CS/MATH 3414 Final Exam
(please fill in the following information)

Name:

ID:

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1. **(2 points)** In one sentence, state a connection between something you learned in the numerical integration chapter and something you learned in the splines chapter.

2. **(2 points)** What does the following functional iteration compute?

$$x_{n+1} = x_n(2 - x_n R)$$

3. **(2 points)** After you have solved the previous problem, establish the above functional iteration by applying Newton's method (of solving non-linear equations) to some equation $f(x) = 0$. State your $f(x)$ for full credit.

4. **(2 points)** It is suspected that the data: $y(-2) = 1$, $y(-1) = 4$, $y(0) = 11$, $y(1) = 16$, $y(2) = 13$, $y(3) = -4$, comes from a cubic polynomial. Prove or disprove this.

5. **(2 points)** Find two different quadratic spline interpolant fits to the following data: $f(1/2) = 0$, $f(1) = 1$, $f(2) = 2$.

6. **(2 points)** Consider a problem with three data points given, just as above (but not the same values as above). Is it possible to have

   - the quadratic interpolating polynomial,
   - a quadratic spline fit, and
   - the least-squares quadratic polynomial fit

   to be the same function? Under what conditions will this happen?

7. **(2 points)** Conduct one-step of Gauss-Seidel iteration to the linear system:

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 6 & -2 \\ 4 & -3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

Will your iteration converge?

8. **(2 points)** By the method of undetermined coefficients, derive a numerical integration formula of the form

$$\int_{-1}^{+1} f(x)dx \approx Af(-\sqrt{\frac{1}{3}}) + Bf(\sqrt{\frac{1}{3}})$$

that is exact for polynomials of as high a degree as possible, i.e., determine A and B. Then, use the integration rule to compute

$$\int_{0}^{\pi/2} sin(t)dt$$

Is the value computed by your integration rule exact (for this integral)?

9. **(2 points)** For the differential equation $x' = t(x^3 - 6x^2 + 15x)$, determine whether the solution curves diverge from one another as $t \to \infty$.

10. **(2 points)** Convert to a first-order system, the following system of equations:

$$\begin{aligned} x'' + x(x^2 + y^2)^{-3/2} &= 0 \\ y'' + y(x^2 + y^2)^{-3/2} &= 0 \end{aligned}$$

with initial conditions $x(0) = 0.5$, $x'(0) = 0.75$, $y(0) = 0.25$, and $y'(0) = 1.0$.

11. **(2 points)** Find the line that best fits the following data in the least-squares sense: $y(0) = 5$, $y(1) = -6$, and $y(2) = 7$. What is the error in your line?

12. **(3 points)** State in English two ways in which we can fit a function of the form $y = e^{cx}$ to the data: $x(0) = a$, $x(1) = b$. Then, actually find such a function using one of the ways.