Homework 4 Solutions
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1. Predicates:

$Student(x)$	True iff x is a student
$Taking(x, y)$	True iff student x is taking the class y
$Failed(x, y)$	True iff student x failed the class y
$Score(x, y, z)$	True iff student x received the score z in the class y
$Greater(x, y)$	True iff $x > y$

   Clauses:

   $\exists x\ Student(x) \wedge (\neg Taking(x, \text{CS4804}) \vee \neg Taking(x, \text{CS5804}))$
   $\exists x\ Failed(x, \text{HIST4804}) \wedge (\forall y\ Failed(y, \text{HIST4804}) \rightarrow x = y)$
   $\exists x\ Failed(x, \text{HIST4804}) \wedge Failed(x, \text{BIOL4804}) \wedge (\forall y\ Failed(y, \text{HIST4804}) \wedge Failed(y, \text{BIOL4804}) \rightarrow x = y)$
   $\exists w, x\ Taking(w, \text{HIST4804}) \wedge Score(w, \text{HIST4804}, x) \wedge$
   $(\forall y, z\ Taking(y, \text{BIOL4804}) \wedge Score(y, \text{BIOL4804}, z) \rightarrow Greater(x, z))$
   $\forall x, y\ Failed(x, y) \rightarrow Taking(x, y)$

2. For this I used two predicates: $Willing(x, y)$ which is true iff x is willing to teach the class y and $Teaching(x, y)$ which is true iff x is teaching the class y. The given statements are which classes are the only ones each teacher is willing to teach:

   (a) $\forall x\ Willing(\text{Flexy}, x) \rightarrow x = \text{CS4104} \vee x = \text{CS4604}$
   (b) $\forall x\ Willing(\text{Accommodating}, x) \rightarrow x = \text{CS4604} \vee x = \text{CS4804}$
   (c) $\forall x\ Willing(\text{Strict}, x) \rightarrow x = \text{CS4104}$

   Observe that we have used = as shorthand for a predicate Equal. We can assert all the obvious true and false instantiations of this predicate, such as CS4104 = CS4104 and $\neg$ (CS4804 = CS4104). We also have further statements: that teachers only teach classes they are willing to teach:

   (d) $\forall x, y\ Teaching(x, y) \rightarrow Willing(x, y)$

   That every class is being taught:

   (e) $\exists x\ Teaching(x, \text{CS4104})$
   (f) $\exists x\ Teaching(x, \text{CS4604})$
   (g) $\exists x\ Teaching(x, \text{CS4804})$

   And that each professor can only teach one course:

   (h) $\forall x, y\ Teaching(x, y) \rightarrow (\forall z\ Teaching(x, z) \rightarrow y = z)$

   Now if this is all that is encoded then it will be impossible to determine who is teaching what course using strict resolution since there may be other professors we are not aware of who could be teaching courses. Thus we want to encode the implicit assumption that if a class is being taught it is being taught by one of the given professors:

   (i) $\forall x, y\ Teaching(x, y) \rightarrow x = \text{Flexy} \vee x = \text{Accommodating} \vee x = \text{Strict}$

   Now to prove who is teaching what introduce the negation of the following clause:

   $\exists x, y, z\ Teaching(\text{x}, \text{CS4104}) \wedge Teaching(\text{y}, \text{CS4604}) \wedge Teaching(\text{z}, \text{CS4804})$

   Converting these into a form suitable for resolution-refutation yields the following clauses:

(1) $\neg Willing(\text{Flexy}, x) \vee x = \text{CS4104} \vee x = \text{CS4604}$
(2) $\neg Willing(\text{Accommodating}, x) \vee x = \text{CS4604} \vee x = \text{CS4804}$
(3) $\neg Willing(\text{Strict}, x) \vee x = \text{CS4104}$
(4) $\neg Teaching(x, y) \vee Willing(x, y)$
(5) $Teaching(S_1, \text{CS4104})$
(6) $Teaching(S_2, \text{CS4604})$
(7) $Teaching(S_3, \text{CS4804})$
(8) $\neg Teaching(x, y) \vee \neg Teaching(x, z) \vee y = z$
(9) $\neg Teaching(x, y) \vee x = \text{Flexy} \vee x = \text{Accommodating} \vee x = \text{Strict}$
(10) $\neg Teaching(\text{x}, \text{CS4104}) \vee \neg Teaching(\text{y}, \text{CS4604}) \vee \neg Teaching(\text{z}, \text{CS4804}))$

