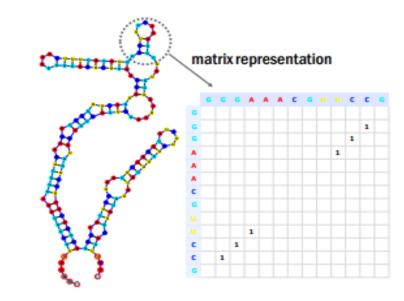
RNA SECONDARY STRUCTURE PREDICTION BY LEARNING UNROLLED ALGORITHMS

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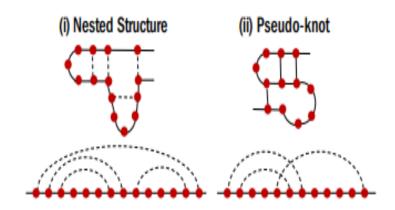
Background

- Ribonucleic acid (RNA) essential in numerous cellular processes and regulating gene expression
- Primary structure of RNA $x := (x_1, \dots, x_L)$ where $x_i \in \{A, G, C, U\}$.
- Secondary structure



Motivation

- Assumption by existing methods
 - energy minimization
 - $A^* = \arg\min_A E_{\boldsymbol{x}}(A).$
- Exponentially large search space
 - Assumption nested structure
 - Optimal substructure O(L³)
- Pseudoknots
 - 1.4% of base-pairs
 - present in around 40% of the RNAs
 - Assist folding into 3D structures



Proposed Method

• Secondary structure is the output of a feed-forward function

Challenges

- RNA secondary structure needs to obey certain hard constraints
- number of RNA data points is limited
- post processing to enforce constraints
- End-to-end network E2Efold
 - Deep Score Network $U_{\theta}(x)$: a transformer-based deep model which represents sequence information useful for structure prediction.

 $A^* = \mathcal{F}_{\theta}(x)$

- Post-Processing Network PP_{ϕ} : a multilayer network which gradually enforces the constraints and restrict the output space

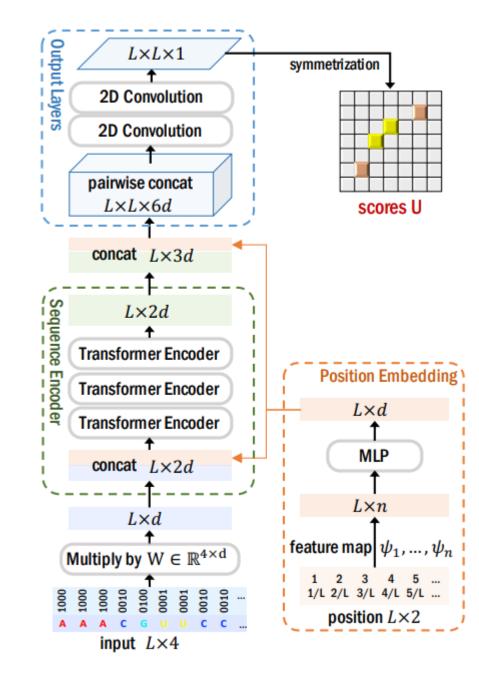
All Binary Structures					
Output Space of E2Efc	bld				
All Valid Structures	with constraints				
Nested Structures (DP applicable)					

Deep Score Network

- L × 4 dimensional one-hot embedding as input
- Position embedding matrix
 - distinguishes each x_i by exact and relative position

 $\boldsymbol{P}_i = \mathrm{MLP}\big(\psi_1(i), \dots, \psi_\ell(i), \psi_{\ell+1}(i/L), \dots, \psi_n(i/L)\big),$

- Transformer Encoders
 - encode the sequence information and the global dependency between nucleotides
- 2D Convolution layers
 - output the pairwise scores
- L × L symmetric matrix as output
 - X_{ii} denotes score of nucleotides x_i and x_i being paired



Post Processing Network

(i) Only three types of nucleotides combinations, $\mathcal{B} := \{AU, UA\} \cup \{GC, CG\} \cup \{GU, UG\}$, can form base-pairs.	$ \forall i, j, \text{ if } x_i x_j \notin \mathcal{B}, \\ \text{then } A_{ij} = 0. $
(ii) No sharp loops are allowed.	$\forall i-j < 4, A_{ij} = 0.$
(iii) There is no overlap of pairs, i.e., it is a matching.	$\forall i, \sum_{j=1}^{L} A_{ij} \leq 1.$

- Target output $A(x) := \{A \in [0,1]^{L \times L}\}$
- Maximize total score $\frac{1}{2}\sum_{i,j}(U_{\theta}(x)_{ij}-s)A_{ij}$
- Nonlinear transformation $\mathcal{T}(\hat{A}) := \frac{1}{2} (\hat{A} \circ \hat{A} + (\hat{A} \circ \hat{A})^{\top}) \circ M(x)$
- L1 penalty term $\|\hat{A}\|_1 := \sum_{i,j} |\hat{A}_{ij}|$

$$\max_{\hat{A} \in \mathbb{R}^{L \times L}} \frac{1}{2} \left\langle U_{\theta}(\boldsymbol{x}) - s, A := \mathcal{T}(\hat{A}) \right\rangle + \rho \|\hat{A}\|_{1} \quad \text{s.t. } A\mathbf{1} \leq \mathbf{1}$$

