Reasoning about Sets using Redescription Mining

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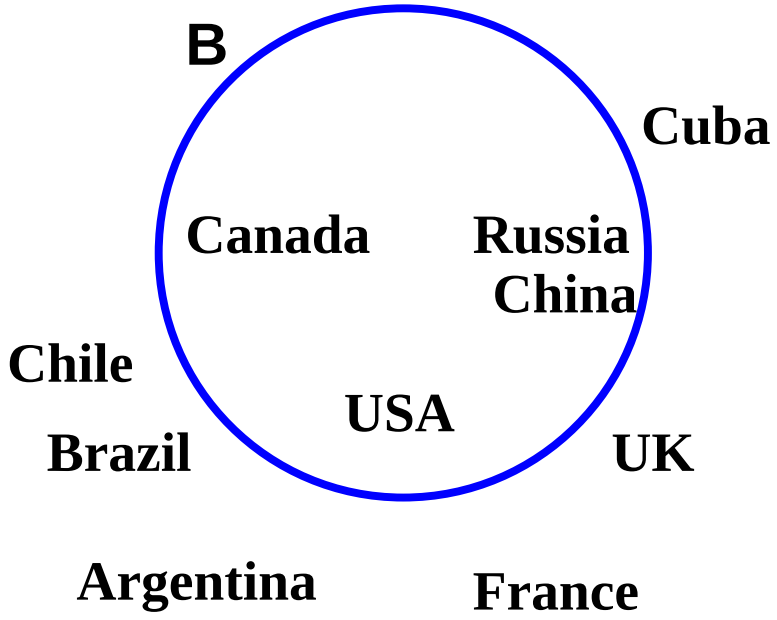
What are redescriptions?

A shift-of-vocabulary, or a different way of communicating a given piece of information.

Input to Redescription Mining

Input to Redescription Mining (contd.)



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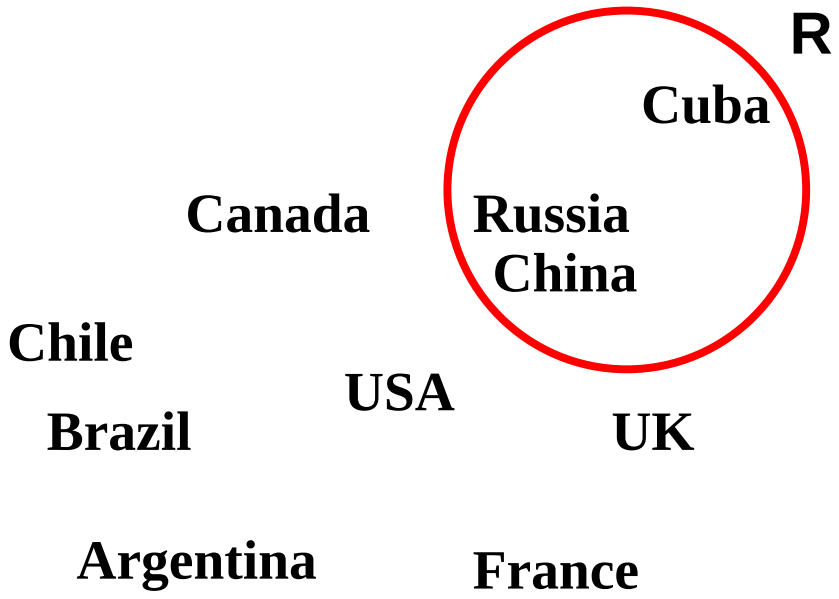
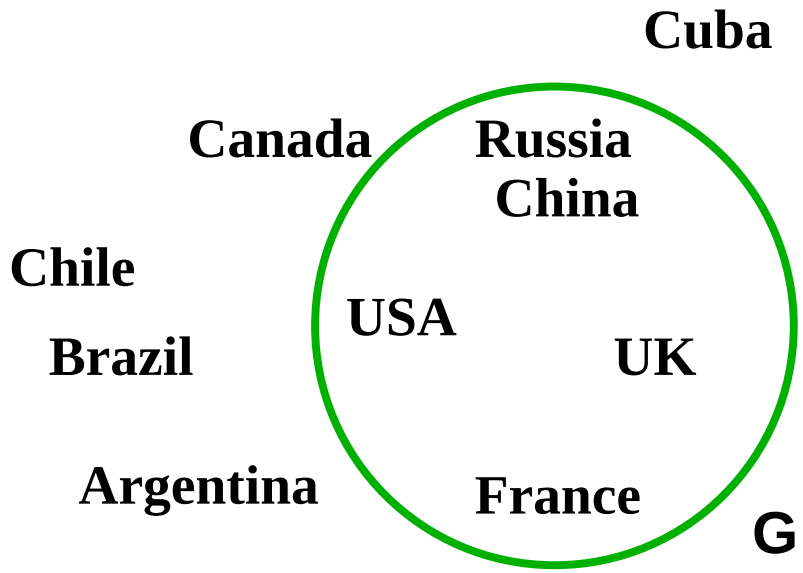


Input to Redescription Mining (contd.)



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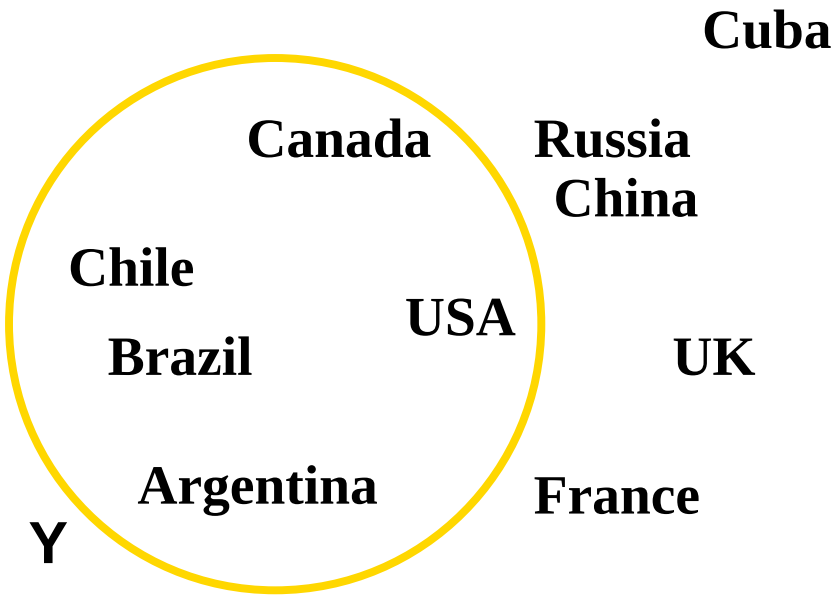

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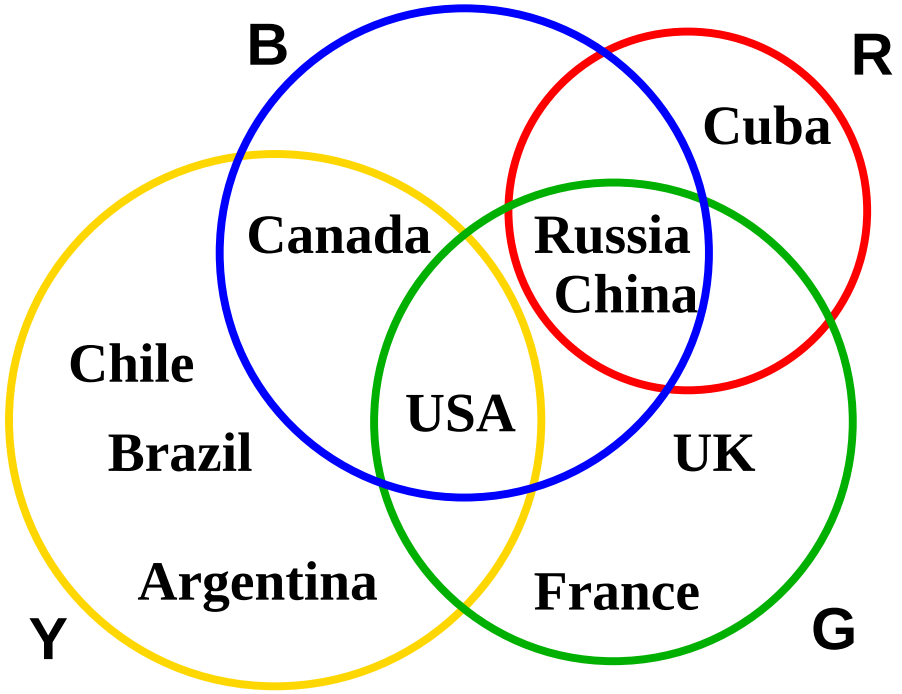


Basic Problem

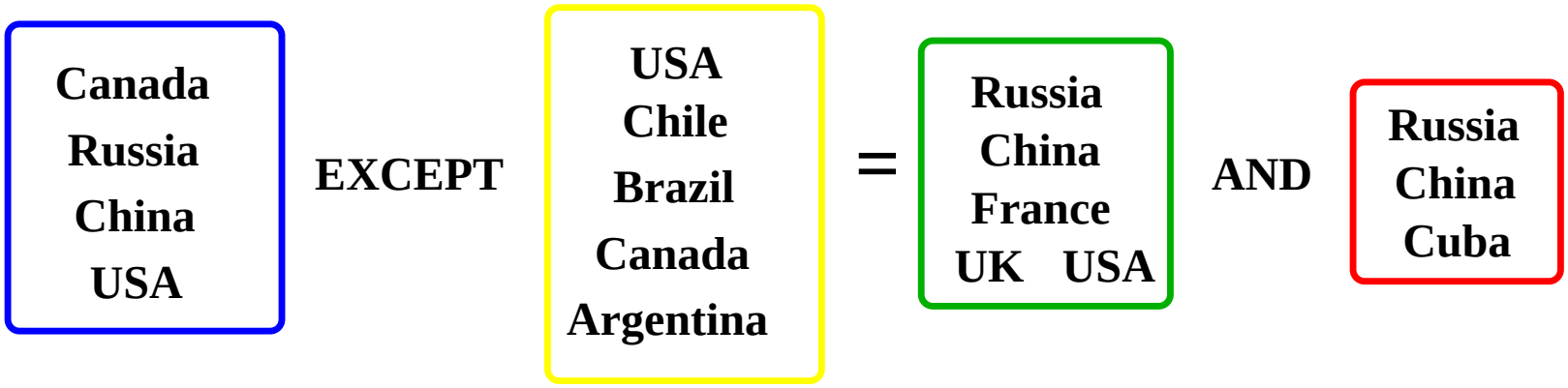
Given

- a set $\mathcal{O}$ of objects (e.g., countries)
- a collection of subsets (*descriptors*) of $\mathcal{O}$

Find

- subsets of $\mathcal{O}$ that can be defined in at least two ways

A Redescription



‘Countries with land area $>$ 3,000,000 sq. miles’ $-$

‘Tourist Destinations in the Americas’

$\Leftrightarrow$

‘Permanent members of U.N. Security Council’ $\cap$

‘Countries with history of communism’

Redescription is sort of like ...

