# illiad: InteLLigent Invariant and Anomaly Detection in Cyber Physical Systems

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Cyber physical systems (CPSs) are today ubiquitous in urban environments. Such systems now serve as the backbone to numerous critical infrastructure applications, from smart grids to IoT installations. Scalable and seamless operation of such CPSs requires sophisticated tools for monitoring the time series progression of the system, dynamically tracking relationships, and issuing alerts about anomalies to operators. We present an online monitoring system (*illiad*) that models the state of the CPS as a function of its relationships between constituent components, using a combination of model-based and data-driven strategies. In addition to accurate inference for state estimation and anomaly tracking, *illiad* also exploits the underlying network structure of the CPS (wired or wireless) for state estimation purposes. We demonstrate the application of *illiad* to two diverse settings: a wireless sensor motes application and an IEEE 33-bus microgrid.

#### 1. INTRODUCTION

It has been projected that by the year 2030, cities will grow by 590,000 square miles and add an additional 1.47 billion people, so that 6 out of every 10 people will live in a city. One of the most consequential changes with this global influx of citizens will be the stress placed on cyber-physical systems across the urban landscape, from smart grids to massively distributed IoT installations to support net zero energy objectives. At the same time, as cities become seen as urban-scale cyber physical systems, a vast amount of data about system management and operation is continuously being harvested and analyzed through sensor networks. Scalable and seamless operation of such cyber-infrastructure requires sophisticated tools for monitoring the progression of the system, dynamically tracking relationships, and issuing anomaly alerts to operators.

Leveraging our prior work [Momtazpour et al. 2015], we develop a system dynamics approach (*illiad*) to invariant and anomaly detection in cyber-physical systems. Our key contributions are:

- A state estimation and anomaly detection algorithm (KASE) that combines modelbased (Recursive Bayesian Filtering) and data-driven (Autoregression with Exogenous Inputs and Exploratory Factor Analysis) approaches. The integration of modelbased and data-driven strategies leverages the selective superiorities of both into a comprehensive system.
- An approach to incorporate the underlying network structure of the cyber-physical system (wired or wireless) into the state estimation process. We demonstrate how this idea significantly improves the computational complexity of inference and renders the approach tractable to large urban settings.
- A visual dashboard application for real-time anomaly detection and alerting in urban-scale cyber-physical systems. We demonstrate the application of *illiad* to two diverse settings: a wireless sensor motes application and an IEEE 33-bus microgrid.



**Fig. 1:** Urban microgrid examples and design and deployment scenarios. (a) Represents an AC grid connected to main grid through a point of common coupling (PCC). (b) Represents a Diesel generator along with other Distributed Generation sources without the main grid.

## 2. BACKGROUND AND RELATED WORK

Availing energy from renewable sources to meet increased energy demands due to increasing electrification in urban environments is a key challenge we are faced with today as alluded to by [Zheng et al. 2014]. An integration of microgrids into the existing power grid has been widely acknowledged as a potential solution. Resilient operation of power and microgrid systems requires constant monitoring of data to extract or predict anomalies, and to support rapid system recovery. Figure 1 represents typical urban microgrids and deployment scenarios. The microgrid consists of several distributed generation units (DGs) such as a photovoltaic (PV) array and diesel generator, as well as energy storage systems and loads. These components are connected together using the power lines, transformers, and feeders. Microgrids can operate in both islanded and grid-tied modes. In the islanded mode the microgrid is as shown in Figure 1 (b).

Energy systems (e.g. PV, battery, load, diesel generator) are typically installed and connected to the microgrid controller via TCP/IP with systems that feature Supervisory Control and Data Acquisition (SCADA), exploiting open communication standards such as OLE for Process Control (OPC) and Modbus RTU/TCP that is a master-slave protocol for use with its programmable logic controllers (PLCs). Measurement data and its collected features vary depending on the type of energy device and its communications protocol such as the format of XML. The measurement data contains active and reactive output power values of each device, voltage and frequency values as microgrid data, and state of charge (SOC) for batteries for every second.

Microgrids, including those studied here, are typically based on traditional power distribution system models, while also incorporating relatively modern urban components such as electric vehicles (EVs) and energy storage (ES) devices along with renewable power generation components such as wind turbines and PV cells [Chowdhury and Crossley 2009]. However, such new additions make the operation of the distribution system more complex [Tsikalakis and Hatziargyriou 2011]. One factor, for example, would be the uncertain and intermittent power output of the renewable distributed generation components, specifically PV panels and wind turbines. Such intermittent power output makes system state estimation a challenging task.

State estimation of the power system is typically accomplished by learning invariant relationships between system components, and such invariants are then used in an energy management system (EMS) to construct a real-time network model [Monticelli 2000]. In Cobelo et al.[2007] the authors proposed a method aimed at providing the

distribution management system controller with real-time information from the microgrid to increase the penetration of the renewable distributed generation. Rana and Li[2015] have proposed a Kalman filter based microgrid state estimation method using an IoT network to acquire information about the distributed generation grid. Wang et al.[2014] also proposed a linear filtering based state estimation method which mainly focused on small signal models. Hu et al. [2011] developed a probabilistic model to conduct real-time linear state estimation through belief propagation. Instead of focusing on a single microgrid, Korres et al.[2011] proposed a multi-microgrid state estimator with limited real-time measurement and investigated the impact of distributed generation in both grid-connected and islanded scenarios. However, most existing research has focused on accuracy of estimation rather than anomaly detection using invariant relationships between components. Even fewer approaches exist which successfully combine model-based and data-driven approaches with the goal of anomaly detection through state estimation. Anomaly detection in microgrids using sensor location and connection information can be considered a spatio-temporal anomaly detection problem and some relevant work has been carried out by [Chawla et al. 2012] in the transportation domain. In [Zheng et al. 2015] the authors propose a spatiotemporal anomaly detection system for detecting collective anomalies by leveraging multi-domain datastreams.

## 3. PROBLEM FORMULATION

As described earlier, our work builds upon the preliminary framework described in [Momtazpour et al. 2015], and we begin by introducing this approach in context.

