CS/Math 3414 Assignment 8
Solution Sketches

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1. Clearly, the given matrix is not diagonally dominant. But that doesn't tell us anything about whether Gauss-Seidel will converge. To determine this, we should calculate the spectral radius of the $M^{-1}N$ matrix (for Gauss-Seidel) and see if it is less than 1. In this case, we write A as $A = D - L - U$, $M = D - L$, $N = U$. This gives:

$$M = \begin{bmatrix} 8 & 0 & 0 \\ 10 & 4 & 0 \\ 50 & 25 & 2 \end{bmatrix} N = \begin{bmatrix} 0 & -2 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

The eigenvalues of $M^{-1}N$ are 0, 0.6838, and 2.2849. We can say that Gauss-Seidel will not converge.

2. A necessary and sufficient condition for SOR to converge (for $0 < w < 2$) is that the matrix A in $Ax = b$ be symmetric and positive definite. The given matrix *is* symmetric. To determine if it is positive definite, we can use one of several conditions:

   - $x^T Ax > 0$ for all nonzero vectors x.
   - All the eigenvalues of A are greater than zero.
   - All the upper left submatrices of A have positive determinants.
   - All the pivots (without row exchanges) are greater than zero.

   It is easy to verify that in this case, the matrix is indeed positive definite. Thus, SOR will converge.

3. Since the matrix is diagonally dominant, both Gauss-Jacobi and Gauss-Seidel will converge.