CS/MATH 3414 FIRST EXAM

Name __________________________________

Show your answers on these pages. Either show your work on these pages or on attached sheets.

1. (10) Give the IEEE standard 64-bit floating point representation of 33.5 (express your answer in hexadecimal).

2. (10) Let $x = (x_1, \ldots, x_n)$ be an arbitrary real n-vector. Find the smallest constant C such that $\|x\|_1 \leq C \|x\|_\infty$ always holds. $C =$________.

3. (20) Let

$$A = \begin{pmatrix} 10^{10} & 1 \\ 0 & 10^{-10} \end{pmatrix}.$$

det $A =$________, $\|A\|_\infty =$________, $\|A^{-1}\|_\infty =$________, cond $A =$________.

4. (20) Compute the LDL^t factorization of A, where L is a unit lower triangular matrix and D is a diagonal matrix:

$$A = \begin{pmatrix} 4 & 0 & -2 \\ 0 & 1 & 1 \\ -2 & 1 & 11 \end{pmatrix}, \qquad L = \qquad\qquad\qquad D =$$

5. (10) Let $A = QR$, where $A \in E^{n \times n}$ is invertible, $Q \in E^{n \times n}$ has the property that $Q^t Q = I$, and $R \in E^{n \times n}$ is upper triangular. Given Q and R, explain how one would solve a linear system $Ax = b$ for x.

6. (20) The Newton form of the polynomial matching the data $f(1) = 9$, $f'(1) = 10$, $f''(1) = 40$, $f(0) = 7$, $f'(0) = 0$, $f(2) = 71$ is

$P(x) =$

7. (10) Let $f(x)$ be a polynomial of degree 7, $x_0 < x_1 < x_2 < \cdots < x_6 < x_7$, $p_6(x)$ the unique polynomial of degree ≤ 6 interpolating $f(x)$ at $x_0, x_1, \ldots, x_6$, and $q_6(x)$ the unique polynomial of degree ≤ 6 interpolating $f(x)$ at $x_1, x_2, \ldots, x_7$. Explain why

$$f(x) = p_7(x) \equiv \frac{(x_0 - x)q_6(x) - (x_7 - x)p_6(x)}{x_0 - x_7}$$

identically in x. (This is Aitken's interpolation formula.)