We are given a set of n timeseries  $\mathcal{D} = x_1(t)..x_n(t)$  in a single or across multiple cyber-physical systems, each timeseries  $x_i(t)$  is modeled as a vector. The values of the vector for a time window  $t_k...t_{k+w}$  are represented as

$$\mathcal{X}_{i}^{k:k+w} = [x_{i}(t_{k}), x_{i}(t_{k+1}), \dots x_{i}(t_{k+w})]^{T}$$
(1)

Each timeseries  $x_i(t)$  is represented by a random variable  $X_i$  and is assumed to be drawn from a distribution represented by  $X_i$ . Every CPS has many direct and latent interactions amongst the components and having sufficient insight into these interactions and relationships is crucial to effectively manage the system.

Definition 3.1. (Approximate Dependency): At time step  $t_m$ , time series  $x_j(t) \in \mathcal{D}$  approximately depends on  $x_i(t) \in \mathcal{D}$  if and only if, there exists a function  $f \colon \mathbb{R} \to \mathbb{R}$  that for appropriately small  $\epsilon > 0$ :

$$\hat{x}_j(t_m) = f(\mathcal{X}_j^{1:m-1}, \mathcal{X}_i^{1:m})$$
 (2)

and

$$|x_j(t_m) - \hat{x}_j(t_m)| < \epsilon \tag{3}$$

This dependency is depicted by  $x_i(t) \xrightarrow{\epsilon} x_i(t)|_{t_m}$ 

**Definition** 3.2. (System Invariants): Two timeseries  $x_j(t) \in \mathcal{D}$  and  $x_i(t) \in \mathcal{D}$ , are system-invariant up to time T within range of  $\epsilon$  if and only if at least one of the following is satisfied:

$$\exists f \colon \mathbb{R} \to \mathbb{R} \quad and \quad \forall \quad t \mid 0 \le t \le T : x_j(t) \xrightarrow{\epsilon} x_i(t)|_{0 \le t \le T}$$
 or 
$$\exists f \colon \mathbb{R} \to \mathbb{R} \quad and \quad \forall \quad t \mid 0 \le t \le T : x_i(t) \xrightarrow{\epsilon} x_j(t)|_{0 < t < T}$$

We denote an invariant time series by  $x_i(t) \rightleftharpoons x_i(t)$ 

Based on the nature of the system, dependencies between time series can be linear or nonlinear and this is modeled by the function f. In complex cyber-physical systems, when we have a large number of time series, it is appropriate to represent the invariants in the form of a graph.

Definition 3.3. (Invariant Graph): Graph G = (V, E) with the set of vertices  $V = \{v_1, ..., v_n\}$  and the set of edges  $E = \{e_1, ..., e_n\}$  is called an invariant graph of a system with observed timeseries  $\mathcal{D} = \{x_1, ..., x_n\}$ , where  $e = (v_i, v_j) \in E$  if and only if  $x_i(t) \rightleftharpoons x_j(t)$ 

From Definition 3.3 we can surmise that vertex  $v_i$  is equivalent to a timeseries  $x_i(t)$ . System invariants and invariant graphs represent features and system-function under normal conditions. In the presence of anomalies, when the function of the system deviates from the norm, these dependencies might disappear. While two timeseries  $x_i(t)$  and  $x_j(t)$  might be invariant under normal conditions, the invariant relationship might disappear in the case of an anomaly.

Definition 3.4. Broken Invariants: We say that system invariant  $x_i(t) \stackrel{\epsilon}{\rightleftharpoons} x_j(t)$  is broken at time  $T = t_m$  if and only if timeseries  $x_i(t)$  and  $x_j(t)$  satisfy the following conditions:

$$\begin{aligned} \exists f : \mathbb{R} \to \mathbb{R} \\ & \text{and} \\ \forall \quad t \mid 0 \leq t < T = t_m : \\ \left( x_j(t) \xrightarrow{\epsilon} x_i(t) |_{t < T} \quad \land \quad |x_j(t_m) - f(\mathcal{X}_j^{1:m-1}, \mathcal{X}_i^{1:m})| \geq \epsilon \right) \\ & \text{or} \\ \left( x_i(t) \xrightarrow{\epsilon} x_j(t) |_{t < T} \quad \land \quad |x_i(t_m) - f(\mathcal{X}_i^{1:m-1}, \mathcal{X}_j^{1:m})| \geq \epsilon \right) \end{aligned}$$

In a CPS the existence of certain unobserved factors has an effect on overall system behavior. But modeling these factors and their effects on all the electro-mechanical devices in a CPS is a non-trivial task. Characterizing these latent effects can aid in state estimation.

Figure 2 shows the network topology of an example CPS with N components  $X_1, ..., X_N$ . Here, the solid lines indicate direct relationships between sensors through physical connections while the dashed lines indicate indirect relationships between sensors. We see five direct and two indirect relationships depicted here. Figures 3 and 4 represent invariant graphs that can be inferred using current state-of-the-art methods (ARX and ARX+ARXL; introduced in [Momtazpour et al. 2015]). ARX is a classical autoregression method (with exogenous inputs) and thus captures only direct relationships, as shown in figure 3. ARX+ARXL, as shown in figure 4 does learn indirect relationships but these relationships are linear and rather simplistic in nature. Our proposed approach (KASE) aims to identify more complex hidden relationships



through hidden factors labeled  $h_1, ..., h_k$  and Kalman state estimates in figure 5. These support the modeling of the system at higher orders.