association rule mining

- generalize from implications to equivalences

conceptual clustering

- find clusters with dual characterizations

constructive induction

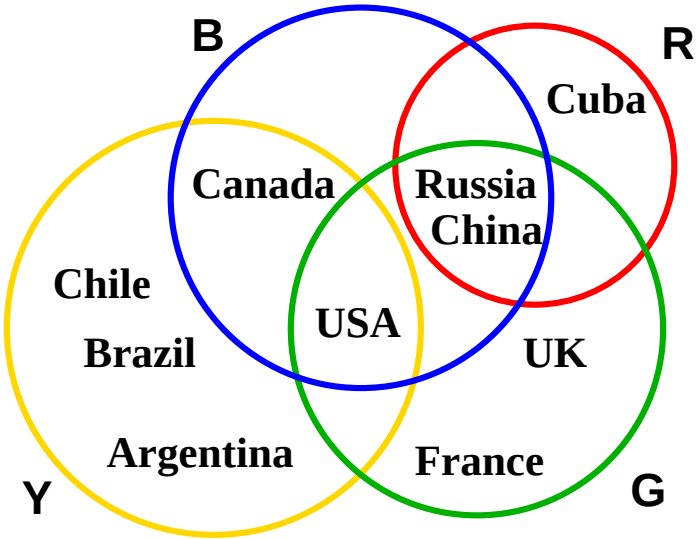
- build features that mutually reinforce each other

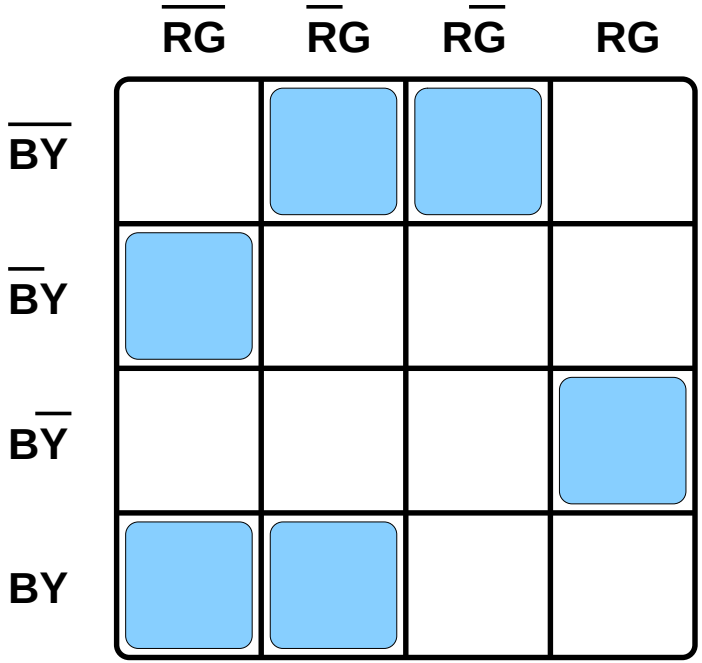
Applications in Bioinformatics

(Gene) subsets galore!

- Genes localized in the mitochondrion
- Genes up-expressed two-fold or more in heat stress
- Genes encoding for proteins forming the immunoglobin complex
- Genes involved in glucose biosynthesis
- Genes handpicked by Prof. Genie for further study
- Genes clustered together by your favorite algorithm
- . . .

How do redescriptions happen?





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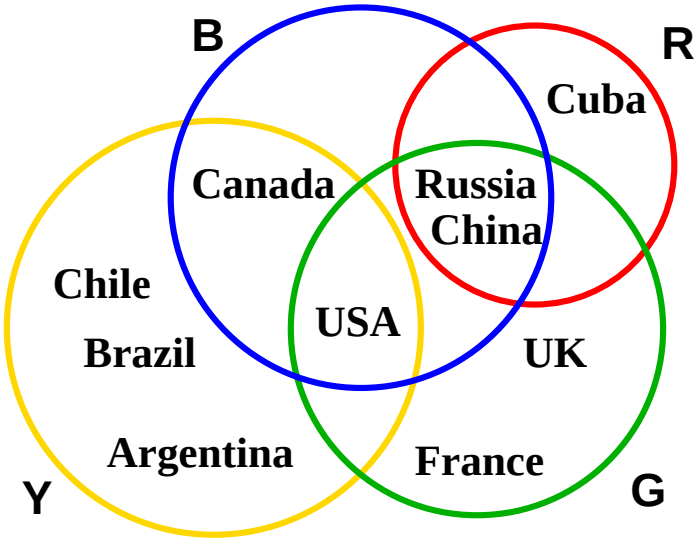
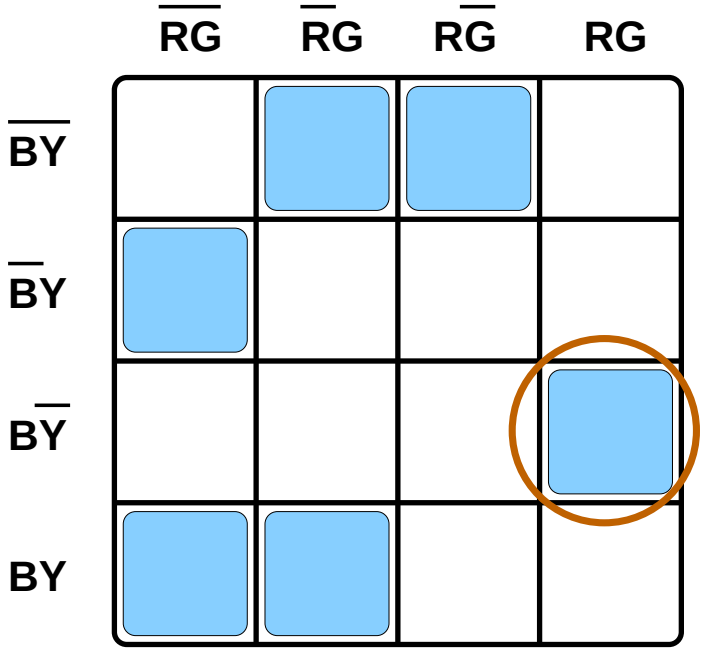
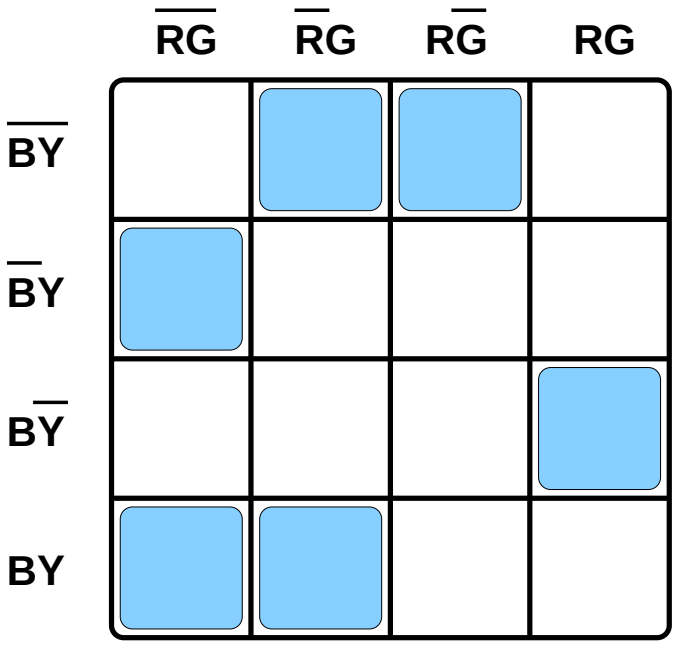
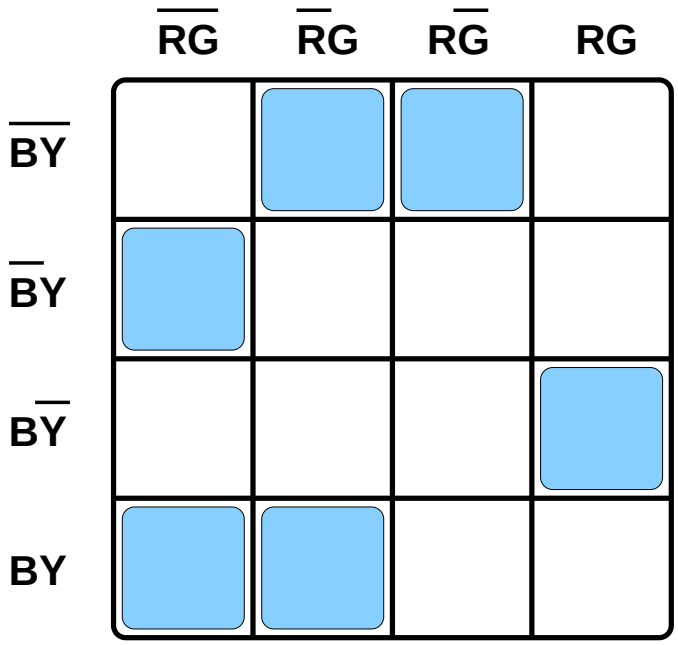




A game on Karnaugh maps





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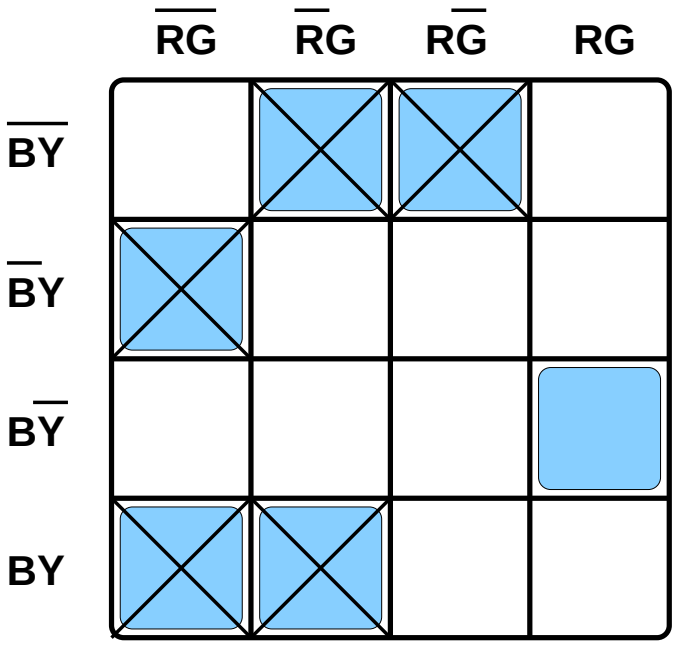

