CS/Math 3414 Assignment 2

Solution Sketches

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1. If you graph this function, you will find that it has roots at $\{0, \pm\pi/2, \pm\pi, \pm 3\pi/2, \pm 2\pi, \ldots\}$. Using bisection, you will be able to find roots only when there is a change in sign. In this case, these are the roots of the form $\{\pm\pi/2, \pm 3\pi/2, \ldots\}$.

2. We have to compute a linear approximation to $sin(x)$ in the vicinity of $x_0 = \pi/4$. The linear function can be given by $f(x) = a(x - \pi/4) + b$. Now, a must be $f^{'}(x_0) = cos(\pi/4) = 1/\sqrt{2}$. Since $f(x_0)$ must also be $1/\sqrt{2}$, we get $b = 1/\sqrt{2}$.

   Now, how is this related to Newton's method for computing roots? It is easy to see that if we make x_0 to be our first estimate of the root, then this linear function gives us the next iterate $x_1 = \pi/4 - 1$, according to Newton's method.

3. We almost worked out this problem in class. If you express the formula in the typical Newton's form, you will notice that it can be written as:

   $$x_{n+1} = x_n - \frac{tan\ x_n - R}{sec^2\ x_n}$$

   which means that $f(x) = tan\ x - R$. So, the iteration is for computing the arctangent of R ($tan^{-1}(R)$).

4. We already solved the previous problem (3.2.25) in class. If you substitute $R = AB$ in that equation, we get an alternate Newton's iteration:

   $$x_{n+1} = \frac{x_n}{2} + \frac{AB}{2x_n}$$

   Running this iteration two times with either A and B will give you the approximation presented in the book. Running it once with $(A+B)/2$ will give you the same approximation. This is merely an exercise in verification.

5. If you coded this up correctly, you should get a root at 1.83928. Your answers will be verified for sound choices of starting points.

6. For some reason, many of you have had trouble with this question. As the hint suggests, you have to rewrite the given equation in the form:

   $$x = \text{some function of (x)}$$

   So, you have to express

   $$10 - 2x + sin\, x = 0$$

   as:

   $$x = 5 + 0.5\, sin\, x$$

   and use the right hand side for iterating. i.e., choose a x_0, plug it in the right side ($5{+}0.5\, sin\, x$) to get x_1, plug it back in, and so on. If you used $10 - 2x + sin\, x$ for iteration, then in reality you are solving the equation $x = 10 - 2x + sin\, x$, for which the old starting point may or may not lead to convergence (in any case, it would be trying to converge to something else, namely the root of $10 - 3x + sin\, x = 0$).

7. If you coded this up correctly, you should get an iterate of 1.36880. We know that the secant method is known to have superlinear, but not quadratic, convergence. To verify if the convergence is quadratic (for any particular problem such as this), you need to calculate the number of correct digits to the right of the decimal point, after each iteration. If this number is 'doubling,' then you have quadratic convergence.

   Suppose we did the calculations with very high significance (a good estimate of the answer is 1.368808107821373), you will notice that the number of significant decimal digits obtained by secant, after each step, are: $1, 1, 3, 5, 10, \ldots$. After the third iterate, the number of decimal digits is approximately doubling. So if you discount the first few steps, we can say that this convergence is quadratic.

   You will not lose points if you didn't observe this, though it is appreciated.