CS/MATH 3414 Exam 2
(please fill in the following information)

Name:

ID:

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1. **(4 points)** Without evaluating the indefinite integral, compute

$$\int_{-1}^{+1} (x^3 + 3x^2 + 5)dx$$

2. **(4 points)** Give an example of a 3-by-3 matrix A that cannot be written in the form $A = LU$, where L is a unit lower triangular matrix and U is an upper triangular matrix.

3. **(3 points)** What would be a good **iterative** method to solve the following linear system?

$$\begin{bmatrix} 7 & -6 \\ -8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Explain your reasons.

4. **(4 points)** Find a **quadratic spline** interpolant to the data: $f(1) = 0$, $f(3/2) = 2$, $f(3) = 3$. Show your calculations clearly and present the final answer systematically.

5. **(3 points)** Is your answer to the previous question a **natural cubic spline**? Why/Why not?

6. **(4+3=7 points)** By the method of undetermined coefficients, derive a numerical integration rule of the form

$$\int_{-1}^{+1} f(x)x^2 dx \approx Af(-\sqrt{\frac{5}{7}}) + Bf(0) + Cf(\sqrt{\frac{5}{7}})$$

that is exact for polynomials of as high a degree as possible, i.e., determine A, B, and C. Also, describe the range of $f(x)$ for which the above rule is exact.