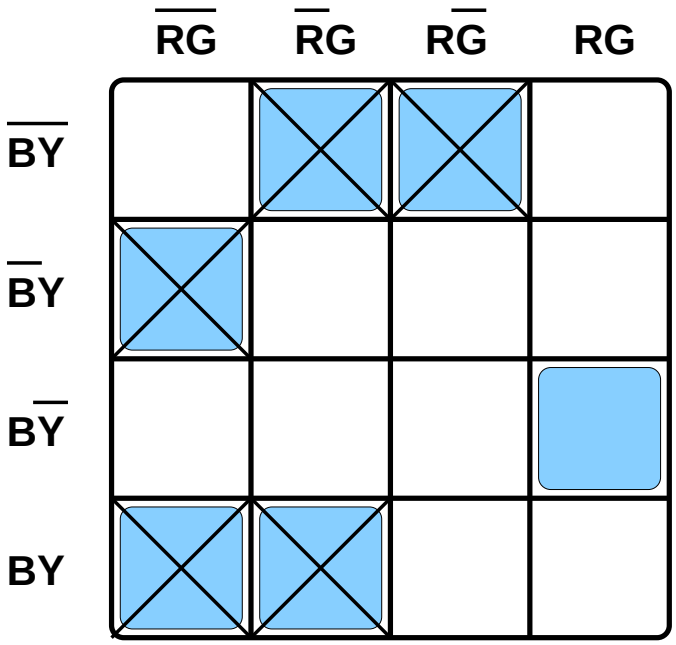

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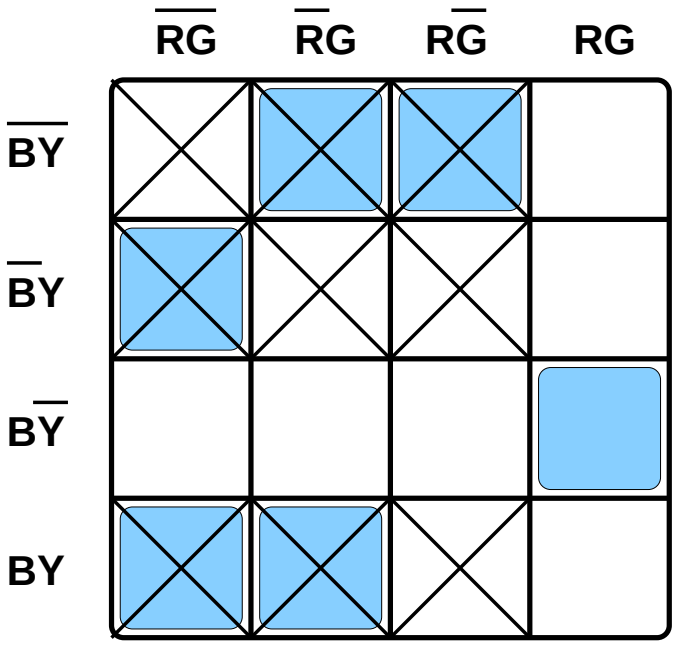

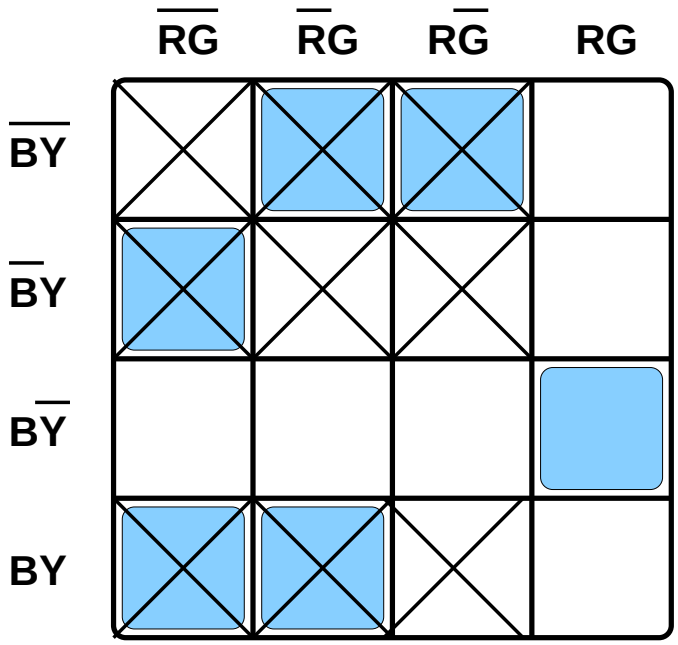

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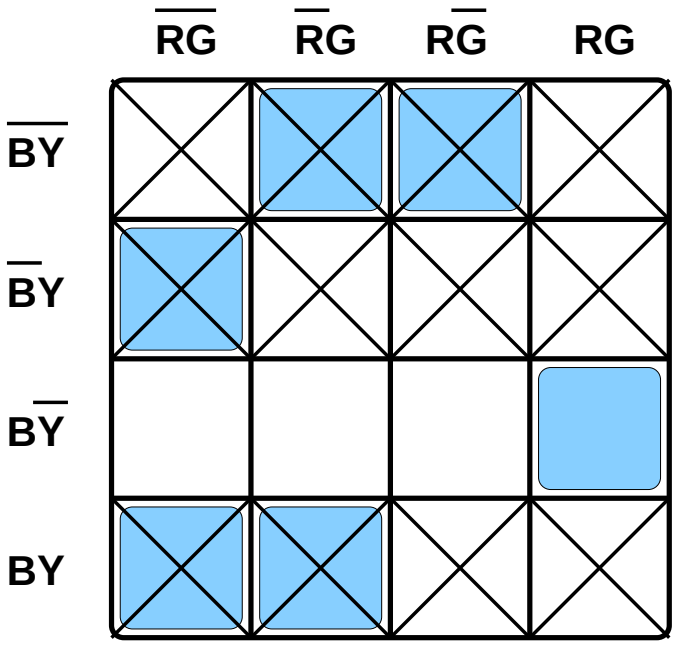

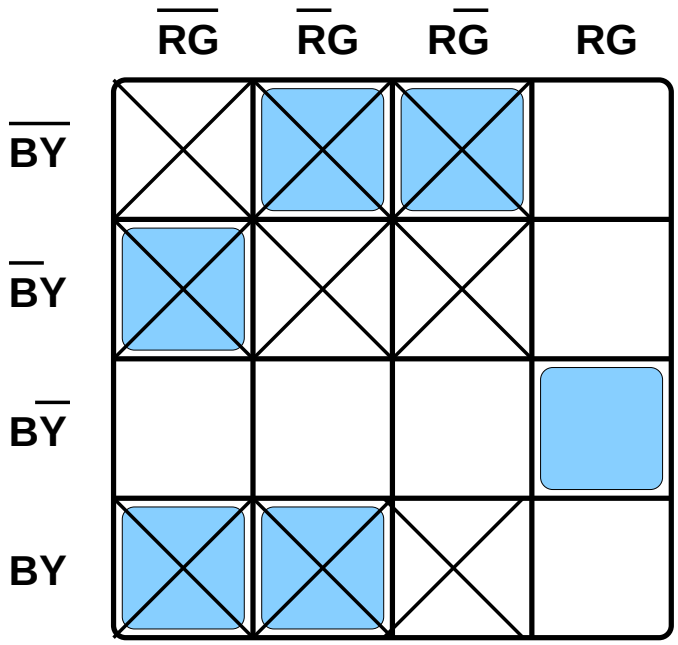

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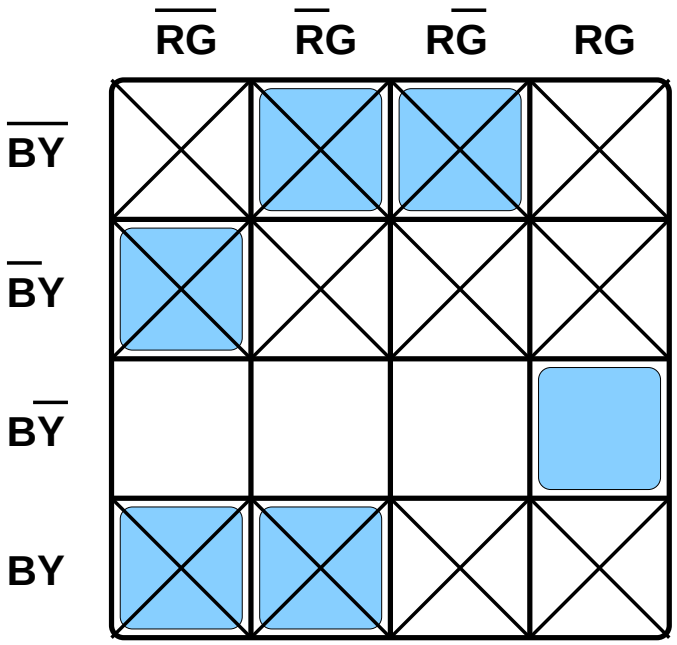
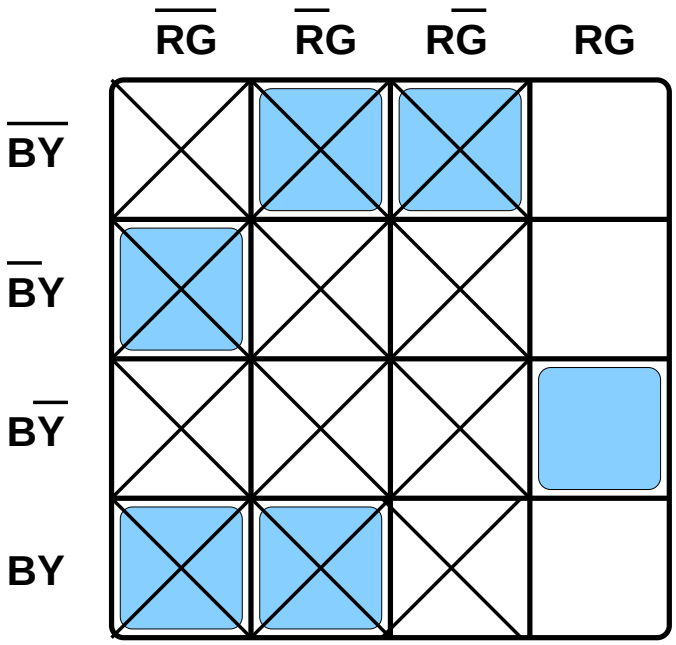
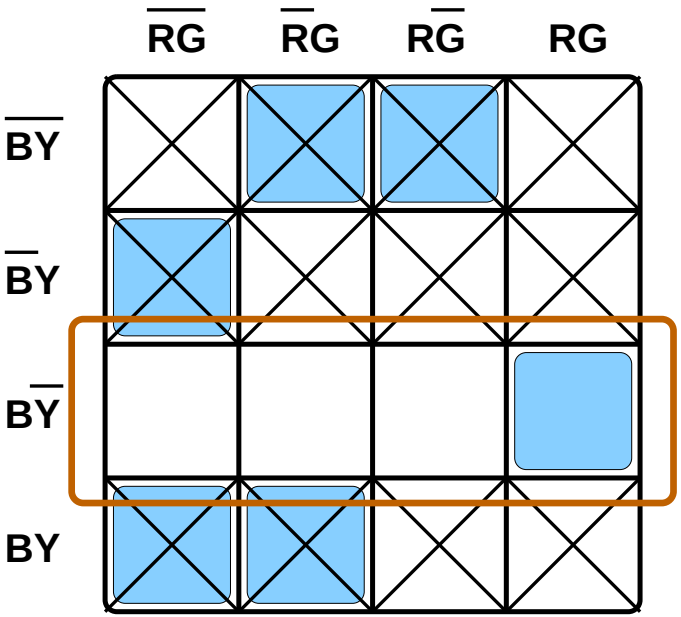
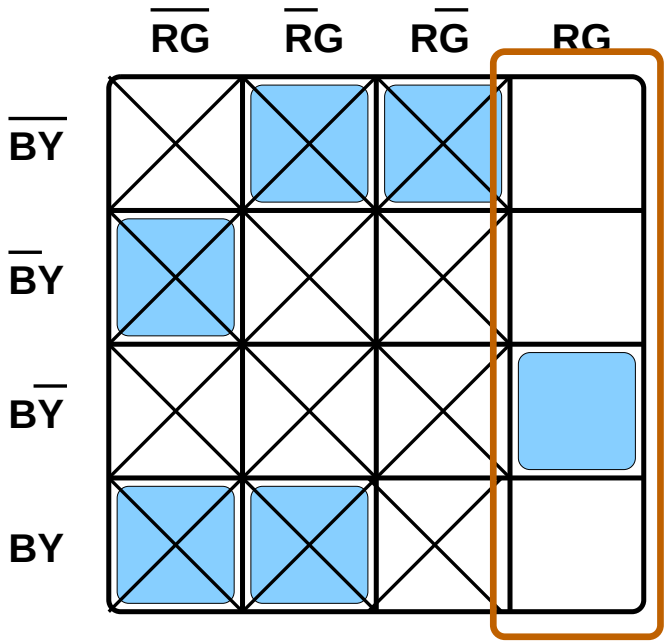