#### 3.1. State Estimation with Kalman Filters

Anomaly detection in sensor networks can be addressed using a dynamic state estimation technique [Nishiya et al. 1982] wherein at any time point t we wish to estimate the state of each component in the system. As a function of these state estimates and corresponding actual state measurements, we can then infer whether the system is stable or if there is an anomaly in one or more components in the system. One of the most robust and widely used methods for state estimation is the Kalman filter [Kalman 1960] which is a special case of recursive Bayesian estimation wherein the data is considered to obey the Gaussian distribution [Chen 2003]. In this paper, we consider the effects of state estimation in CPS using the linear Kalman filter. We model the Kalman filter as described by Welch and Bishop[2001]. It attempts to estimate the state  $x \in \mathbb{R}^n$  at time step k of a discrete-time controlled process with a measurement  $z \in \mathbb{R}^n$ , The Kalman filtering process assumes that the matrices A, H, Q, R which represent the process transition matrix, the measurement transition matrix, the process covariance matrix and measurement covariance matrix respectively and the initial mean and covariance of the data are known at the outset. In our work, we estimate A, Q and R using expectation maximization [Borman 2004]. The matrix H is the identity matrix. The initial state mean and covariance ( $\mu_0$  and  $\Sigma_0$ ) are estimated from historical data.

#### 3.2. Leveraging the Neighborhood Assumption

Section 3 outlines the basic problem framework and provides definitions about invariants, and anomalies. Although the original (ARX + ARXL) method attempts to find all direct and indirect dependencies in addition to learning complex latent variables, we hypothesize that considering the network topology of the CPS to learn only a set of direct relationships for each component yields equal or stronger state estimation and anomaly detection capabilities. We term this approach SAIL for *Structure Aware Invariant Learning*.

Let us first consider the case when the topology of the CPS is known, e.g., consider the sample CPS depicted in figure 2. Here, the system consists of N components  $X_1, ..., X_N$ . If we consider  $X_1$ , it has two direct connections, one to  $X_3$  and the other to  $X_2$  along with an indirect relationship with  $X_N$ . Our hypothesis in the context of  $X_1$  claims that the state of  $X_1(t)$  can be estimated simply using only the values  $X_1^{1:t-1}$ ,  $X_3^{1:t}$ ,  $X_2^{1:t}$  and the Kalman filter *a posteriori* state estimate  $k_1(t)$ . Hence set  $\{X_2, X_3\}$ 

forms the SAIL neighborhood of  $X_1$  denoted  $S_n^1$ . In the context of wireless sensor networks that lack a connected network topological structure, we exploit the physical locations of sensors to obtain a proximal set of nearest neighbors for each component in the wireless network of CPS components.

The SAIL approach requires us to augment our previously stated definition 3.1 as follows:

Definition 3.5. (Structure Aware Approximate Dependency): At time step  $t_m$ , time series  $x_j(t) \in \mathcal{D}$  with SAIL neighborhood  $S_n^j$  approximately depends on  $x_i(t) \in \mathcal{D}$  if and only if,  $x_i(t) \in S_n^j$  and there exists a function  $f \colon \mathbb{R} \to \mathbb{R}$  that for appropriately small  $\epsilon > 0$ :

$$\hat{x}_j(t_m) = f(\mathcal{X}_j^{1:m-1}, \mathcal{X}_i^{1:m})$$
(4)

and

$$|x_j(t_m) - \hat{x}_j(t_m)| < \epsilon \tag{5}$$

This dependency is depicted by  $x_j(t) \xrightarrow{\epsilon} x_i(t)|_{t_m}$ 

The aforementioned structure informed invariant learning methods help scale the invariant learning procedure to large systems as they reduce the quadratic complexity of the algorithm. This is because, although the complexity of the algorithm technically is still quadratic in the worst case, the complexity of the newly proposed KASE (Kalman Autoregressive State Estimation with Latent Factors and Exogenous Inputs) invariant learning algorithm is a function of the average cardinality of the SAIL neighborhood for all the components. Since it is highly unlikely that all or even most of the components of a CPS will be directly connected, the SAIL neighborhood of each component will be relatively sparse hence achieving the scalability and speeding up the algorithm relative to benchmarks that do not exploit the neighborhood assumption.

We now outline the proposed KASE approach and explain the algorithm in detail in section 4. In figure 5 we observe different colored nodes and edges. If we consider one node say  $X_2(t)$  we see that the SAIL neighborhood of  $X_2(t)$  denoted  $S_n^2$  contains  $\{X_1(t), X_3(t)\}$ . The blue colored incoming edges from  $X_1(t)$  and  $X_3(t)$  to  $X_2(t)$  denote the invariants calculated for state estimation of  $X_2(t)$ . The self-loop labelled  $k_2(t)$  indicates the Kalman filter state estimate for  $X_2(t)$  at time t. The latent factors  $h_1, ...h_k$  are the same as stated previously. Hence the KASE algorithm performs state estimation as a function of the current Kalman filter state estimate, the current measurements of proximal components as well as the system-wide relationships learned through the latent factors. The system hence seamlessly combines both model-based (Kalman filters) as well as data-driven approaches (latent factor based autoregression) for state estimation.

#### 4. METHODS

In this section we describe the KASE algorithm for invariant learning and anomaly detection. We also outline the working of the entire **illiad** application.

#### 4.1. Factor Analysis

Let  $X_1, ..., X_n$  denote random variables and  $H_1, ..., H_k$  denote k hidden factors, and assume that the latent variables are generated using *factor analysis* with the assumption that they can be expressed as linear combinations of the observed variables. Factor analysis attempts to model the variation of the data and hence models the latent

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sources that cause said variation [De Winter and Dodou 2016]. Factor analysis although similar to PCA is more generalizable due to consistency of factor loadings for different feature subsets [Suhr 2005]. Details of the exact factor analysis procedure we have used can be obtained from [Momtazpour et al. 2015]. Other resources for factor analysis are also available [Jöreskog 1967; Harman 1976].

#### 4.2. *KASE* - Kalman Autoregressive State Estimation with Latent Factors and Exogenous Inputs

If we were to once again consider the set  $\mathcal{D} = \{x_1(t), x_2(t), \dots, x_n(t)\}$  then according to the ARX model in [Jiang et al. 2006], the state estimate for a timeseries at time *t* is given by

$$\hat{x}_j(t) = \sum_{p=1}^u a_p x_j(t-p) + \sum_{p=0}^v b_p x_i(t-l-p)$$
(6)

Here  $\hat{x}_j(t)$  represents the state estimate of the timeseries  $x_j$  at time t and this is calculated as a function of the previous values of timeseries  $x_j$  and the values of the exogenous timeseries  $x_i$ . The parameters u, v, l represent the order of the model and control the number of previous timesteps that affect the state estimate at the current timestep.  $a_p$  and  $b_p$  represent weight parameters that control the effect of each of the historical values on the current timestep. In our model, u = v and the values are estimated using cross-validation. In our case we assume that l = 0 as there is no lag.

