CS 4804 Midterm Exam

Name:

ID:

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1. (5 points) Give an example of a search space (i.e., a diagram with circles for states, edges for state transitions, and edge costs labeled on each edge) where $f(n) = h^*(n)$ leads to the optimal solution. Recall that $h^*(n)$ gives the true (optimal) cost of the path from n to the goal state.

2. (5 points) Give an example of a search space where $f(n) = h^*(n)$ leads to a sub-optimal solution.

3. (30 points) You are given the following logic puzzle: In five houses, each with a different color, live 5 persons of different nationalities, each of whom prefer a different brand of cigarette, a different drink, and a different pet. Given the following facts, the question to answer is: "Where does the zebra live, and in which house do they drink water?"

   - The Englishman lives in the red house.
   - The Spaniard owns the dog.
   - The Norwegian lives in the first house on the left.
   - Kools are smoked in the yellow house.
   - The person who smokes Chesterfields lives in the house next to the person with the fox.
   - The Norwegian lives next to the blue house.
   - The Winston smoker owns snails.
   - The Lucky Strike smoker drinks orange juice.
   - The Ukrainian drinks tea.
   - The Japanese smokes Parliaments.
   - Kools are smoked in the house next to the house where the horse is kept.
   - Coffee is drunk in the green house.
   - The Green house is immediately to the right (your right) of the ivory house.
   - Milk is drunk in the middle house.

   You do **NOT** have to solve the puzzle! Simply explain how you will formulate this problem as a constraint satisfaction problem so that solving the CSP will help answer the question above. For full credit, carefully define the variables, domains, and constraints of the problem. How many constraints are there? Are they binary, 3-way, or more generally n-way constraints?

4. (20 points) You are given a satisfiability (SAT) problem in CNF form (i.e., a conjunction of clauses, where each clause is a disjunction). Further assume that each clause consists of exactly 4 literals, such as $x_1 \vee \neg x_2 \vee x_5 \vee x_7$. Such a problem is called a 4-CNF-SAT problem.

   You are now given a solver that can solve 3-CNF-SAT problems, i.e., where each clause consists of exactly 3 literals. An example of a formula in 3-CNF form is $(x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_5)$. Explain how you will re-formulate the 4-CNF-SAT problem into a 3-CNF-SAT problem so that you can use this solver and extract the solution to the original 4-CNF-SAT problem.

5. (12 points) Explain how you would use the DPLL algorithm to quickly conclude that the following propositional formula (given as a collection of clauses) is unsatisfiable:

$$\begin{aligned} \neg p \vee \neg q \vee r \\ \neg s \vee t \\ \neg t \vee p \\ s \\ \neg r \\ \neg s \vee u \\ \neg u \vee q \end{aligned}$$

For full credit, identify in a step-by-step fashion how DPLL concludes whether a given variable must be true or false and the names of heuristics ('tricks') applied at each step.

6. (8 points) Prove by resolution-refutation that

$$(p \rightarrow q) \rightarrow [(r \vee p) \rightarrow (r \vee q)]$$

is a tautology, where p, q, and r are propositional logic symbols.

7. (20 points) Sam, Clyde, and Oscar are elephants. We are given the following facts about them:

   (a) Sam is pink.

   (b) Clyde is gray and likes Oscar.

   (c) Oscar is either pink or gray (but not both) and likes Sam.

   Use resolution-refutation in predicate logic to prove that some gray elephant likes some pink elephant. For full credit, state the given sentences (including the goal) in first order predicate logic (5 points), convert to clauses suitable for resolution-refutation (10 points), and prove a contradiction (5 points).