Simulation Modeling

• Imitation of the operation of a real-world process or system over time
• Objective: to collect data as if a real system were being observed
• Data collected from the simulation are used to estimate the performance/dependability measures of the system
Discrete Event Simulation

• modeling of the system as it evolves over time by a representation in which the state variables change only at a countable number of points in time

• terminology:
  – simulation clock: a variable that gives the current value of the simulated time
  – event: an instantaneous occurrence which may change the state of the system
Simulation Terminology

– **event list**: a list (data structure) consisting of event records with each record containing the time of occurrence of a particular event, e.g., the arrival time, the departure time of a client

– **timing routine**: a subroutine which determines and removes the most imminent event record from the event list and advances the simulation clock to the time when the corresponding event is to occur

– **event routine**: a subroutine which updates the state of the system when a particular type of event occurs
  - one event routine for each type of event
Event Scheduling

- Determine the number of event types in the system, e.g., 1: arrival, 2: request for service, 3: service completion, 4: timer, etc.
- Place one or more initial event records in the event list, each containing
  - event time, event type, customer class, etc.
- Determine the most imminent event in the event list (by the timing routine) in a loop until a specified stopping rule is satisfied
  - update the simulation clock when an event record is removed from the event list
Event Scheduling (cont.)

– Pass the control to the event routine corresponding to the event type
– Update the state of the system
– Gather the statistics if necessary

• Report the simulation results when the simulation is completed
  – For example
    • the average response time per client
    • the loss probability of calls
    • the system throughput
    • the average number of clients served over a time period
Simulation using smpl

• In the smpl view of systems, there are three types of entities:
  – **resources**: facilities
    • smpl provides functions to define, request, release and preempt (queueing) facilities
  – **tokens**: active entities of the systems, e.g., tasks, users (indistinguishable or distinguishable)
  – **events**: a change of state of any system entity is an event
    • smpl provides functions for scheduling and for selecting events in the order of event occurrence time
Structure of An smpl Program

Initialization routine;

timing control routine to select the most imminent event from the event list (event clock is updated implicitly)
{
    event type 1: event routine for event type 1;
    event type 2: event routine for event type 2;
    ...
    event type n: event routine for event type n;
}

statistics reporting routine;
Initialization Routine

smpl(m, s)
int m=0; /* always 0 */
char *s;

smpl provides seeds for 15 streams for generating random numbers. To collect a set of 15 sample values of a particular performance measure, one can invoke smpl() 15 times:

    loop: repeat 15 times
        { smpl(0, "hw1"); }

One can also use stream(1), stream(2), etc. to specify the stream number to be used in a simulation run
Facility Definition and Control

\[ fd = \text{facility}(s, n) \]
\[ \text{char } *s; \]
\[ \text{int } n; /* \# \text{ of servers } */ \]
\[ \Rightarrow \text{define a queueing server with “n” servers; } \]
\[ \text{smpl automatically manages enqueueing/dequeueing activities} \]

\[ r = \text{request}(fd\_id, \text{token}\_id, \text{pri}) \]
\[ \text{int } fd\_id; \text{ int } \text{token}\_id; \text{ int } \text{pri}; \]
\[ \Rightarrow \text{request a server of facility “fd\_id” be reserved for the token designated by “token\_id” with priority “pri” (higher is better)} \]

\[ r=0: \text{facility is reserved} \]
\[ r=1: \text{facility is busy and the request is blocked in the queue ordered on priority} \]
Facility Definition and Control

\[ r = \text{preempt(fd\_id, tkn\_id, pri)} \]

\text{int fd\_id, tkn\_id, pri;} => same as request() except that it will preempt the server if it is busy serving a task with priority \(< \text{“pri”} \)

\[ \Rightarrow \text{the event record corresponding to the preempted token (for the service completion event) is removed from the event list and a queue entry with the residual time is created} \]

\[ r=0: \text{facility is reserved} \]

\[ r=1: \text{facility is busy and the request is blocked in the queue ordered on priority} \]

\text{release(fd\_id, tkn\_id)}

\text{int fd\_id; int tkn\_id; => release the facility and if the queue is not empty, reschedule an event with the event occurrence time at NOW for a blocked task, and reschedule an event with the event occurrence time at NOW+ the residual time for a preempted task.}