Incorporating the above assumptions, the corresponding equation for the Latent Factor ARX model (ARX + ARXL) as outlined in [Momtazpour et al. 2015] is

$$\hat{x}_j(t) = \sum_{p=1}^u a_p x_j(t-p) + \sum_{p=0}^u b_p x_i(t-p) + \sum_{p=0}^u \sum_{q=1}^k c_{pq} h_k(t-p)$$
(7)

Here k indicates the number of latent factors and u indicates the size of the sliding window we use (the number of previous values we consider). For our experiments we have set u = 10.

We modify Eqn. 7 to incorporate the Kalman filter state estimates step as follows:

$$\hat{x}_j(t) = \sum_{p=1}^u a_p x_j(t-p) + \sum_{p=0}^u b_p x_i(t-p) + \sum_{p=0}^u \sum_{q=1}^k c_{pq} h_k(t-p) + \sum_{p=0}^u d_p k_{ji}(t-p)$$
(8)

Equation 8 incorporates the Kalman state estimates for timeseries  $x_j$ , represented by the symbol  $k_{ji}(t)$ . The  $d_p$ 's represent the corresponding weights of each historical state estimate. The symbol  $k_{ji}(t)$  indicates the Kalman filter state estimate for  $x_j(t)$ calculated using the measurements of timeseries  $x_j$  and  $x_i$  at time t.

## 4.2.1. Learning Invariant Based State Estimates.

In the context of two timeseries  $x_i$  and  $x_j$  wherein  $x_j$  is the output timeseries and  $x_i$  is the input timeseries, we calculate the state estimate of  $x_i(t)$  as follows:

Let  $\bar{\mathbf{x}}_k \in \mathbb{R}^{2X1}$  indicate the Kalman prior state mean of the state at time k, then the transition equations of the Kalman filter is given by

$$\bar{\mathbf{x}}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_k + \mathbf{w}_{k-1} \tag{9}$$

Here  $\mathbf{A} \in \mathbb{R}^{2X^2}$  is the transition matrix.  $\mathbf{x}_{k-1} \in \mathbb{R}^{2X^1}$  indicates the *a posteriori* state estimate at time k-1. We ignore the optional control input term  $\mathbf{Bu}_k$  and make the as-

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sumption as stated previously that  $\mathbf{w}_{k-1} \in \mathbb{R}^{2X1}$  is a white-noise process that indicates the error at time k-1.

$$\mathbf{y}_k = \mathbf{z}_k - \mathbf{H}\bar{\mathbf{x}}_k \tag{10}$$

 $\mathbf{H} \in \mathbb{R}^{2X^2}$  is the measurement matrix. This in our case is the identity matrix.  $\mathbf{z}_k \in \mathbb{R}^{2X^1}$  are the actual measurements at time k. In the case of the two timeseries  $x_i$  (input) and  $x_j$  (output)

$$\mathbf{z}_k = \begin{bmatrix} x_j(k) \\ x_i(k) \end{bmatrix}$$

 $\mathbf{y}_k \in \mathbb{R}^{2X1}$  indicates the residual at time k.

$$\mathbf{x}_k = \bar{\mathbf{x}}_k + \mathbf{K} \mathbf{y}_k \tag{11}$$

 $\mathbf{K} \in \mathbb{R}^{2X2}$  represents the Kalman gain and is calculated as a function of the process and measurement covariance matrices  $\mathbf{P}$  and  $\mathbf{Q}$  and the measurement matrix  $\mathbf{H}^1$  $\mathbf{x}_k \in \mathbb{R}^{2X1}$  represents the posterior state estimate. In the context of the time series  $x_i$ and  $x_i$ , where  $x_i$  is the output timeseries whose state at time t is to be estimated,

$$\mathbf{x}_t = \begin{bmatrix} k_{ji}(t) \\ k_{ij}(t) \end{bmatrix}$$

Here  $k_{ji}(t) \in \mathbb{R}$  indicates the aposteriori state estimate (mean) for timeseries  $x_j$  using the exogenous input  $x_i$  for time t. If we now refer back to figure 5, and consider the node  $X_2(t)$  in blue, the self-loop labelled  $k_2(t) \in \mathbb{R}^{CX1}$  represents a vector of aposterior state means where  $C = |S_n^2|$ . Where the SAIL set  $S_n^2$  of timeseries  $X_2 = \{X_1, X_3\}$ .

$$k_2(t) = \begin{bmatrix} k_{21}(t) \\ k_{23}(t) \end{bmatrix}$$

**A**, **P**, **Q** are estimated (and periodically re-estimated) from historical data using the expectation maximization algorithm [Borman 2004]. The initial state mean and co-variance  $\mu_0$  and  $\Sigma_0$  are estimated from the data.

As acknowledged by [Jiang et al. 2006; Ge et al. 2013; Chen et al. 2010, 2008; Sharma et al. 2013], ARX only takes into account direct linear relationships. Hence we retain the latent factor model to take into account the complex underlying relationships between the components in the system. We also consider the model-based Kalman filtering approach owing to the advantages and improvement of system performance in fault detection and health monitoring of hybrid-approaches (model-based + data-driven) discussed in [Tidriri et al. 2016].

Algorithm 1 represents the newly designed KASE algorithm that combines both model-based and data-driven approaches in learning the invariant graph. The algorithm defined in [Momtazpour et al. 2015] has been augmented to incorporate topological structural assumptions (line 6). At each timestep t, the **UpdateKalman** function (line 16) takes in the measurements of a pair of timeseries  $x_i(t)$ ,  $x_j(t)$  and updates the model using equations 9, 10, 11. The posterior state estimate of the updated model is then used to learn  $\theta_{ji}^{KASE}$ . The  $F_{ji}^*$ 's are calculated and superiority threshold ( $\Delta$ ),

<sup>&</sup>lt;sup>1</sup>The notation for equations 9,10 and 11 has been taken from [Labbe Jr 2014; Welch and Bishop 2001].