Reading off a redescription





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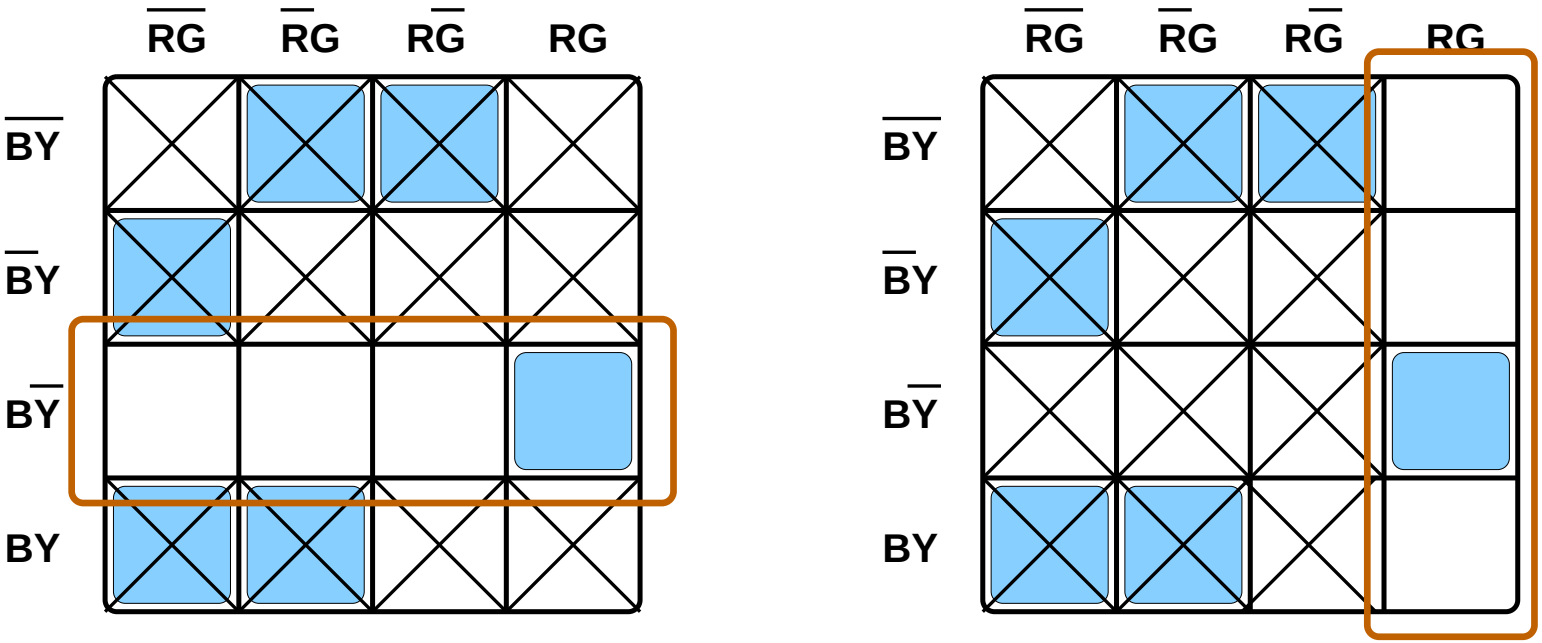
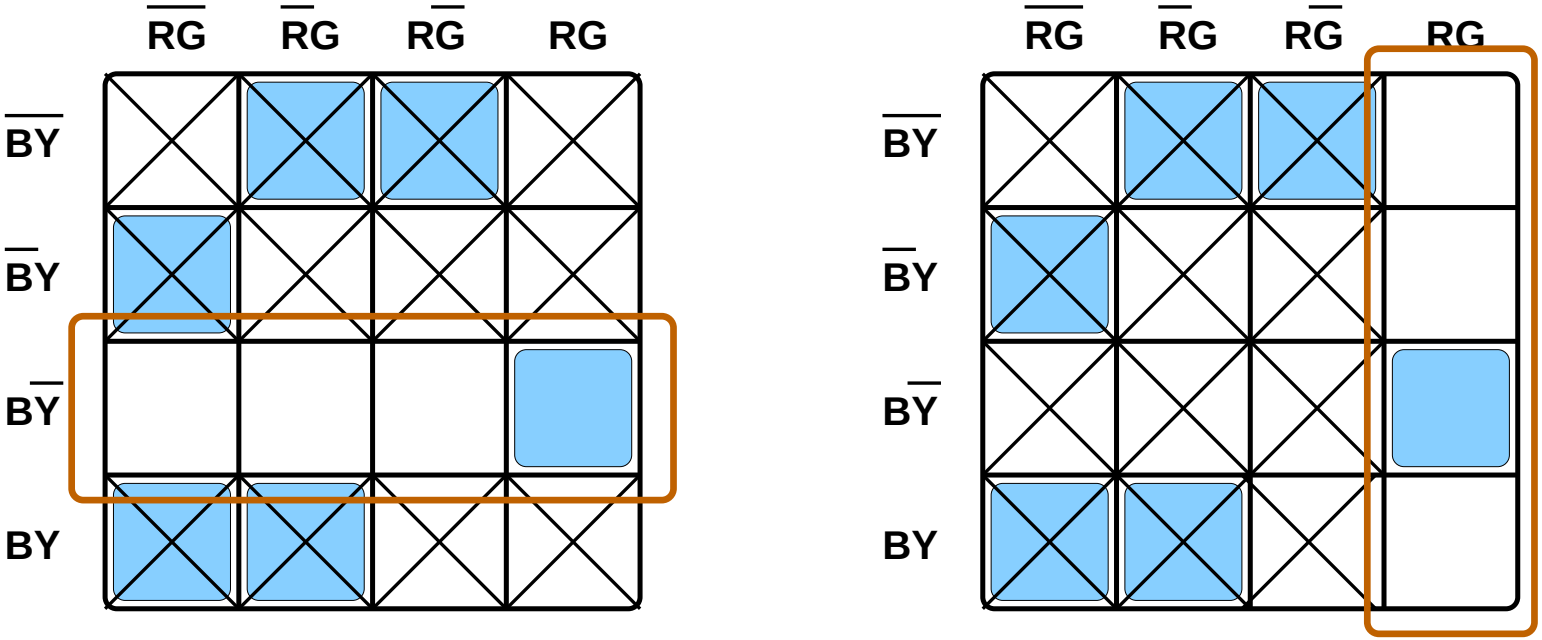


$$(B\overline{Y}\,\overline{RG} \;\vee\; B\overline{Y}\,\overline{R}G \;\vee\; B\overline{Y}R\overline{G} \;\vee\; B\overline{Y}RG)$$

Reading off a redescription

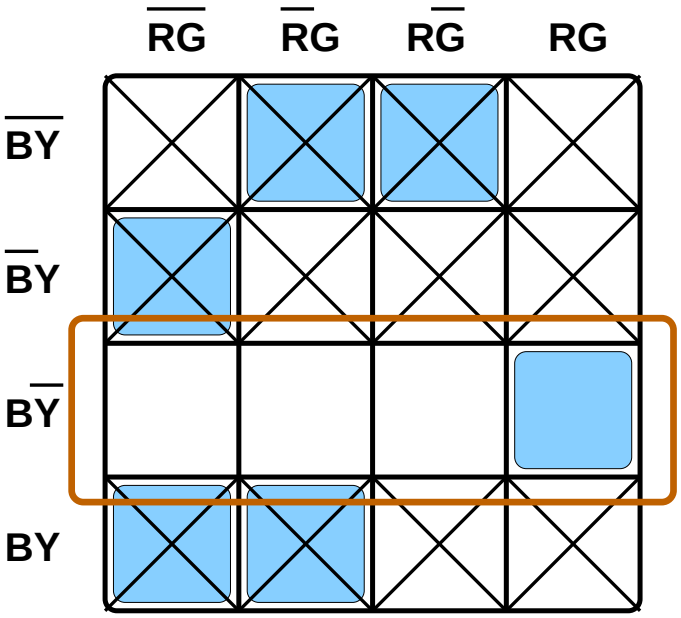


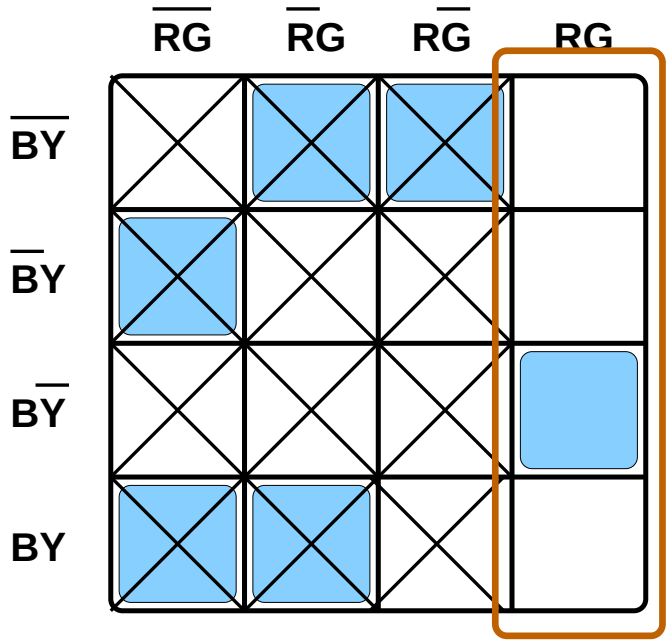
$$(B\overline{Y}\,\overline{RG} \;\vee\; B\overline{Y}\,\overline{R}G \;\vee\; B\overline{Y}R\overline{G} \;\vee\; B\overline{Y}RG)$$

$$\Leftrightarrow$$

$$(\overline{B}\overline{Y}\,\overline{RG} \;\vee\; \overline{B}\overline{Y}RG \;\vee\; B\overline{Y}RG \;\vee\; BYRG)$$

Reading off a redescription



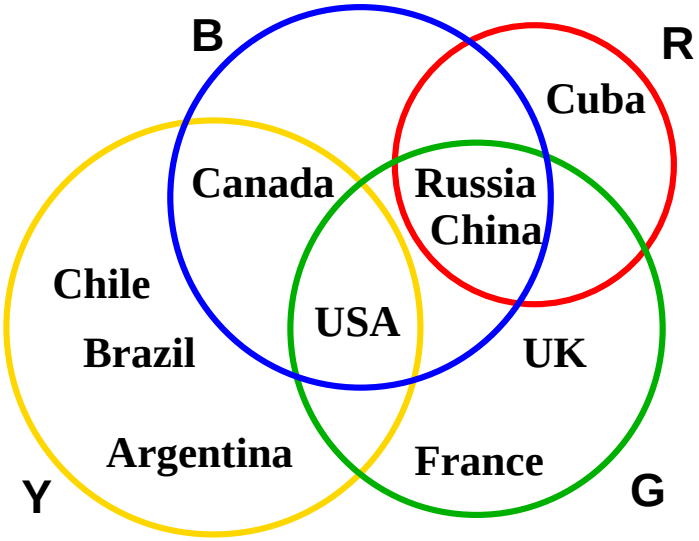


$$(B\overline{Y})$$

$$\Leftrightarrow$$

$$(RG)$$

Redescriptions help reason about sets



Q: How can B be made equal to R?