• Lagrange multiplier λ

$$\min_{\boldsymbol{\lambda} \ge \mathbf{0}} \max_{\hat{A} \in \mathbb{R}^{L \times L}} \underbrace{\frac{1}{2} \langle U_{\theta}(\boldsymbol{x}) - s, A \rangle - \langle \boldsymbol{\lambda}, \operatorname{relu}(A\mathbf{1} - \mathbf{1}) \rangle}_{f} - \rho \|\hat{A}\|_{1}.$$

Algorithm

- proximal gradient for maximization and gradient descent for minimization
- Final deep model

E2Efold :
$$\{A_t\}_{t=1}^T = \overbrace{PP_{\phi}(\underbrace{U_{\theta}(x)}_{Deep \ Score \ Network}, M(x))}^{Post-Process \ Network}$$

• Continuous function to mimic TP, TN, FP, FN

$$\mathrm{TP} = \langle A, A^* \rangle, \ \mathrm{FP} = \langle A, 1 - A^* \rangle, \ \mathrm{FN} = \langle 1 - A, A^* \rangle, \ \mathrm{TN} = \langle 1 - A, 1 - A^* \rangle.$$

• Differentiable F1 loss

 $\mathcal{L}_{-\mathrm{Fl}}(A,A^*) := -2\langle A,A^* \rangle / \left(2\langle A,A^* \rangle + \langle A,1-A^* \rangle + \langle 1-A,A^* \rangle \right)$

Overall loss function

$$\min_{\theta,\phi} \frac{1}{|\mathcal{D}|} \sum_{(x,A^*)\in\mathcal{D}} \frac{1}{T} \sum_{t=1}^T \gamma^{T-t} \mathcal{L}_{-\mathrm{F1}}(A_t,A^*)$$

Algorithm 1: Post-Processing Network $PP_{\phi}(U, M)$

Parameters $\phi := \{w, s, \alpha, \beta, \gamma_{\alpha}, \gamma_{\beta}, \rho\}$ $U \leftarrow \text{softsign}(U - s) \circ U$ $\hat{A}_{0} \leftarrow \text{softsign}(U - s) \circ \text{sigmoid}(U)$ $A_{0} \leftarrow \mathcal{T}(\hat{A}_{0}); \quad \lambda_{0} \leftarrow w \cdot \text{relu}(A_{0}\mathbf{1} - \mathbf{1})$ For $t = 0, \dots, T - 1$ do $[\lambda_{t+1}, A_{t+1}, \hat{A}_{t+1} = \text{PPcell}_{\phi}(U, M, \lambda_{t}, A_{t}, \hat{A}_{t}, t)]$ return $\{A_{t}\}_{t=1}^{T}$

Algorithm 2: Neural Cell PPcell_{\phi}Function PPcell_{\phi} (U, M, \lambda, A, \hat{A}, t) : $G \leftarrow \frac{1}{2}U - (\lambda \circ \operatorname{softsign}(A1 - 1)) \mathbf{1}^{\mathsf{T}}$ $\dot{A} \leftarrow \hat{A} + \alpha \cdot \gamma_{\alpha}{}^{t} \cdot \hat{A} \circ M \circ (G + G^{\mathsf{T}})$ $\dot{A} \leftarrow \operatorname{relu}(|\dot{A}| - \rho \cdot \alpha \cdot \gamma_{\alpha}{}^{t})$ $\hat{A} \leftarrow 1 - \operatorname{relu}(1 - \hat{A})$ [i.e., $\min(\hat{A}, 1)$] $A \leftarrow \mathcal{T}(\hat{A}); \lambda \leftarrow \lambda + \beta \cdot \gamma_{\beta}{}^{t} \cdot \operatorname{relu}(A1 - 1)$ $\operatorname{return} \lambda, A, \hat{A}$

$$A_t\}_{t=1}^T = \mathsf{PP}_{\phi}(U_{\theta}(\boldsymbol{x}), M(\boldsymbol{x}))$$

Related Work

- Classical RNA folding methods
 - energy minimization through DP
 - Run time O(L³) and space O(L²)
 - RNAstructure, Vienna RNAfold, UNAFold
 - LinearFold: run time O(L)
 - Heuristic algorithms: HotKnots, Probknots
- Learning-based RNA folding methods
 - rely on DP-based method
 - ContraFold, ContextFold, CDPfold
- Learning with differentiable algorithms
 - differentiable unrolled algorithms as a building block in neural architectures
 - OptNet : cubic complexity