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ALGORITHM 1: KASE Invariant Search Algorithm

**Input:**  $x_i, i \in \{1, ..., n\}$ : set of timeseries,  $\Delta$ : ARX superiority threshold,  $\tau$ : minimum acceptable score,  $t_s$  and  $t_e$ : start and end time of training dataset. Output: G: Invariant Graph **1**  $S_{ARX} = \{\};$ **2**  $S_{ARXL} = \{\};$ **3**  $S_{KASE} = \{\};$ 4 for i = 1 to n do 5 **for** *j* **=** 1 *to n* **do** if  $((i == j) \text{ or } (x_i \notin S_n^j))$  then 6 continue; 7 end 8 foreach  $t_s \leq t \leq t_e$  do 9 Learn an ARX model,  $\theta_{ji}^{ARX}$ , using Eq. 6; 10 Calculate  $\hat{x}_{j}^{ARX}(t)$  using  $\theta_{ji}^{ARX}$ ; 11 Compute  $\mathcal{F}_{ji}^{ARX}(t)$ ; 12 Learn an ARXL model,  $\theta_{ii}^{ARXL}$ , using Eq. 7; 13 Calculate  $\hat{x}_{j}^{ARXL}(t)$  using  $\theta_{ji}^{ARXL}$ ; Compute  $\mathcal{F}_{ji}^{ARXL}(t)$ ; 14 15 **UpdateKalman**  $(x_i(t), x_j(t))$  /\*Add Measurements\*/ 16 Learn a KASE model,  $\theta_{ii}^{KASE}$ , using Eq. 8; 17 Calculate  $\hat{x}_{i}^{KASE}(t)$  using  $\theta_{ii}^{KASE}$ ; 18 Compute  $\mathcal{F}_{ii}^{KASE}(t)$ ; 19 end 20  $\mathbf{if}\left(\sum_{t=t_s}^{t_e} \mathcal{F}_{ji}^{ARX}(t) \ge max(\sum_{t=t_s}^{t_e} \mathcal{F}_{ji}^{ARXL} and \sum_{t=t_s}^{t_e} \mathcal{F}_{ji}^{KASE}) - \Delta\right) and$ 21  $\begin{array}{l} (\min_{t}(F_{ji}^{ARX}(t)) \geq \tau) \text{ then} \\ \mid S_{ARX} = S_{ARX} \cup \{x_i \rightleftharpoons x_j\}; \\ \text{else if } \left(\sum_{t=t_s}^{t_e} \mathcal{F}_{ji}^{ARXL} \geq (\sum_{t=t_s}^{t_e} \mathcal{F}_{ji}^{KASE} - \Delta)\right) \text{ and } (\min_{t}(F_{ji}^{ARXL}(t)) \geq \tau) \text{ then} \\ \mid S_{ARXL} = S_{ARXL} \cup \{x_i \rightleftharpoons x_j\}; \end{array}$ 22 23 24 else 25 /\*Indicates that  $\mathcal{F}_{ji}^{KASE}$  has highest score\*/  $S_{KASE} = S_{KASE} \cup \{x_i \rightleftharpoons x_j\};$ 26 27 end 28 29 end

minimum acceptable score ( $\tau$ ) are enforced similar to [Momtazpour et al. 2015]. We also adopt the same alerting and anomaly ranking and localization procedure with the *RMSE* evaluation metric. In section 5, we provide a comparison of the anomaly detection ranking scores of both the original algorithm (ARX + ARXL) proposed in [Momtazpour et al. 2015] and our newly proposed KASE algorithm.

## 4.3. illiad System Architecture

We outline the functioning of the *illiad* anomaly detection system in this section. Figure 6 depicts the process flow/ architecture diagram of the *illiad* system. The system periodically re-trains the Kalman filter and the factor analysis model using historical data while also continuously updating the state estimate of the Kalman filter using new data instances. The system also has a front-end dashboard with the invariant graph depicted on screen aiding the system maintainer to easily glean whether the system has anomalous components. Figures 7 and 8 represent the dashboard of the



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Fig. 6: illiad System Architecture

board - Sensormotes



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wireless sensor motes temperature sensor network and the IEEE 33 bus microgrid network described later in Section 5.1. The dashboard is interactive and allows for the user to click on nodes in the network to view various historical data and metrics about them. In case of an anomaly, an alert is issued (indicated by a red box under the invariant graph) on the dashboard and the corresponding broken invariants are highlighted in red as shown in the figures. It is apparent from the broken invariants in figure 7 that Sensor 4 has an anomaly. The time series graphs at the bottom of the dashboard represent the residuals of each of the invariants of the anomalous sensor (sensor 4 in figure 7 and Bus 30 in figure 8). In addition to residuals, the actual temperature readings and the voltage magnitudes , phase angles and PV , Wind Generation statistics are also included in their respective dashboards. The *illiad* system can thus be used for system health monitoring and fault detection both for wired and wireless cyber-physical systems.

## 5. EXPERIMENTAL RESULTS

We present the application of our methodology to two diverse datasets in urban computing. Through these experiments we seek to showcase that our system is applicable to both wireless and wired sensor networks in real-world settings. We will showcase how structural and proximal relationships are inferred automatically in the case of wireless sensor networks. Most importantly, we wish to demonstrate how our system leads to an improvement in the quality of anomaly and invariant detection over existing benchmarks. To this end, we conduct experiments to showcase the strength of invariant graphs learned by *illiad* as well as its ability to accurately identify anomalies even with a relatively sparse invariant graph. We also discuss the benefits of the *SAIL* invariant learning approach over the combinatorial invariant learning approach used in prior work by analyzing the runtime behavior of the two systems.