\text{create an event of the same type and put it in the event list}
Scheduling Events

schedule(event_id, te, tkn_id)
int event_id;
real te; /* time interval relative to the current time */
int tkn_id;
=> schedule the event with id “event_id” to occur at NOW+te
=> this essentially inserts an event record with the event occurrence time NOW+te into the event list
=> part of the information in the event record is event_id, tkn_id and the event occurrence time NOW+te

Example: schedule(2, 0.0, token_id)
=> schedule event type #2 associated with token id “token_id” to occur NOW
cause(event_id, tkn_id)
int *event_id;
int *tkn_id;

=> remove the most imminent event from the event list and automatically advance the simulation clock to the event occurrence time

=> return the event number (type) and token id to the caller

Typically in the smpl program, we use a select statement on the event_id returned, so as to transfer the control to the appropriate event routine.
Canceling Events

cancel(event_id)
int event_id;
=> search the event list and remove the first event with the event number = event_id

Get Current Simulation Time

t = real time()
=> return the current simulation clock value
=> real is a predefined type; it is the same as double in C
Status Functions

\( n = \text{int } \text{inq}(fd); \)
\( \Rightarrow \) returning \# of tokens currently in queue (not including the ones in service)

\( r = \text{int } \text{status}(fd) \)
\( \Rightarrow r=0: \) facility is free; \( r=1: \) facility is busy

\( u = \text{real } \text{U}(fd) \)
\( \Rightarrow \) mean \# of tokens in service

\( n = \text{real } \text{Lq}(fd) \)
\( \Rightarrow \) mean \# of tokens in queue excluding the ones in service

\( b = \text{real } \text{B}(fd) \)
\( \Rightarrow \) mean busy period = accumulated busy time/release counts
Random Variate Generation (rand.c)

\[ r = \text{real } \text{drand48}(); /* \text{available on UNIX machines} */ \]
\[ \Rightarrow \text{return } r \text{ in the range of } (0,1) \]

\[ r = \text{real } \text{expntl}(x) \]
\[ \text{double } x; \]
\[ \Rightarrow \text{return an exponentially distributed sample value with mean } x \]

\[ r = \text{real } \text{uniform}(a,b) \]
\[ \text{double } a, b; \Rightarrow \text{return a real number } r \text{ in the range of } (a,b) \]

\[ k = \text{int } \text{random}(i,j) \]
\[ \text{int } i, j; \Rightarrow \text{return an integer } k \text{ in the range of } (i,j) \]

\[ r = \text{real } \text{normal}(x,s) \]
\[ \Rightarrow \text{return a normally distributed sample value with mean } x \text{ and standard deviation } s \]
Traces and Debugging

trace(n)
int n;
=> generate trace messages when a facility is defined, requested, or released, or whenever an event is scheduled or caused

n=0: trace is off
n=1: free-running, i.e., trace messages are generated continuously
n=2: screen by screen running (press any key to resume tracing)
n=3: message by message running (press any key to resume)
M/M/1 smpl program

#include “smpl.h”
main()
{
real Ta=200, Ts=100,  te=200000;
int customer=1, event, server;
smpl(0, “M/M/1 Queue”);
server = facility(“server”,1);
schedule(1, 0.0, customer);
while (time()< te)
{
    cause(&event, &customer);
    switch(event)
    {
    case 1: /* arrival */
        schedule(2,0.0, customer);
        schedule(1, expntl(Ta), customer);
        break;
    case 2: /* request server */
        if (request(server, customer,0)==0)
            schedule(3, expntl(Ts), customer);
        break;
    case 3: /* completion */
        release(server, customer);
        break;
    }
}
report();}
Confidence Interval and Level

• Suppose we collect N sample values $Y_1, Y_2, \ldots, Y_N$ from N simulation runs

• sample mean $Y = (Y_1 + Y_2 + \ldots + Y_N)/N$

• true mean is $\mu$

• Define $1-\alpha$ as the probability that the absolute value of the difference between $Y$ and $\mu$ is equal to or less than $H$\]

\[ \text{Confidence interval half-width} \]

\[
\text{that is, } \text{prob}[Y-H \leq \mu \leq Y+H] = 1-\alpha
\]

Confidence interval
Confidence level
Confidence Interval and Level (cont.)

