

CS 4804 Homework 4  
Solution Sketches

---

1. (10 points)

$$\exists x : Barber(x) \wedge (\forall y : \neg Shaves(y, y) \wedge Man(y) \Rightarrow Shaves(x, y))$$

2. (10 points) To prove that an implication  $X \Rightarrow Y$  is valid is the same as showing that  $X \models Y$ . We proceed to do this using resolution refutation (negating  $Y$  and resolving with  $X$  to get a null clause).

$$\begin{aligned} \text{(a) 1) } & (\exists x P(x)) \Rightarrow Q(A) \\ & \equiv \neg(\exists x P(x)) \vee Q(A) \\ & \equiv (\forall x \neg(P(x))) \vee Q(A) \\ & \equiv \neg(P(x)) \vee Q(A) \\ \text{G) } & \neg(\forall x P(x) \Rightarrow Q(A)) \\ & \equiv \exists x \neg(P(x) \Rightarrow Q(A)) \\ & \equiv \exists x \neg(\neg(P(x)) \vee Q(A)) \\ & \equiv \exists x P(x) \wedge \neg(Q(A)) \\ & \equiv P(B) \wedge \neg(Q(A)) \end{aligned}$$

Now calling the first goal clause  $G1$  and the second  $G2$ , the resolution proceeds as follows

G1  
|--1  
|--G2  
NIL

$$\begin{aligned} \text{(b) 1) } & (\forall x P(x)) \Rightarrow Q(A) \\ & \equiv \neg(\forall x P(x)) \vee Q(A) \\ & \equiv (\exists x \neg(P(x))) \vee Q(A) \\ & \equiv \neg(P(B)) \vee Q(A) \\ \text{G) } & \neg(\exists x P(x) \Rightarrow Q(A)) \\ & \equiv \forall x \neg(P(x) \Rightarrow Q(A)) \\ & \equiv \forall x \neg(\neg(P(x)) \vee Q(A)) \\ & \equiv \forall x P(x) \wedge \neg(Q(A)) \\ & \equiv P(x) \wedge \neg(Q(A)) \end{aligned}$$

Now calling the first goal clause  $G1$  and the second  $G2$ , the resolution proceeds as follows

G1  
|--1  
|--G2  
NIL

3. (20 points) Predicates:

$Blue(x) :=$  Object  $X$  is blue

$Green(x) :=$  Object  $X$  is green

$Pushable(x) :=$  Object  $X$  is pushable

- (a) If pushable objects are blue then non-pushable ones are green.  
 $(\forall x : Pushable(x) \Rightarrow Blue(x)) \Rightarrow (\forall y : \neg Pushable(y) \Rightarrow Green(y))$   
 $\equiv \neg(\forall x : \neg Pushable(x) \vee Blue(x)) \vee (\forall y : Pushable(y) \vee Green(y))$   
 $\equiv (\exists x : Pushable(x) \wedge \neg Blue(x)) \vee Pushable(y) \vee Green(y)$   
 $\equiv (Pushable(S) \wedge \neg Blue(S)) \vee Pushable(y) \vee Green(y)$   
 $\equiv (Pushable(S) \vee Pushable(y) \vee Green(y)) \wedge (\neg Blue(S) \vee Pushable(y) \vee Green(y))$
- (b) All objects are either blue or green but not both.  
 $(\forall x : Green(x) \vee Blue(x)) \wedge (\forall x : \neg Blue(x) \vee \neg Green(x))$   
 $\equiv (Green(x) \vee Blue(x)) \wedge (\neg Blue(x) \vee \neg Green(x))$
- (c) If there is a non-pushable object, then all pushable ones are blue  
 $(\exists x : \neg Pushable(x)) \Rightarrow (\forall y : Pushable(y) \Rightarrow Blue(y))$   
 $\equiv \neg(\exists x : \neg Pushable(x)) \vee (Pushable(y) \Rightarrow Blue(y))$   
 $\equiv (\forall x : (Pushable(x))) \vee (\neg Pushable(y) \vee Blue(y))$   
 $\equiv Pushable(x) \vee \neg Pushable(y) \vee Blue(y)$
- (d) Object O1 is pushable  
 $Pushable(O1)$
- (e) Object O2 is non-pushable  
 $\neg Pushable(O2)$
- (f) The negated goal: There is not a green object  
 $\forall x : \neg Green(x) \equiv \neg Green(x)$

Now calling the first clause of a A1 and the second A2, and similarly for b, the resolution proceeds as follows

```
F
|--A2
|--B1
|--F
|--E
|
NIL
```

4. (20 points)

- (a)  $\{x/A, y/B, z/B\}$   
(b) No unifier  
(c)  $\{y/John, x/John\}$   
(d) No unifier, Occurs check

5. (40 points)

For simplicity, only needed statements are shown. Predicate definitions are also omitted and must be obvious from the context.