We first describe the application of *illiad* to a wireless sensor network and subsequently to a microgrid system.

## 5.1. Sensor Motes

**Dataset Description:** The sensor motes dataset contains measurements from wireless sensors at Intel Berkeley Research lab<sup>2</sup>. There are a total of 54 sensors located at a lab measuring temperature, humidity, light, and voltage between February  $28^{th}$  and April 5th, 2004. Each sensor was able to record different variables every 31 seconds. Figure 9 shows the location of each sensor in different parts of the lab. We focus on temperature recordings between February  $28^{th}$  and March  $10^{th}$  for the purpose of our study.

**Data Processing:** We cleanse the data by eliminating sensors with a large number of missing entries. The missing temperature readings in the remaining set of sensors were addressed using linear interpolation [Meijering 2002]. The dataset was then down-sampled to 10 minute intervals with the mean as the sampling rule. Figure 10 depicts a representation of all the filtered set of sensors used in our analysis, clustered according to their temperature readings. Figure 11 shows the temperature values of a group of sensors that are part of the same cluster. We can see that all the sensors in the figure showcase similar patterns of temperature variation.

**Results and Discussion:** In figure 12 we plot the average normalized relationship strength  $(1 - avg\_prediction\_error)$  per temperature sensor. The relationship strength is indicative of how well the invariants that a sensor is involved in are able to estimate the current state of the sensor. We observe that the relationship strengths of sensors obtained using KASE is higher than that obtained using the original (ARX + ARXL)

<sup>&</sup>lt;sup>2</sup>http://db.csail.mit.edu/labdata/labdata.html

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Fig. 9: Lab Layout Sensor Motes



**Fig. 10:** Clustering of Sensors Based on Original Temperature Readings. A fair degree of spatial correlation can be observed.



Fig. 11: Temperature Trends of Sensors in the top right yellow cluster in figure 10

method indicating that KASE is able to infer a higher number of invariant relationships that are stronger. This theory is further corroborated by figure 13 which depicts the average residuals per temperature sensor; in this case as well, we observe that KASE is able to generalize better on the test set and produce more accurate temperature state estimates. The network structure of the wireless sensor network is derived using the *SAIL* method (using sensor locations) described in section 3 and is shown in figure 14. The invariant graph learned using this network from the KASE algorithm is depicted in figure 15. We observe that the invariant graph is identical to the original network structure shown in figure 14. Figure 16 indicates a similar invariant graph obtained by executing the original (ARX + ARXL) algorithm with the *SAIL* neighborhood constraint on the same temperature sensormotes dataset. We observe that the number of edges learned in the invariant graph is lower than in the case of KASE. The missing edges are depicted in red in figure 16.

**Anomaly Detection:** Apart from the invariant discovery and state estimation accuracy, we also conducted experiments to test the anomaly detection capabilities of the KASE algorithm in comparison to the original (ARX + ARXL) algorithm. The experiment was carried out by first injecting anomalies into temperature sensor number 4 between 11 AM and 23:59 pm on 03/09/2004 and then running the two algorithms (original ARX + ARXL and the KASE algorithm) to understand the behavior of the two systems when faced with an anomaly. The anomalous snippet of the time series of sensor 4 and the original timeseries have been shown in figure 17. The anomalies were generated by adding Gaussian noise with 0 mean and 2.0 standard deviation to the sensor 4 temperature values at certain pre-determined timepoints between 11 AM and 23:59 PM on 03/09/2004. These anomalies have been indicated in red in figure 17. We







Fig. 13: Average Residual Per Temperature Sensor.

use the anomaly ranking algorithm used in [Momtazpour et al. 2015] to quantify and compare the anomaly detection quality of both methods. The results of the anomaly detection are indicated in Table I. Here we observe that although a significant number of invariants concerning the anomalous sensor (sensor 4) are broken, there are almost an equivalent number of invariants still existing in the system indicating that these invariants do not register the anomaly. The KASE algorithm however recognizes the anomaly by breaking all the existing invariants (in this case sensor 4 initially has 2 invariants which are both broken) and hence obtaining a higher anomaly score and also providing for a more interpretable detection mechanism.

**Discussion:** State estimation in the current *illiad* system could suffer if the sensor distribution is significantly skewed spatially. Since we consider the spatially closest neighbors to a particular sensor as candidates for invariant learning, a significant skew in that distribution could make estimation challenging due to sparse data in some locations. This in turn could lead to fewer invariants inferred and thus diminished capacity for anomaly detection. The use of Gaussian processes for dynamic sensor placement [Ramakrishnan et al. 2005; Krause et al. 2008] can aid in judicious placement of sensors. A second issue in generalization has to do with potential mobility of sensors in practical settings (in which case, some violations of invariants could be normal). In IoT settings, we need to inherently model sensor mobility into the analytics engine.

Table I: Anomaly Detection Score Sensormotes

Component	Method Name	Remaining Links	Broken Links	Ranking Score
Sensor 4	Original (ARX + ARXL)	14	15	0.517
Sensor 4	KASE	0	2	1.0

# 5.2. Electric Power Microgrid

**Dataset Description:** This simulated dataset is based on the (real) IEEE 33-bus standard distribution power system, a commonly used example distribution network. It is



**Fig. 14:** Original Nearest-Neighbor Graph (based on sensor location information.)



**Fig. 15:** Invariant Graph Learned from KASE algorithm



22.26 21.0 22.6 21.0 12.00 14.00 16.00 18.00 20.00 22.60 10.00

**Fig. 16:** Invariant Graph Learned from ARXL + ARX method (with SSAIL assumption)

**Fig. 17:** Temperature Sensor Number 4, Original Readings vs. Anomaly Injection Readings Snapshot.

composed of 33 electric buses or nodes. Bus 1 is a transformer connecting the distribution level system and bulk transmission power system. It is the feeder to the whole system, through which all the electric power flows into the network. It serves as the regular energy source. The other 32 buses are all load buses initially, which means that they only consume energy. We model all energy consumers and not the energy source in Bus 1 during our experiments as the voltage and power of the transformer is assumed constant throughout the dataset generation process. In order to imitate a typical mircrogrid, we have integrated multiple non-traditional elements into the system, including two types of renewable distributed energy sources: PV and wind turbines, electric vehicles, and an energy storage device.