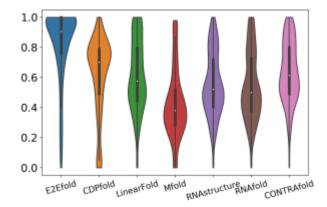
Result: dataset and training

Туре	Arch	niveII	RNAStralign		
Type	length	length #samples		#samples	
All	28~2968	3975	30~1851	30451	
16SrRNA	73~1995	110	54~1851	11620	
5SrRNA	102~135	1283	104~132	9385	
tRNA	54~93	557	59~95	6443	
grp1	210~736	98	163~615	1502	
SRP	28~533	928	30~553	468	
tmRNA	102~437	462	102~437	572	
RNaseP	120~486	454	189~486	434	
telomerase	382~559	37	382~559	37	
23SrRNA	242~2968	3 35	-	-	
grp2	619~780	11	-	-	

Table 1: Dataset Statistics

Table 2: Results on RNAStralign test set. "(S)" indicates the results when one-position shift is allowed.

Method	Prec	Rec	F1	Prec(S)	Rec(S)	F1(S)
E2Efold	0.866	0.788	0.821	0.880	0.798	0.833
CDPfold	0.633	0.597	0.614	0.720	0.677	0.697
LinearFold	0.620	0.606	0.609	0.635	0.622	0.624
Mfold	0.450	0.398	0.420	0.463	0.409	0.433
RNAstructure	0.537	0.568	0.550	0.559	0.592	0.573
RNAfold	0.516	0.568	0.540	0.533	0.587	0.558
CONTRAfold	0.608	0.663	0.633	0.624	0.681	0.650



Result: no re-training and running time

Prec	Rec	F1	Prec(S)	Rec(S)	F1(S)		
0.734	0.66	0.686	0.758	0.676	0.704		
0.557	0.535	0.545	0.612	0.585	0.597		
0.641	0.617	0.621	0.668	0.644	0.647		
0.428	0.383	0.401	0.450	0.403	0.421		
0.563	0.615	0.585	0.590	0.645	0.613		
0.565	0.627	0.592	0.586	0.652	0.615		
0.607	0.679	0.638	0.629	0.705	0.662		
	Prec 0.734 0.557 0.641 0.428 0.563 0.565	PrecRec0.7340.660.5570.5350.6410.6170.4280.3830.5630.6150.5650.627	1	Prec Rec F1 Prec(S) 0.734 0.66 0.686 0.758 0.557 0.535 0.545 0.612 0.641 0.617 0.621 0.668 0.428 0.383 0.401 0.450 0.563 0.615 0.585 0.590 0.565 0.627 0.592 0.586	Prec Rec F1 Prec(S) Rec(S) 0.734 0.66 0.686 0.758 0.676 0.557 0.535 0.545 0.612 0.585 0.641 0.617 0.621 0.668 0.644 0.428 0.383 0.401 0.450 0.403 0.563 0.615 0.585 0.590 0.645 0.565 0.627 0.592 0.586 0.652		

Table 3: Performance comparison on ArchiveII

Table 4: Inference time on RNAStralign

Method		total run time	time per seq	
E2Efold (Pyto	rch)	19m (GPU)	0.40s	
CDPfold (Pyto	orch)	440m*32 threads	300.107s	
LinearFold	(C)	20m	0.43s	
Mfold	(C)	360m	7.65s	
RNAstructure	(C)	3 days	142.02s	
RNAfold	(C)	26m	0.55s	
CONTRAfold	(C)	1 day	30.58s	

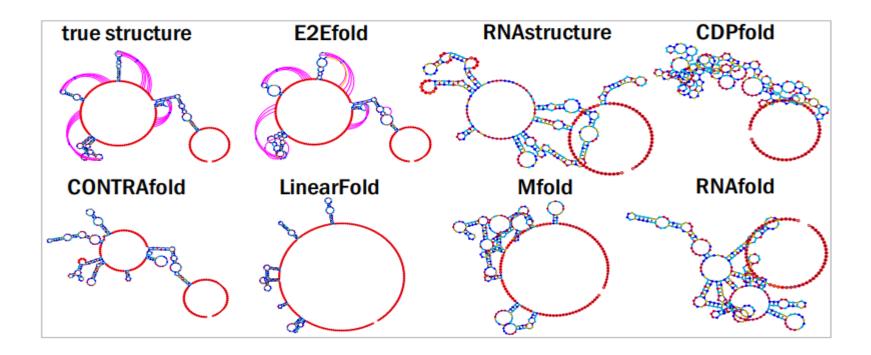
Table 5: Evaluation of pseudoknot prediction

Method	Set F1	TP	FP	TN	FN
E2Efold	0.710	1312	242	1271	0
RNAstructure	0.472	1248	307	983	286

Result: visualization and ablation study

- Naive post processing
 - Choose offset s and set $A_{ii} = 1$ if $U_{\theta}(x)_{ij} > s$.

Table 6: Ablation study							
Method	Prec	Rec	F1	Prec(S)	Rec(S)	F1(S)	
E2Efold							
U_{θ} +PP	0.755	0.712	0.621	0.782	0.737	0.752	



Comments

- Cross validation
- performance per RNA category
- Ablation study
 - Variation in deep score network