- (a) A professor is either a Assistant, Associate, or Full.  
 $\forall x : \text{Professor}(x) \Rightarrow \text{Rank}(x, \text{Assistant}) \vee \text{Rank}(x, \text{Associate}) \vee \text{Rank}(x, \text{Full})$   
 $\equiv \forall x : \neg \text{Professor}(x) \vee \text{Rank}(x, \text{Assistant}) \vee \text{Rank}(x, \text{Associate}) \vee \text{Rank}(x, \text{Full})$   
 $\equiv \neg \text{Professor}(x) \vee \text{Rank}(x, \text{Assistant}) \vee \text{Rank}(x, \text{Associate}) \vee \text{Rank}(x, \text{Full})$
- (b) If someone has a rank, then they are a professor.  
 $\forall x : (\exists y : \text{Rank}(x, y)) \Rightarrow \text{Professor}(x)$   
 $\equiv \forall x : \neg(\exists y : \text{Rank}(x, y)) \vee \text{Professor}(x)$   
 $\equiv \forall x : (\forall y : \neg \text{Rank}(x, y)) \vee \text{Professor}(x)$   
 $\equiv \neg \text{Rank}(x, y) \vee \text{Professor}(x)$
- (c) No one can evaluate a Full professor.  
 $\forall xy : \text{Rank}(x, \text{Full}) \Rightarrow \neg \text{Evaluate}(y, x)$   
 $\equiv \forall xy : \neg \text{Rank}(x, \text{Full}) \vee \neg \text{Evaluate}(y, x)$   
 $\equiv \neg \text{Rank}(x, \text{Full}) \vee \neg \text{Evaluate}(y, x)$
- (d) If someone is an Assistant professor, then an Associate professor can evaluate them.  
 $\forall xy : \text{Rank}(x, \text{Assistant}) \wedge \text{Rank}(y, \text{Associate}) \Rightarrow \text{Evaluate}(y, x)$   
 $\equiv \forall xy : \neg \text{Rank}(x, \text{Assistant}) \vee \neg \text{Rank}(y, \text{Associate}) \vee \text{Evaluate}(y, x)$   
 $\equiv \neg \text{Rank}(x, \text{Assistant}) \vee \neg \text{Rank}(y, \text{Associate}) \vee \text{Evaluate}(y, x)$
- (e) If someone is an Assistant professor, then a Full professor can evaluate them.  
 $\forall xy : \text{Rank}(x, \text{Assistant}) \wedge \text{Rank}(y, \text{Full}) \Rightarrow \text{Evaluate}(y, x)$   
 $\equiv \forall xy : \neg \text{Rank}(x, \text{Assistant}) \vee \neg \text{Rank}(y, \text{Full}) \vee \text{Evaluate}(y, x)$   
 $\equiv \neg \text{Rank}(x, \text{Assistant}) \vee \neg \text{Rank}(y, \text{Full}) \vee \text{Evaluate}(y, x)$
- (f) There are at least two people who can evaluate Dr. Strangelove.  
 $\exists xy : \text{Evaluate}(x, \text{Strangelove}) \wedge \text{Evaluate}(y, \text{Strangelove}) \wedge \neg(x = y)$   
 $\equiv \text{Evaluate}(S1, \text{Strangelove}) \wedge \text{Evaluate}(S2, \text{Strangelove}) \wedge \neg(S1 = S2)$   
 i.  $\text{Evaluate}(S1, \text{Strangelove})$   
 ii.  $\text{Evaluate}(S2, \text{Strangelove})$   
 iii.  $\neg(S1 = S2)$
- (g) There are at most two people who can evaluate Dr. Strangelove.  
 $\forall xyz : \text{Evaluate}(x, \text{Strangelove}) \wedge \text{Evaluate}(y, \text{Strangelove}) \wedge \text{Evaluate}(z, \text{Strangelove}) \Rightarrow$   
 $x = y \vee x = z \vee y = z$   
 $\equiv \forall xyz : \neg(\text{Evaluate}(x, \text{Strangelove}) \wedge \text{Evaluate}(y, \text{Strangelove}) \wedge \text{Evaluate}(z, \text{Strangelove})) \vee$   
 $x = y \vee x = z \vee y = z$   
 $\equiv \forall xyz : \neg \text{Evaluate}(x, \text{Strangelove}) \vee \neg \text{Evaluate}(y, \text{Strangelove}) \vee \neg \text{Evaluate}(z, \text{Strangelove}) \vee$   
 $x = y \vee x = z \vee y = z$   
 $\equiv \neg \text{Evaluate}(x, \text{Strangelove}) \vee \neg \text{Evaluate}(y, \text{Strangelove}) \vee \neg \text{Evaluate}(z, \text{Strangelove}) \vee$   
 $x = y \vee x = z \vee y = z$
- (h) Dr. Strangelove is a professor.  
 $\text{Professor}(\text{Strangelove})$
- (i) Dr. Knowitall is an Associate professor.  
 $\text{Rank}(\text{Knowitall}, \text{Associate})$
- (j) Dr. Gray is an Full professor.  
 $\text{Rank}(\text{Gray}, \text{Full})$