**Data Processing:** The hourly operating conditions of this distribution system are simulated for a year with a 1-hour sampling rate, which is 8760 cases in total, using the MatPower package [Zimmerman and Gan 1997]. We conducted 8760 power flow calcu-



(a) IEEE 33-bus system microgrid.



(c) KASE Algorithm Broken Invariants Representation





ARXL) vs. Original (ARX+ARXL) + SAIL

lations based on hourly scales of the regular and irregular load and the hourly outputs of the renewable distributed generation. The load consumption, solar radiation and wind intensity values required as input for data generation were all considered for the city of Richmond, Virginia USA, to ensure consistency between the electric load, and the energy produced using distributed generation. The 8760-hour load data has been taken from a dataset containing commercial and residential hourly load profiles for all TMY3 locations in the United States<sup>3</sup>. For different load buses, we use 5% uniformly distributed deviation to generate different load curves that follow the same general

<sup>&</sup>lt;sup>3</sup>https://en.openei.org/datasets/dataset/commercial-and-residential-hourly-load-profiles-for-all-tmy3-locations-in-the-united-states

pattern. The load at Bus 33 has a solar panel attached to it. The solar radiation data used to generate the synthetic PV output is available from the National Solar Radiation Data Base<sup>4</sup>. One wind turbine is attached to Bus 15, and the power output of a wind farm located close to Richmond the data for which was obtained from the National Renewable Energy Laboratory Wind Prospector Data Set<sup>5</sup> was used as a reference for the simulation. We have also attached an EV charging station to Bus 10. A one year charging load for this station was obtained from Idaho National Laboratory EV Project Quarterly and Annual Report Data<sup>6</sup>. An energy storage battery is attached to Bus 2. For simplicity, we adopt a charging/discharging strategy that assumes that the battery is charged during the load valley from 00:00 am to 8:00 am and that it is discharged from 9:00 am to 11:00 pm. The physical connections among the various components of the microgrid system have been specified in Figure 18 (a). It must be noted that the energy storage device depicted at Bus 2 hasn't been modeled as an explicit separate component; instead the load at Bus 2 has been modified appropriately to depict behavior of having an energy storage device attached to it. Hence we do not model the battery as a separate component in our experiments.

**Results and Discussion:** Figure 18 (a) represents the circuit diagram of the IEEE 33 Bus network used in our experiments. We use the network structure depicted in the figure (excluding the dashed lines that depict open switches) to learn the SAIL neighborhoods of each of the 35 components. The Wind turbine, PV Cell and the Electric Vehicle Load have been renamed Bus 35, Bus 36 and Bus 37 respectively. The SAIL neighborhood learned by the KASE algorithm has been depicted in figure 18 (b) and the broken links due to anomalies have been depicted in figure 18 (c). We see that the invariants learned by the KASE algorithm correspond to the original circuit diagram. The KASE has three sub-components, (ARX, ARXL, Kalman State Estimation). These three algorithms each try to learn relationships between each pair of invariants. From Algorithm 1 (lines 21 to 29), it is clear that the invariant learning process attempts to select the strongest relationship between a pair of components learned by ARX, ARXL and Kalman state estimation. Hence we can conclude that the algorithm that contributes the most number of invariants to the graph has the property of learning stronger relationships between components. Table II depicts the number of invariants learned as a function of Maximum Acceptable Error (MAE). The MAE is the maximum prediction error (per invariant) below which invariant-relationships learned by any of the aforementioned procedures qualify for selection (this can also be considered to be 1 -  $\tau$  where  $\tau$  indicates the minimum acceptable invariant-relationship strength) during the invariant learning process. The original\_SAIL column represents the (ARX + ARXL) original algorithm run using the SAIL neighborhood assumption. The KASE column represents algorithm 1 and KASE\_ARX, KASE\_ARXL, KASE\_KSE represent the number of invariants learned by each of ARX, ARXL and Kalman state estimation sub-components of KASE, respectively. We observe that for any particular value of Maximum Acceptable Error, KASE learns a greater subset of the true set of invariants (the IEEE 33 Bus network has a total of 68 invariants) than the Original\_SAIL algorithm. Even amongst the KASE sub-components, we can see that a majority of edges are contributed by the KSE procedure indicating that the Kalman filtering procedure plays a major role leading to the KASE algorithm learning a more diverse set of edges and stronger invariant relationships as compared to the Original (ARX + ARXL) algorithm. We provide quantitative proof of our claim that the KASE algorithm is indeed a better state-estimator by recording the average values for residual, RMSE (root mean

 $<sup>^{4}</sup> http://rredc.nrel.gov/solar/old_data/nsrdb/1991-2005/tmy3/by\_state\_and\_city.html$ 

<sup>&</sup>lt;sup>5</sup>https://mapsbeta.nrel.gov/wind-prospector/?visible=wind\_3tier\_site\_metadata

<sup>&</sup>lt;sup>6</sup>https://avt.inl.gov/project-type/downloads

Max. Acceptable Error (MAE)	Original_SAIL	KASE	KASE_ARX	KASE_ARXL	KASE_KSE
0.0001	0	0	0	0	0
0.001	11	12	0	3	9
0.002	20	21	0	6	15
0.003	22	27	0	6	21
0.004	34	38	0	8	30
0.005	46	48	0	15	33
0.006	66	67	0	17	50
0.0851	68	68	0	18	50

**Table II:** Number of Invariants Learned as a function of MAE.

squared error), RSE (relative squared error), RAE (relative absolute error) on the test set in Table III and figures 19,20,21. We find that the KASE algorithm achieves a significant percentage improvement over the Original (ARX + ARXL) algorithm in all cases. We must note that we have left out the componenents (Bus20,Bus30,Bus7) in which anomalies were injected in the test set as they will skew the results with high error values <sup>7</sup>. Since this table specifically has been included to showcase the state-estimation capability of the KASE algorithm, we feel excluding the anomalous nodes will have no effect as we have already demonstrated the anomaly detection capabilities in Table IV.