Then resolution on 1 with $\neg\text{CS4804} = \text{CS4604}$ and $\neg\text{CS4804} = \text{CS4104}$ gives:

(11) $\neg Willing(\text{Flexy}, \text{CS4804})$

Now resolution on 4 and 11 gives:

(12) $\neg Teaching(\text{Flexy}, \text{CS4804})$

Similarly for 2 with $\neg\text{CS4104} = \text{CS4604}$ and $\neg\text{CS4104} = \text{CS4804}$ and 4:

(13) $\neg Teaching(\text{Accommodating}, \text{CS4104})$

Now resolution on 3 with $\neg\text{CS4604} = \text{CS4104}$ gives:

(14) $\neg Willing(\text{Strict}, \text{CS4604})$

Which again resolves with 4 to give:

(15) $\neg Teaching(\text{Strict}, \text{CS4604})$

Similarly resolving 3 with $\neg\text{CS4804} = \text{CS4104}$ and 4 gives:

(16) $\neg Teaching(\text{Strict}, \text{CS4804})$

Now resolving 5 with 9 gives:

(17) $S_1 = \text{Flexy} \vee S_1 = \text{Accommodating} \vee S_1 = \text{Strict}$

Now introduce an equality rule $\forall x, y, z\ x = y \rightarrow Teaching(x, z) \rightarrow Teaching(y, z)$ which translates to $\neg x = y \vee \neg Teaching(x, z) \vee Teaching(y, z)$ and resolve with 5 to get:

(18) $\neg S_1 = y \vee Teaching(y, \text{CS4104})$

Resolving 17 with 18 with $\theta = \{y/\text{Flexy}\}$:

(19) $Teaching(\text{Flexy}, \text{CS4104}) \vee S_1 = \text{Accommodating} \vee S_1 = \text{Strict}$

Repeating this twice more with $\theta = \{y/\text{Accommodating}\}$ and $\theta = \{y/\text{Strict}\}$

(20) $Teaching(\text{Flexy}, \text{CS4104}) \vee Teaching(\text{Accommodating}, \text{CS4104}) \vee Teaching(\text{Strict}, \text{CS4104})$

Similarly beginning with the clauses 6 and 7 rather then 5:

(21) $Teaching(\text{Flexy}, \text{CS4604}) \vee Teaching(\text{Accommodating}, \text{CS4604}) \vee Teaching(\text{Strict}, \text{CS4604})$
(22) $Teaching(\text{Flexy}, \text{CS4804}) \vee Teaching(\text{Accommodating}, \text{CS4804}) \vee Teaching(\text{Strict}, \text{CS4804})$

Using resolution on 12, 13, 15, and 16 combined with 20, 21, and 22:

(23) $Teaching(\text{Flexy}, \text{CS4104}) \vee Teaching(\text{Strict}, \text{CS4104})$
(24) $Teaching(\text{Flexy}, \text{CS4604}) \vee Teaching(\text{Accommodating}, \text{CS4604})$
(25) $Teaching(\text{Accommodating}, \text{CS4804})$

Using resolution on 8 and 25 with $\theta = \{x/\text{Accommodating}, y/\text{CS4804}\}$:

(26) $\neg Teaching(\text{Accommodating}, z) \vee \text{CS4804} = z$

Resolution with 24 and 26 gives:

(27) $Teaching(\text{Flexy}, \text{CS4604}) \vee \text{CS4804} = \text{CS4604}$

Resolving 27 with the equality axiom $\neg\text{CS4804} = \text{CS4604}$ gives:

(28) $Teaching(\text{Flexy}, \text{CS4604})$

Now, as for 8 and 25, using resolution on 8 and 28:

(29) $\neg Teaching(\text{Flexy}, z) \vee \text{CS4604} = z$

And then 23 with 29:

(30) $Teaching(\text{Strict}, \text{CS4104}) \vee \text{CS4604} = \text{CS4104}$

Once again using an axiom $\neg\text{CS4604} = \text{CS4104}$:

(31) $Teaching(\text{Strict}, \text{CS4104})$

Now resolve 25, 28, and 31 with the negated goal (10) and $\theta = \{x/\text{Strict}, y/\text{Flexy}, z/\text{Accommodating}\}$ to derive the null clause, thus indicating that Professor Strict teaches CS4104, Professor Flexy teaches CS4604, and Professor Accommodating teaches CS4804.