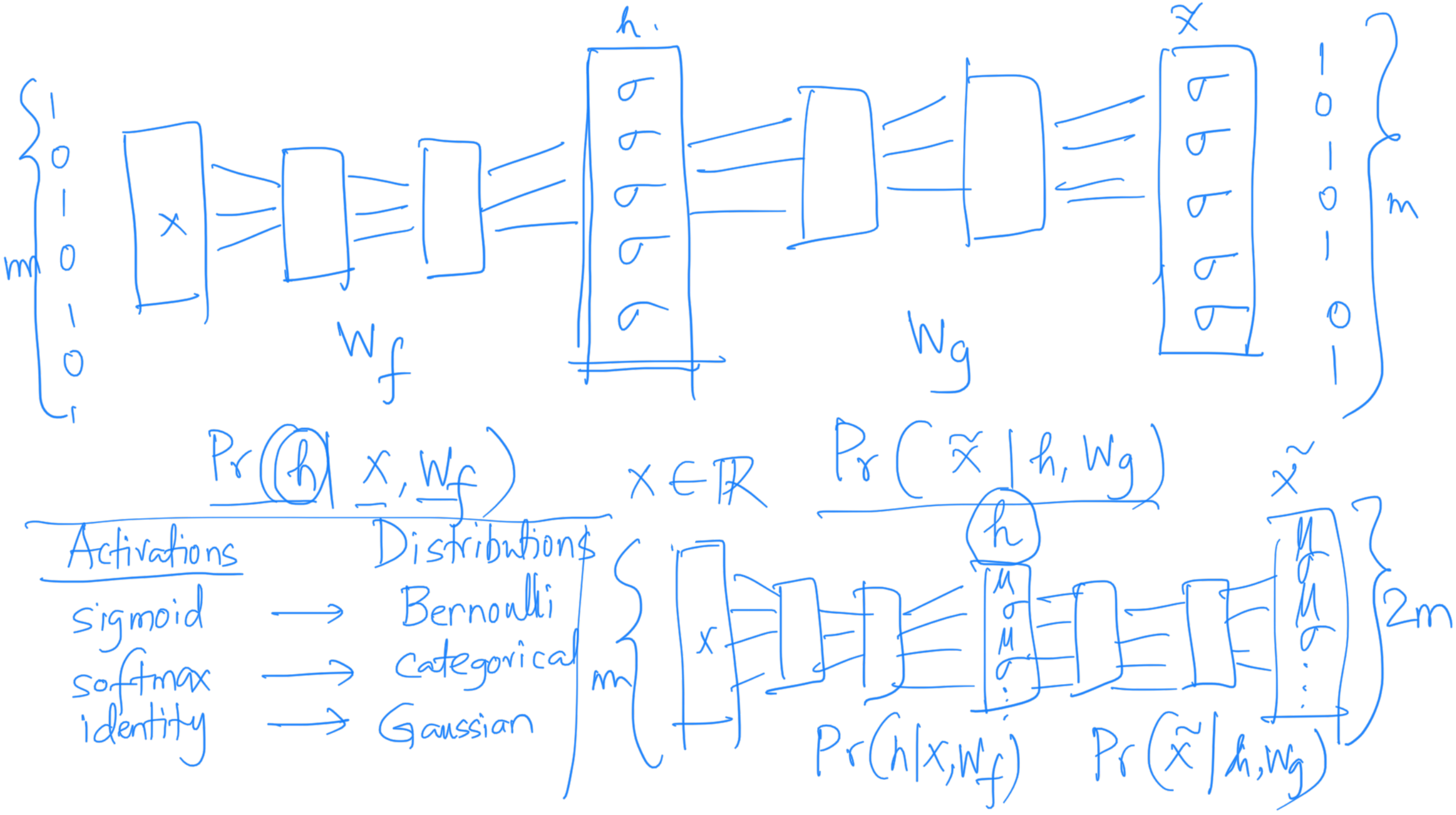


Probabilistic Autoencoder

- Let f and g represent conditional distributions
  - $f : Pr(\mathbf{h} \mid \mathbf{x}; \mathbf{W}_f)$ and $g : Pr(\tilde{\mathbf{x}} \mid \mathbf{h}; \mathbf{W_g})$
  - by using sigmoid, softmax or linear units at the hidden and output layers
- Schematic



Probabilistic Autoencoder



Generative Model

- Sample **h** from some distribution Pr(**h**)
- sample **x** from the decoder: $Pr(\mathbf{h} \mid \mathbf{x}; \mathbf{W}_g)$