**Anomaly Detection:** We introduce anomalies in the data to simulate a load surge at certain time steps by decreasing the voltage at three different buses. We decrease the voltage magnitude at Bus 7 between time steps 7500 and 7565. We also decrease the voltage at Bus 20 from time step 8500 to 8550 and at Bus 30 from time step 8500 to 8545 and showcase the effect of these anomalies on the anomaly detection mechanism. These anomalies test the ability of the system not only to detect and report the anomalies, but also the localization capability of the system in the presence of a single as well as multiple anomalous components. The voltages of the three buses around the time steps of their respective anomalies have been depicted in figures 22, 23,24. In each figure the anomalous region has been bound by vertical red lines. Since the invariant relationships learned are stronger, the anomaly detection procedure is also able to perform in a more robust manner when it encounters an anomaly. This can be seen in figure 18 (c) where all the invariants of Buses 7, 20, and 30 are broken in response to the injected anomalies. We also show that the KASE algorithm is better at ranking anomalies; on average a higher score is learned using the ranking procedure from [Momtazpour et al. 2015]. Table IV shows that for each of the three buses, all invariants involving the anomalous bus are broken for the KASE algorithm yielding a higher anomaly score relative to the Original (ARX + ARXL) algorithm, further enforcing our belief that the KASE algorithm proposed in this paper learns stronger and more robust invariants most or all of which are violated in case an anomaly occurs in the system.

Finally, we depict the advantage of the SAIL methods towards increasing the scalability of the invariant learning algorithms discussed in this paper thereby enabling the algorithms to be run with larger datasets. In order to conduct this experiment we generated a random matrix  $X \in \mathbb{R}^{mXn}$  where we varied *n* between 10–100. Here *n* represents the number of unique timeseries (or unique components in the CPS). We then ran both the Original and the Original method with the SAIL assumption. We simulated the SAIL neighborhood by assuming that the underlying network of the CPS

 $<sup>^{7}</sup>$  if there is an anomaly, the system under / overestimates the state significantly, which is what is indicative of the anomaly

 Table III: IEEE 33 Bus Avg. Residual, Avg. Errors and percentage Improvement of KASE over

 Original

Name	Original	KASE	% Improvement
RMSE	0.004153	0.003964	4.5639
RSE	0.148742	0.134409	9.6363
RAE	0.365744	0.347662	4.944
Residual	0.002909	0.0027889	4.14

Component	Method	Remaining Links	Broken Links	Score
Bus7	Original(ARX+ARXL)	6	14	0.7
	KASE	0	2	1.0
Bus20	Original(ARX+ARXL)	11	21	0.66
	KASE	0	2	1.0
Bus30	Original(ARX+ARXL)	4	13	0.76
	KASE	0	2	1.0

Table IV: Ranking and Anomaly Localization

had a 0.3 graph density (each component is connected to 30% of the other components in the network) which is much denser than either of our examples depicted in this paper and is much denser than most connected CPSs. Figure 25 shows the significant advantage in terms of running time of incorporating the network information in the invariant learning. The figure clearly shows that the Original (ARX + ARXL) algorithm with the SAIL assumption scales much better than its non SAIL counterpart.

**Discussion:** Although we have taken great care to model our microgrid design using actual load profiles, and real solar and wind intensity data to simulate the generation of the PV and wind turbines, the lack of open datasets for microgrids is a serious bottle-neck in this area of research. Similarly, our injection of anomalies can be generalized into a greater taxonomy of faults, e.g., component failure, cyber-attacks, and changing environmental situations. We aim to build upon the work here to develop a broader framework for microgrid analytics.

## 5.3. System Function - Wireless vs. Wired Networks

It is instructive to compare and contrast our wired and wireless network studies. While we have shown that our method works in both settings, wireless networks exhibit broader profiles of fault-tolerance not witnessed in wired networks (e.g., a sensor malfunction in a wireless network might not affect the overall system dynamics as much as a component failure in a microgrid might, e.g., potentially leading to cascading failures). The illiad system has a built-in rule to raise an alert when a a threshold number of samples from some component of the CPS have failed to register but more elaborate fault models can be explored within our framework.

## 5.4. System Characteristics and Minimum Requirements

The proposed system has been shown to work with both wireless and wired sensor networks and has been tuned to work with power systems like microgrids as well as sensors that measure physical indicators such as temperature. Further, the system is able to perform with either a static dataset or streaming data, for both wired and wireless sensor networks. The minimum requirement for the system is either an absolute physical network diagram of the sensor network or a logical connectivity diagram so that the invariant learning can leverage this prior information as domain knowledge and further contribute to interpretability.

#### 6. CONCLUSIONS AND FUTURE WORK

We have presented a system for anomaly detection and state estimation for wireless and wired sensor networks. We have tested our models on multiple datasets (both real-world and synthetic) and demonstrated the improvement in performance in comparison to the baseline method. Our application successfully combines model based (Kalman filter) and data-driven (auto-regression and latent factor based methods) approaches to learn a better state representation of the system under surveillance. This higher accuracy in state estimation is achieved through learning stronger relationships between various components in the networks while also being sensitive to potential violations in these relationships resulting in anomalies. The sparse network structure of invariant relationships learnt through the network-structure aware invariant learning procedures makes for a scalable system wherein the state of the system is easily interpretable by human experts tasked with overseeing system maintenance. The real-time dashboard and the alerting system aid further in this regard. The field of invariant discovery is vast as there are many varieties of latent, direct, indirect, simple and complex relationships amongst components in cyber-physical systems. Power systems are rife with non-linearities, like voltage phase-angles. Incorporating these components into the invariant learning is a logical next step to improve the state estimation capabilities of the system. Further, each component in a power system interacts with other components in compliance with certain well known laws of physics and electricity; thus, augmenting the anomaly detection procedure to be cognizant of these relationships would be a useful direction of future work.

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