Ans: Subtract Y from B; intersect G with R, yielding $B\overline{Y} \Leftrightarrow RG$.

Some Definitions

Given a collection of objects $\mathcal{O}$ and descriptors $\mathcal{D}$:

- A redescription $X \iff Y$ ($X, Y \subseteq \mathcal{D}$) holds when
  - $X \cap Y = \emptyset$ and
  - X and Y induce the same set of objects.

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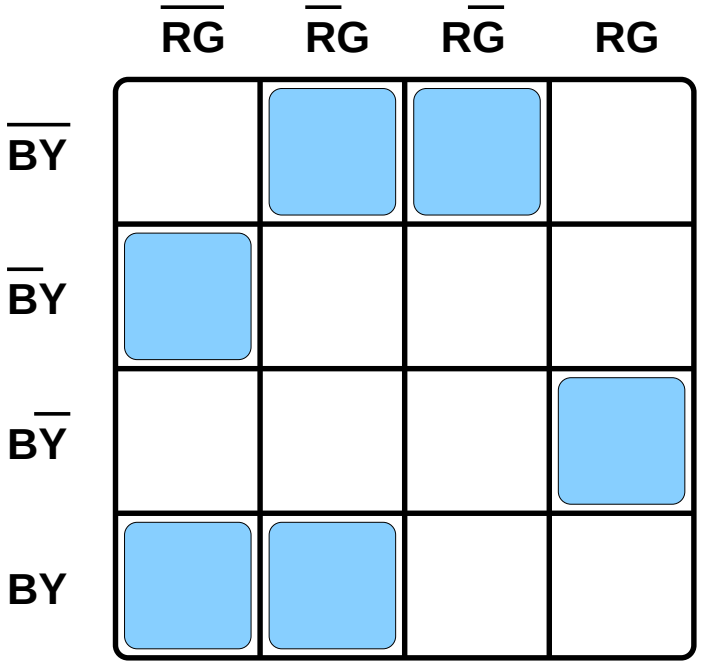
  - $X \cap Z$ and $Y \cap Z$ induce the same set of objects.
- A redescription $X \iff Y$ is a non-redundant redescription iff there does not exist another redescription $X' \iff Y'$ for the same set of objects, such that $X' \subseteq X$ and $Y' \subseteq Y$

Connections to Association Rule Mining



Objects	=	Transactions
Descriptors	=	Items

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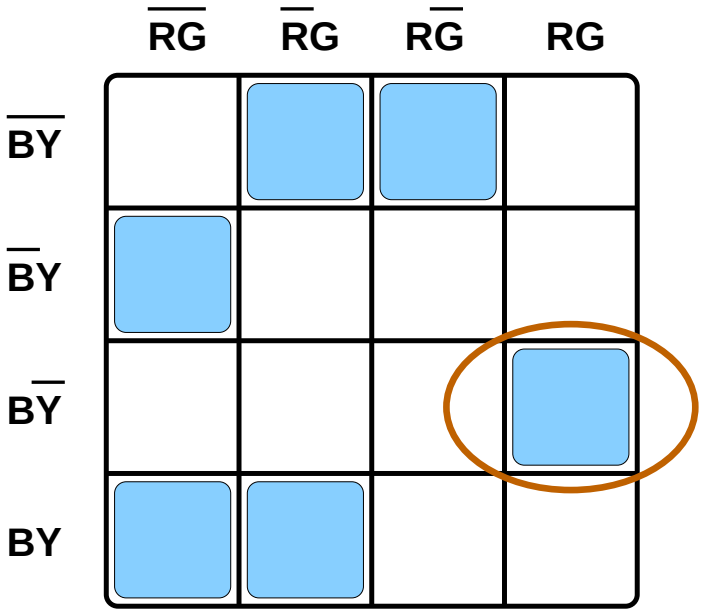

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Colored cell = closed itemset (e.g., $B\overline{Y}RG$)

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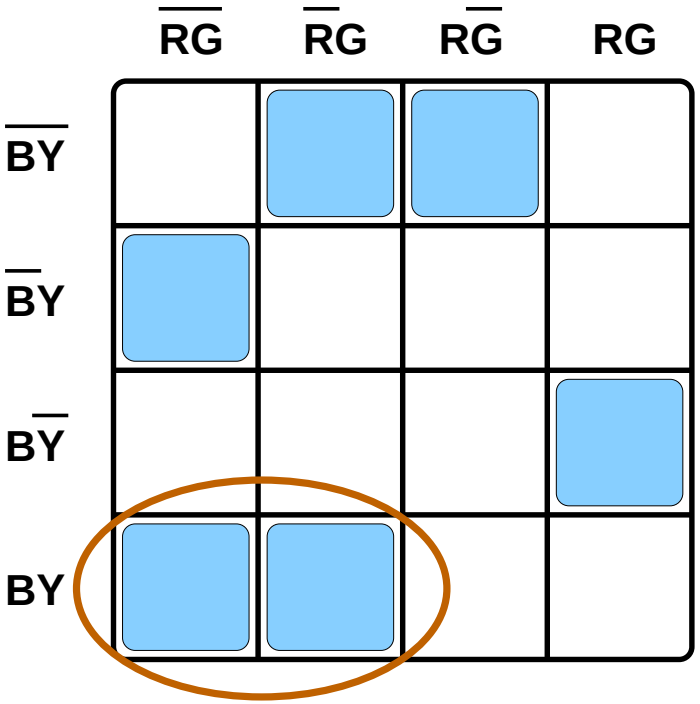
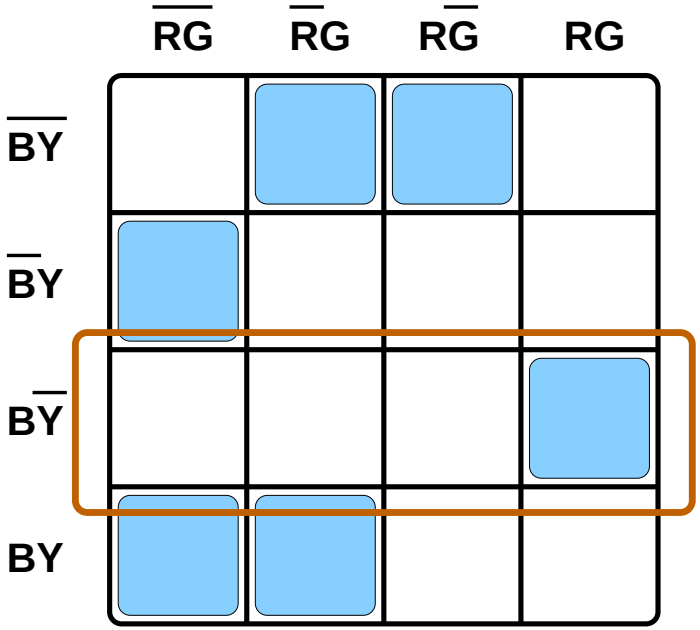


Objects	=	Transactions
Descriptors	=	Items

Reducible cluster of colored cells = closed itemset (e.g., $BY\overline{R}$)

Connections to Association Rule Mining



Objects	=	Transactions
Descriptors	=	Items

Reducible cluster of mixed cells = non-closed itemset (e.g., $B\overline{Y}$)

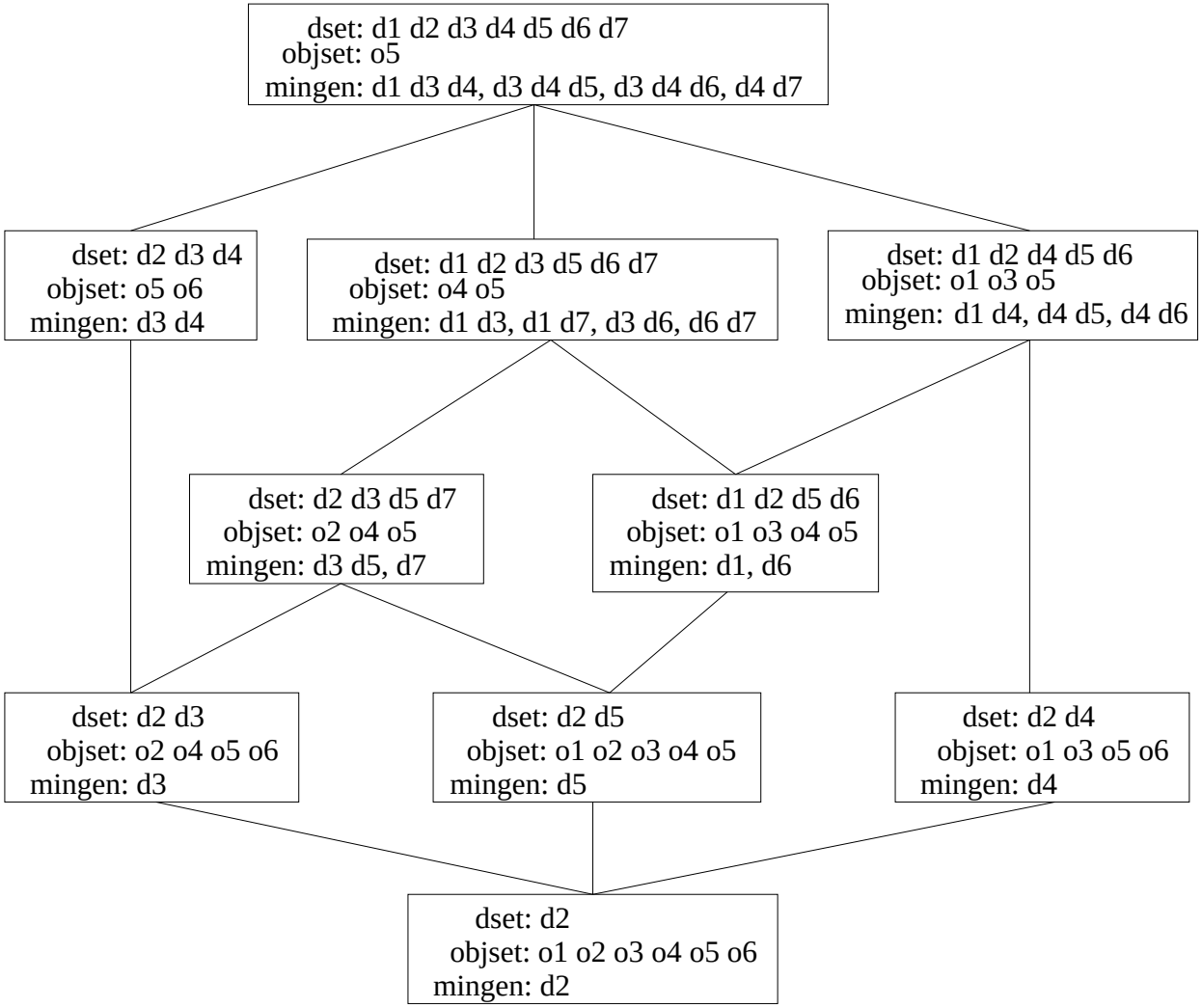
Adapting association mining algorithms

Mining redescriptions reduces to:

- mining closed itemsets (descriptor sets)
- obtain submatrices reducible to these closed sets (generators)

Object	Descriptors
o_1	$d_1 d_2 d_4 d_5 d_6$
o_2	$d_2 d_3 d_5 d_7$
o_3	$d_1 d_2 d_4 d_5 d_6$
o_4	$d_1 d_2 d_3 d_5 d_6 d_7$
o_5	$d_1 d_2 d_3 d_4 d_5 d_6 d_7$
o_6	$d_2 d_3 d_4$

Lattice of Closed Sets



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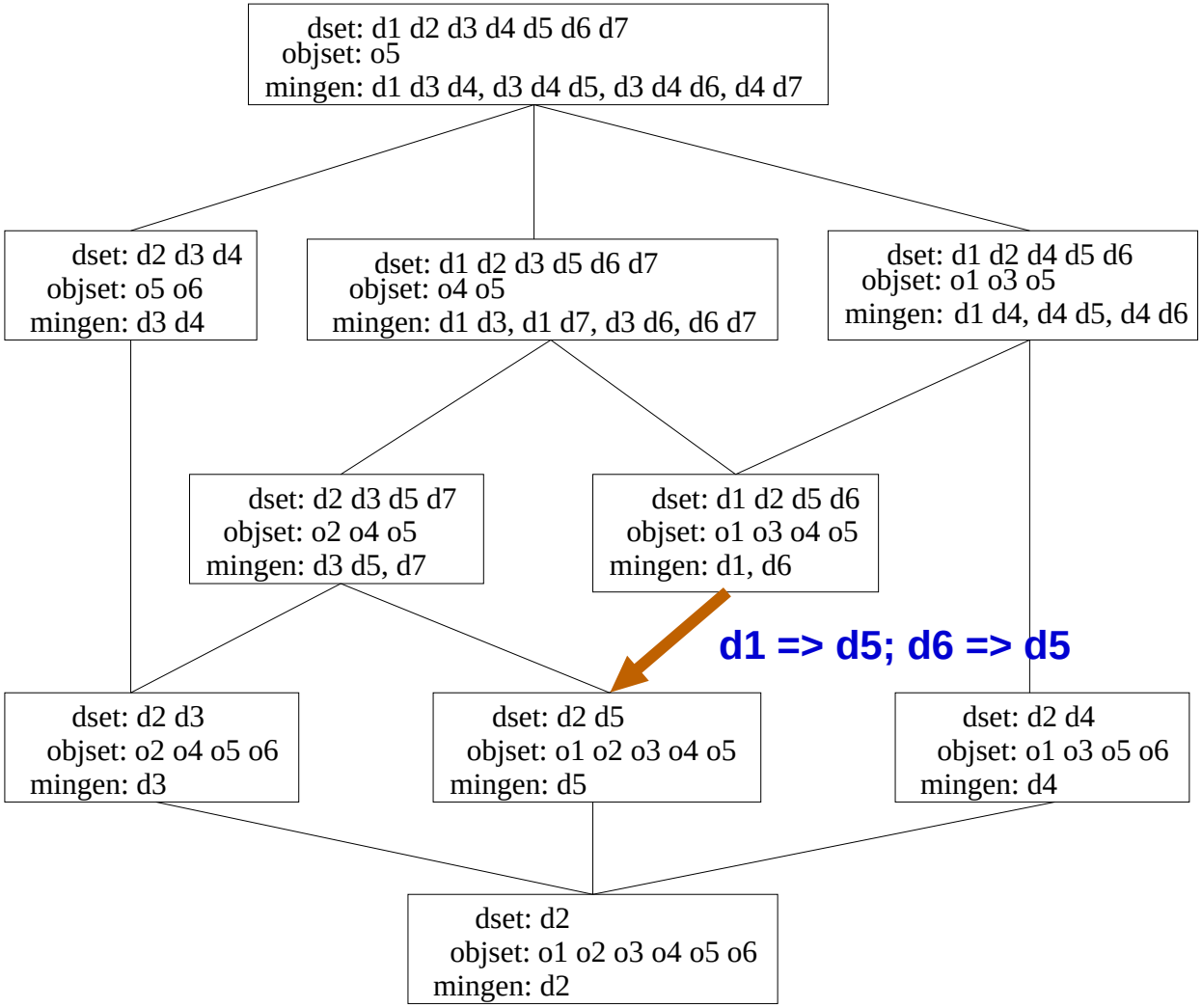

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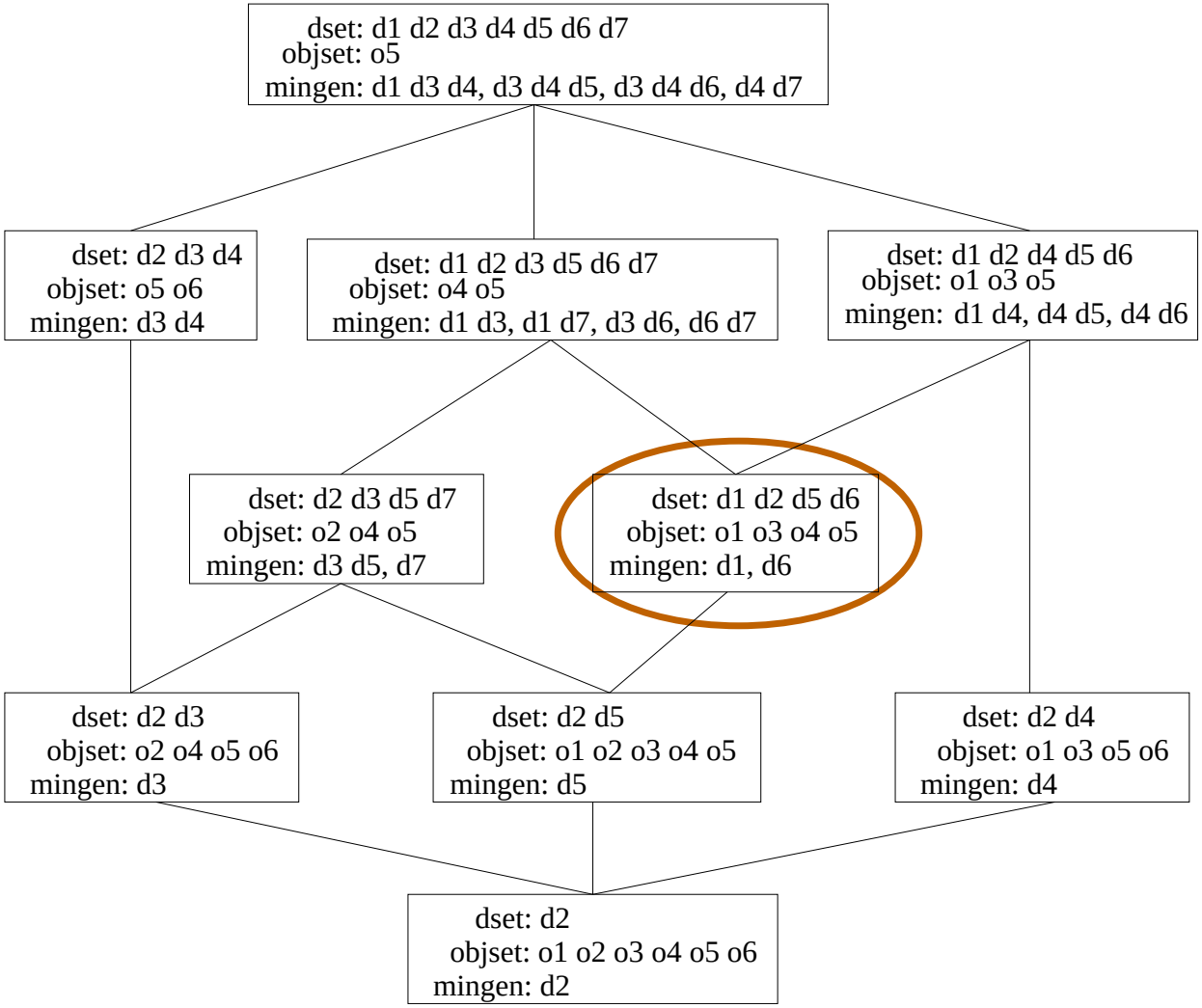
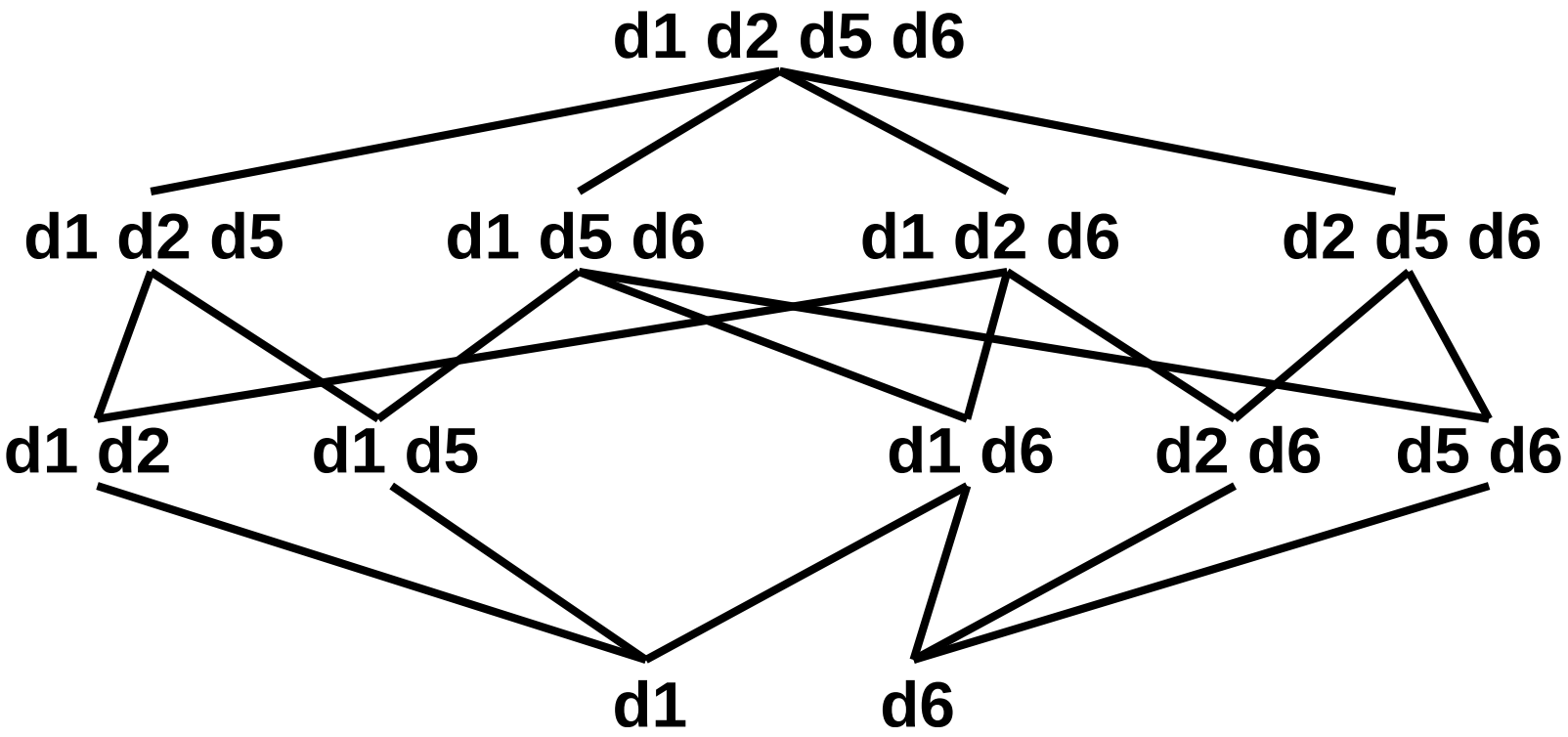