- When $Y_1, Y_2, \ldots, Y_N$ are independent random variables from a normal distribution with the mean $\mu$, $H$ is defined by $H = t_{\alpha/2; N-1}^{*} \frac{\sigma}{\sqrt{N}}$ where $t$ is the student’s $t$ distribution and $\sigma^2$ is the sample variance given by $\sigma^2 = \sum(Y_i - Y)^2 / (N-1)$ (and thus $\sigma$ is the standard deviation).
Batch Mean Analysis by smpl

- Use a batch size $m$ around 2000 observations to collect a sample value $Y_i$ to justify the normal distribution assumption (by central limit theorem).
- Delete $d = 0.1 \times m$ initial observations
- Collect $k = 10$ batches and compute the confidence interval half-width $H$
- If the desired accuracy has not been reached, collect another batch and compute $H$ again. Repeat as necessary.
BMA: stat.c and bmeans.c

• Based on 95% confidence level ($\alpha = 0.05$) with 10% confidence accuracy ($H/Y = 10\%$)

• The following three routines are provided:

• init_bm(d, m): d is number of initial observations to be discarded and m is the number of observations to collect one sample $Y_i$

• obs(y): y is the observation value generated out of a simulation run
  – if the returning value is 1, it means that the required confidence level and accuracy have been reached; otherwise, need to continue calling this function obs(y)

• civals(Y, H, k): Y, H and k are passed in by reference. This function returns the final result.
M/M/1 smpl program with BMA

```c
#include "smpl.h"
#define TOKENS 1000
#define TRUE 1
#define FALSE 0

main()
{
    real Ta=200.0,Ts=100.0,mean,hw;
    int tk_id=0,customer=0,event,server,nb;
    real ts[TOKENS]; /* start time stamp */
    int cont=TRUE;
    smpl(0,"M/M/1 Queue with BMA");
    init_bm(200,2000); /* d=200; m=2000 */
    server=facility("server",1);
    schedule(1,0.0,tk_id);
    while (cont)
    {
        cause(&event,&customer);
        switch(event)
        {
            case 1: /* arrival */
                ts[customer] = time();
                schedule(2,0.0, customer);
                if (++tk_id >= TOKENS) tk_id=0;
                schedule(1,expntl(Ta),tk_id);
                break;
            case 2: /* request server */
                if (request(server, customer,0)==0)
                    schedule(3,expntl(Ts),customer);
                break;
            case 3: /* release server */
                release(server, customer);
                if (obs(time()-ts[customer]) == 1)
                    cont = FALSE;
                break;
        }
    } /* end while */
    civals(&mean, &hw, &nb);
    printf("Y= %f; H= %f after %d batches\n", mean, hw, nb);
```
#include "smpl.h"
#include "stat.c"

static int d,k,m,n;
static real smy,smY,smY2,Y, h;

init_bm(m0,mb)
int m0,mb;
{ /* set deletion amount & batch size */
d=m0; m=mb; smy=smY=smY2=0.0;
k=n=0;
}

obs(y)
real y;
{
int r=0; real var;
if (d) then {d--; return(r);}
smy+=y; n++;
if (n==m) then{ /* batch complete: update sums & counts */
smy/=n; smY+=smy; smY2+=smy*smy; k++;
printf("batch %2d mean = %.3f",k,smy);
smy=0.0; n=0; /* reset batch variables */
if (k>=10) then
{ /* compute grand mean & half width */
   Y=smY/k; var=(smY2-k*Y*Y)/(k-1);
h=T(0.025,k-1)*sqrt(var/k);
printf(" , rel. HW = %.3f",h/Y);
if (h/Y<=0.1) then r=1;
}
printf("\n");
}
return(r);
}

civals(mean,hw,nb)
real *mean,*hw; int *nb;
{
 /* return batch means analysis results */
*mean=Y; *hw=h; *nb=k;
}