- (k) Dr. White is an Full professor.  
 $\text{Rank}(\text{White}, \text{Full})$
- (l) The goal is the rank that Dr. Strangelove holds.  
 $\neg(\exists x : \text{Rank}(\text{Strangelove}, x))$   
 $\equiv \neg\text{Rank}(\text{Strangelove}, x)$

The refutation then proceeds as follows (the keyword *AND* is used to denote the resolution at each step):

$\neg\text{Rank}(\text{Strangelove}, x)$   
AND  $\neg\text{Professor}(y) \vee \text{Rank}(y, \text{Assistant}) \vee \text{Rank}(y, \text{Associate}) \vee \text{Rank}(y, \text{Full})$   
 $\equiv \neg\text{Professor}(\text{Strangelove}) \vee \text{Rank}(\text{Strangelove}, \text{Assistant}) \vee \text{Rank}(\text{Strangelove}, \text{Full})$   
AND  $\text{Professor}(\text{Strangelove})$   
 $\equiv \text{Rank}(\text{Strangelove}, \text{Assistant}) \vee \text{Rank}(\text{Strangelove}, \text{Full})$   
AND  $\neg\text{Rank}(x, \text{Full}) \vee \neg\text{Evaluate}(y, x)$   
 $\equiv \text{Rank}(\text{Strangelove}, \text{Assistant}) \vee \neg\text{Evaluate}(y, \text{Strangelove})$   
AND  $\text{Evaluate}(S1, \text{Strangelove})$   
 $\equiv \text{Rank}(\text{Strangelove}, \text{Assistant})$   
AND  $\neg\text{Rank}(x, \text{Assistant}) \vee \neg\text{Rank}(y, \text{Associate}) \vee \text{Evaluate}(y, x)$   
 $\equiv \neg\text{Rank}(y, \text{Associate}) \vee \text{Evaluate}(y, \text{Strangelove})$   
AND  $\text{Rank}(\text{Knowitall}, \text{Associate})$   
 $\equiv \text{Evaluate}(\text{Knowitall}, \text{Strangelove})$ .

Save this fact for later and lets continue on a different track. Lets go back to the statement (proven above) that Dr. Strangelove must be an Assistant professor and proceed from there.

$\text{Rank}(\text{Strangelove}, \text{Assistant})$   
AND  $\neg\text{Rank}(x, \text{Assistant}) \vee \neg\text{Rank}(y, \text{Full}) \vee \text{Evaluate}(y, x)$   
 $\equiv \neg\text{Rank}(y, \text{Full}) \vee \text{Evaluate}(y, \text{Strangelove})$   
AND  $\text{Rank}(\text{Gray}, \text{Full})$   
 $\equiv \text{Evaluate}(\text{Gray}, \text{Strangelove})$ .

Agan, save this fact for later. Continue:

$\text{Rank}(\text{Strangelove}, \text{Assistant})$   
AND  $\neg\text{Rank}(x, \text{Assistant}) \vee \neg\text{Rank}(y, \text{Full}) \vee \text{Evaluate}(y, x)$   
 $\equiv \neg\text{Rank}(y, \text{Full}) \vee \text{Evaluate}(y, \text{Strangelove})$   
AND  $\text{Rank}(\text{White}, \text{Full})$   
 $\equiv \text{Evaluate}(\text{White}, \text{Strangelove})$ .

At this point, we have identified three people who can evaluate Dr. Strangelove. This clearly violates one of the statements given in the problem, so we can aim to arrive at a contradiction.

$\text{Evaluate}(\text{White}, \text{Strangelove})$   
AND  $\neg\text{Evaluate}(x, \text{Strangelove}) \vee \neg\text{Evaluate}(y, \text{Strangelove}) \vee \neg\text{Evaluate}(z, \text{Strangelove}) \vee$   
 $x = y \vee x = z \vee y = z$   
 $\equiv \neg\text{Evaluate}(y, \text{Strangelove}) \vee \neg\text{Evaluate}(z, \text{Strangelove}) \vee \text{White} = y \vee \text{White} = z \vee y = z$   
AND  $\text{Evaluate}(\text{Gray}, \text{Strangelove})$   
 $\equiv \neg\text{Evaluate}(z, \text{Strangelove}) \vee \text{White} = \text{Gray} \vee \text{White} = z \vee \text{Gray} = z$   
AND  $\text{Evaluate}(\text{Knowitall}, \text{Strangelove})$

$$\begin{aligned}
&\equiv White = Gray \vee White = Knowitall \vee Gray = Knowitall \\
&\text{AND } \neg(White = Gray) \\
&\equiv White = Knowitall \vee Gray = Knowitall \\
&\text{AND } \neg(White = Knowitall) \\
&\equiv Gray = Knowitall \\
&\text{AND } \neg(Gray = Knowitall) \\
&\equiv false
\end{aligned}$$

Since the unification  $\{x/Associate\}$  was made for the negated goal statement, Dr. Strangelove is an Associate professor. This proof uses set-of-support resolution, with the initial set consisting of the negated goal. Linear resolution could also be used as it is complete.