Up Closed and Personal



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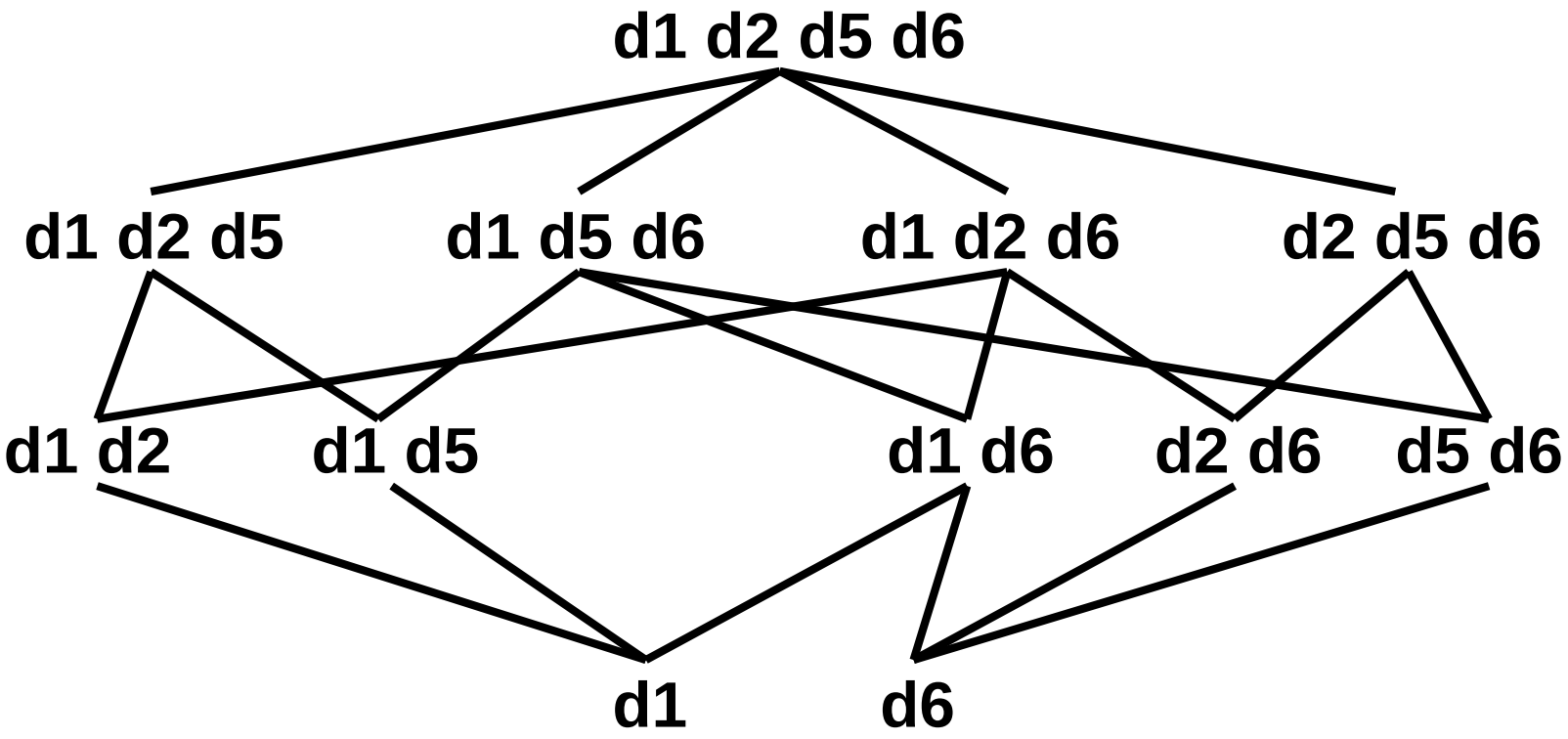
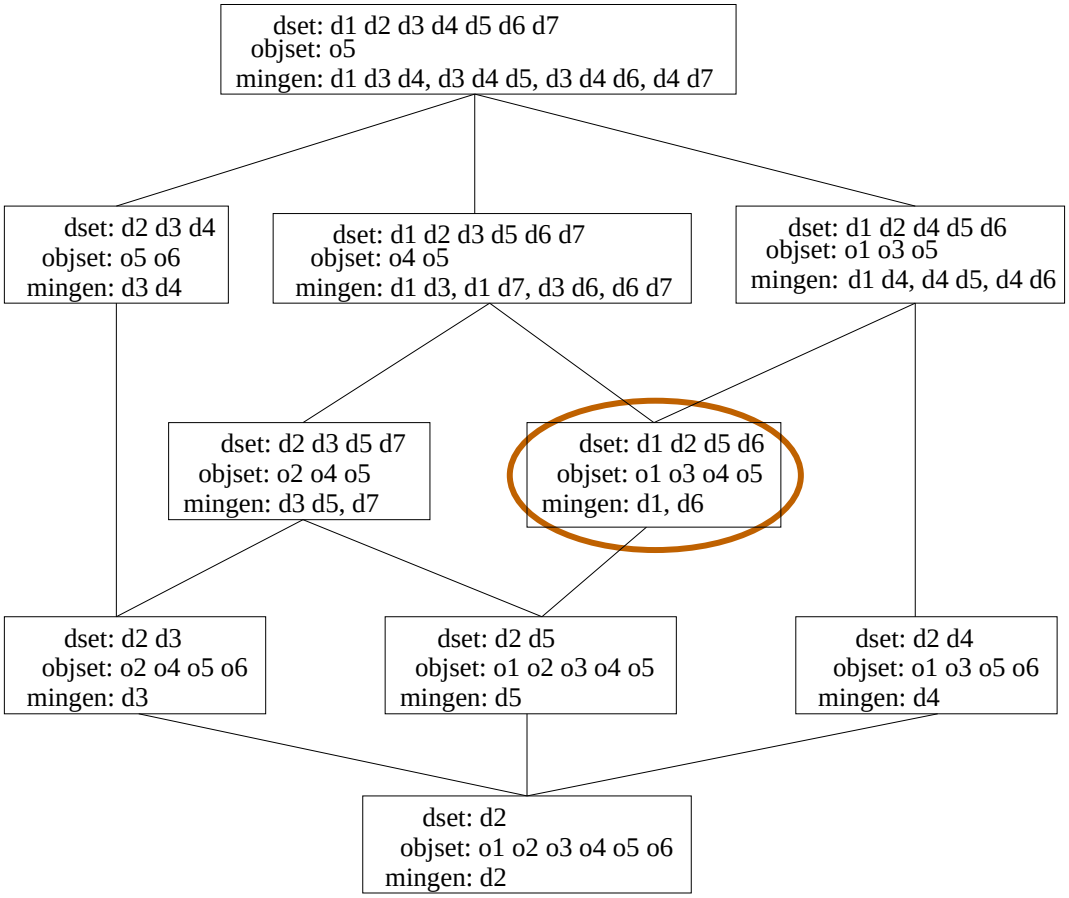


