CS/Math 3414 Assignment 7

Solution Sketches

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1. We can show this either by contradiction (as done in class) or by actually working out Gaussian Elimination to reveal that the pivot is zero. Recall that the LU factorization corresponds to Gaussian Elimination (in its naive form). If we chose the latter approach, we will get the following matrix for the second step:

$$\begin{bmatrix} 2 & 2 & 1 \\ 0 & 0 & \frac{1}{2} \\ 0 & -1 & -\frac{1}{2} \end{bmatrix}$$

   If the rows are re-ordered as third, first, and second, the matrix will permit an LU factorization:

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

2. The code for GE and back substitution is available in the textbook. To find the inverse of a matrix A, we solve three linear systems $Ax = b$ with b assuming the values of the three unit vectors.

3. The solution should identically be $[1, -1, 1, -1]^T$. SOR should take the fewest number of iterations, followed by Gauss-Seidel, followed by Gauss-Jacobi.