d1 <=> d6

Finding Minimal Generators



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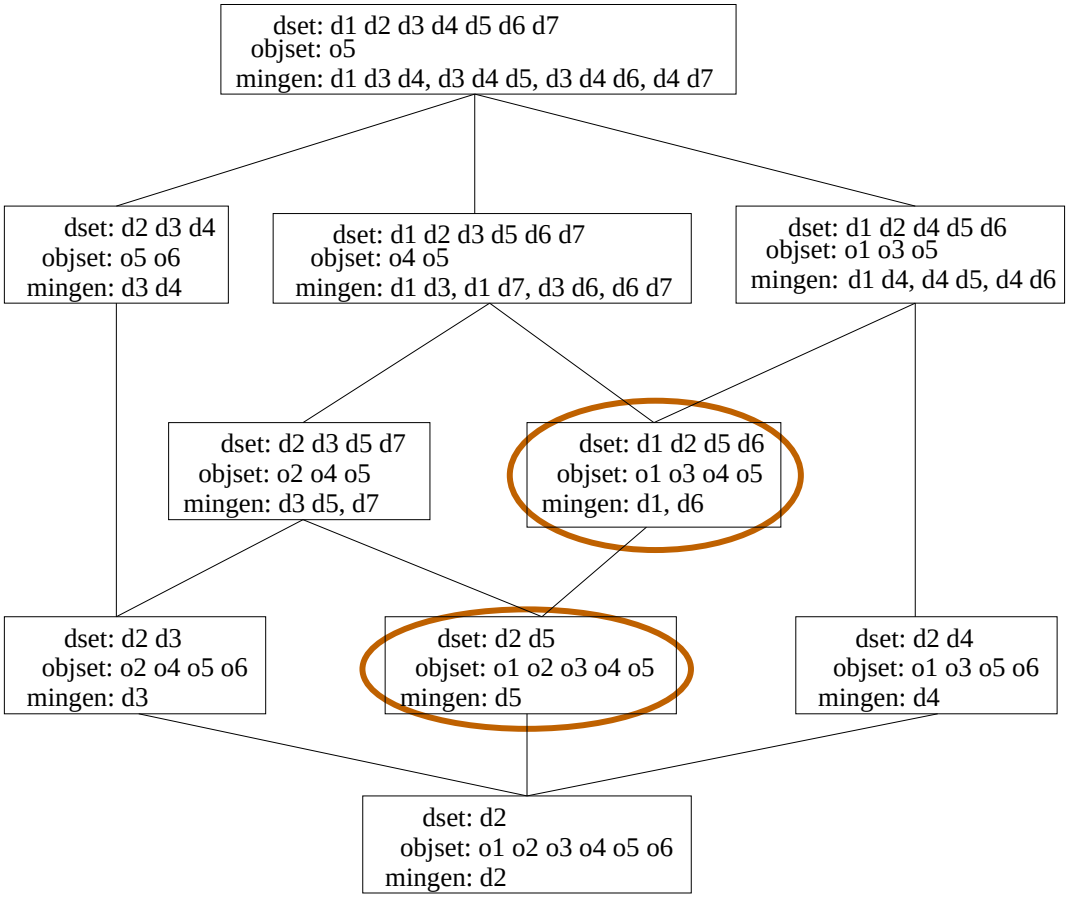
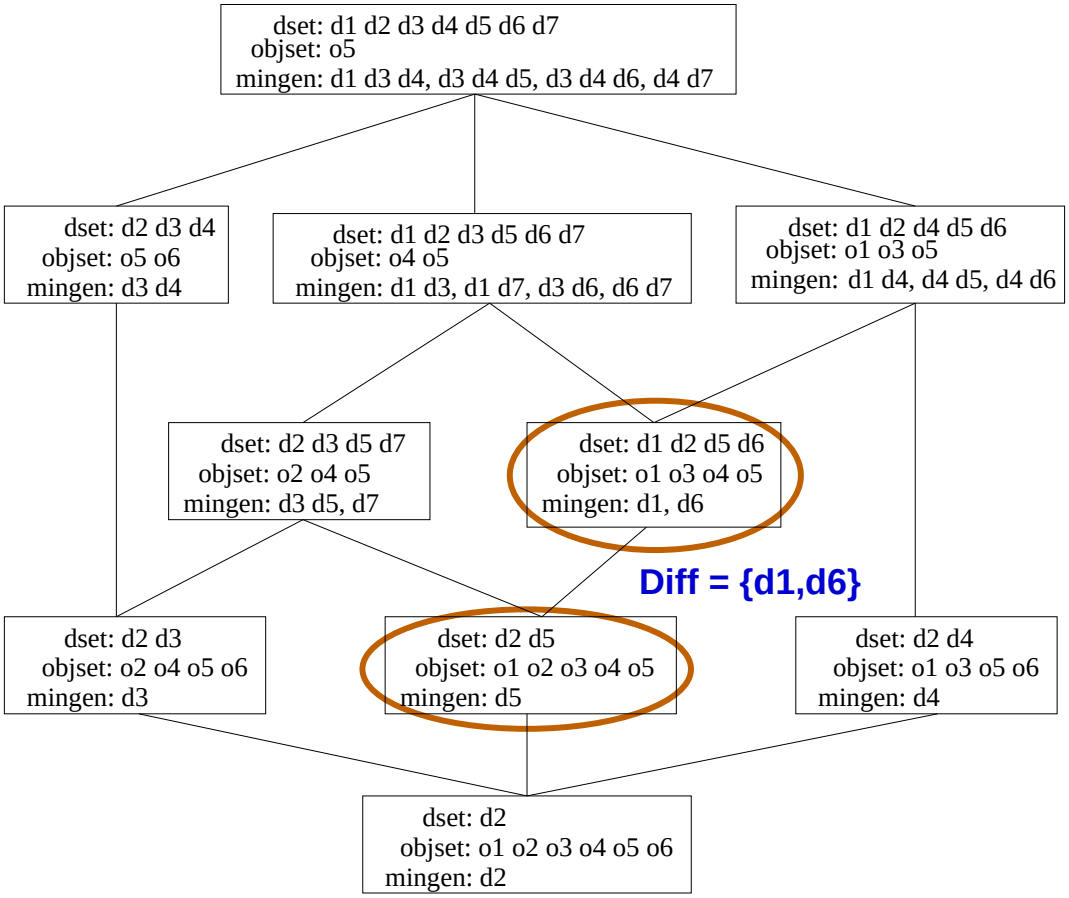


Finding Minimal Generators



Not so fast...

Datasets are 50% dense!

- cannot rely on pruning to help handle large datasets

Solution approach

- CHARM-L: Mining with constraints
  - Only expand lattice around objects/descriptors of interest

Exploring Gene Sets in Bioinformatics

Vocabularies

- GO functional categories (BIO, CEL, and MOL)
- Expression range buckets in specific microarray experiments
- Gene clusters

Interactive Exploration w/ CHARM-L

What is the relationship between ...

- $d183$ (ORFs ≥ 5 expressed in 15 minutes of heat shock)
- $d184$ (ORFs ≥ 5 expressed in 20 minutes of heat shock)

Answer:

- $d183 - d388 - d460 - d515 \Leftrightarrow d184 - d309$
  - $d388$: (GO MOL mannose transporter)
  - $d460$: (GO CEL external protective structure)
  - $d515$: (GO BIO fructose metabolism)
  - $d309$: (GO MOL molecular function unknown)

Another example

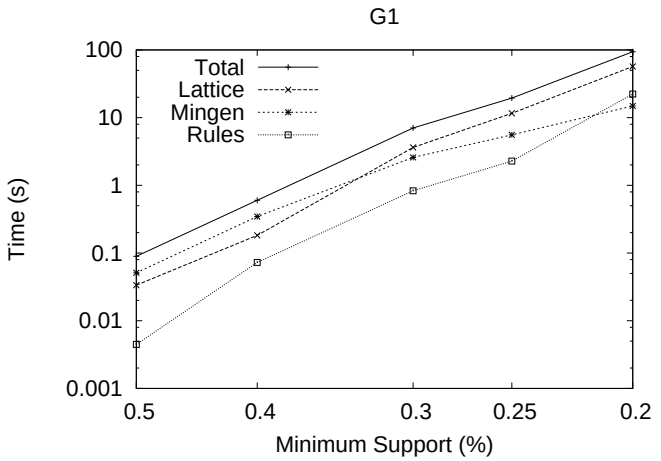
What is the relationship between ...

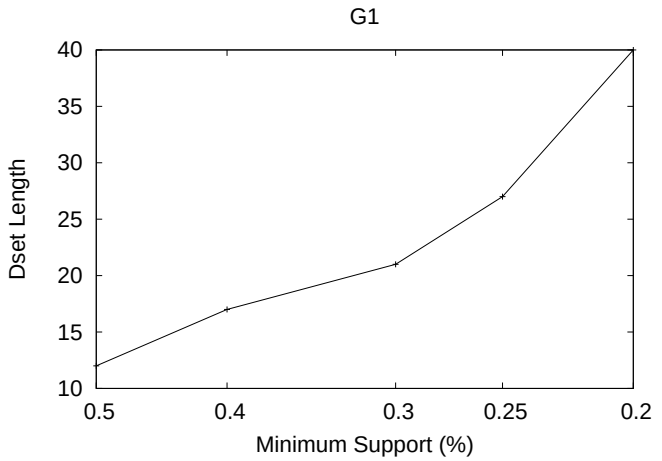
- $d141$ (ORFs ≥ 2 expressed in 10 minutes of heat shock)
- $d184$ (ORFs ≥ 5 expressed in 20 minutes of heat shock)

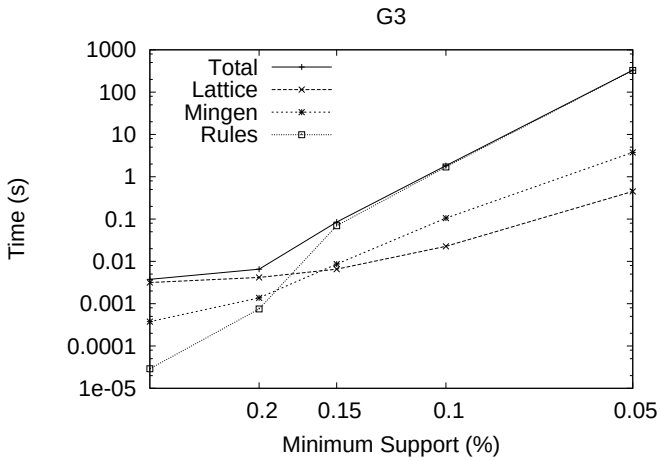
Answer:

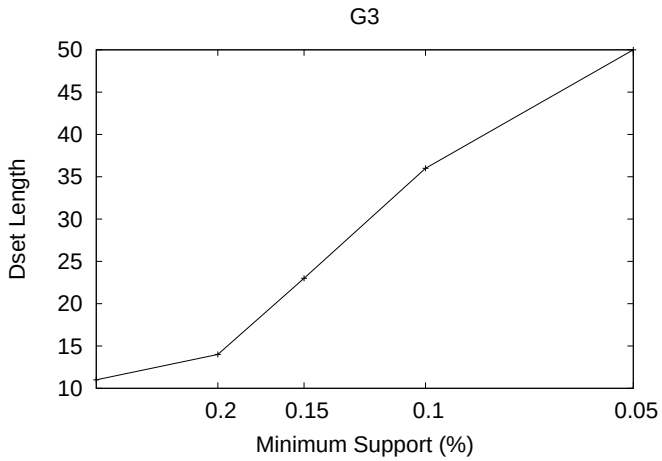
- $d141 - d515 - d608 \Leftrightarrow d184|d183$
  - $d515$: (GO BIO fructose metabolism)
  - $d608$: (ORFS ≥ 4 expressed in histone depletion)
  - $d183$: (ORFs ≥ 5 expressed in 15 minutes of heat shock)

Performance Results









Recap

Redescriptions help reason about set collections

- Conjunctive forms handle set intersections and negations
- Empowers biologist to create and work with vocabularies

Algorithmic innovations

- Lattice mining, finding minimal generators, constraint propagation
- Established connections to boolean formula manipulation

Future Work

Story telling

- Find a sequence of redescriptions connecting disjoint sets X and Y

Schema matching

- $X \subseteq O_1$, $Y \subseteq O_2$, O_1 and O_2 are related by relation R

Generalized boolean expressions

- Mine redescriptions in more expressive forms

Acknowledgements

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- NSF grants IBN-0219332 and EIA-0103660 (Ramakrishnan)

Questions?

For related work, see:

N. Ramakrishnan *et al.*, Turning CARTwheels: An Alternating Algorithm for Mining Redescriptions, in *Proceedings of KDD'04*, pages 266–275, 2004.

- L. Parida and N. Ramakrishnan, Redescription Mining: Structure Theory and Algorithms, in *Proceedings of AAAI'05*, pages 837-844, July 2005.

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