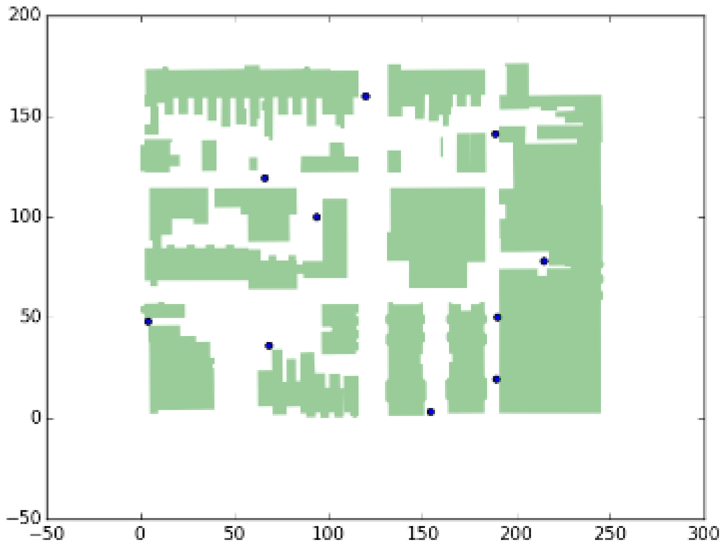
STOCHASTIC AND DETERMINISTIC TECHNIQUES FOR LOCATING SOURCES FROM MULTIPLE DETECTOR RESPONSES

RAZVAN STEFANESCU, KATIE SCHMIDT AND RALPH SMITH
NORTH CAROLINA STATE UNIVERSITY

PROBLEM SETUP



- **10 detectors**
- **10 observations from detector**

Source Location Techniques

- **Hybrid Variational Data Assimilation**
  - SA (Simulated annealing)- [Metropolis,1953, Kirkpatrick,1983]
  - PS (Particle swarm) – [Eberhar and Kennedy 1995]
  - GA (Genetic algorithm) - [Golberg 1989]
  - IF (Implicit Filtering) - [Kelley 2011]

- **Stochastic Bayesian**
  - DRAM (Delayed Rejection Adaptive Metropolis) - [Haario, et al., 2006]

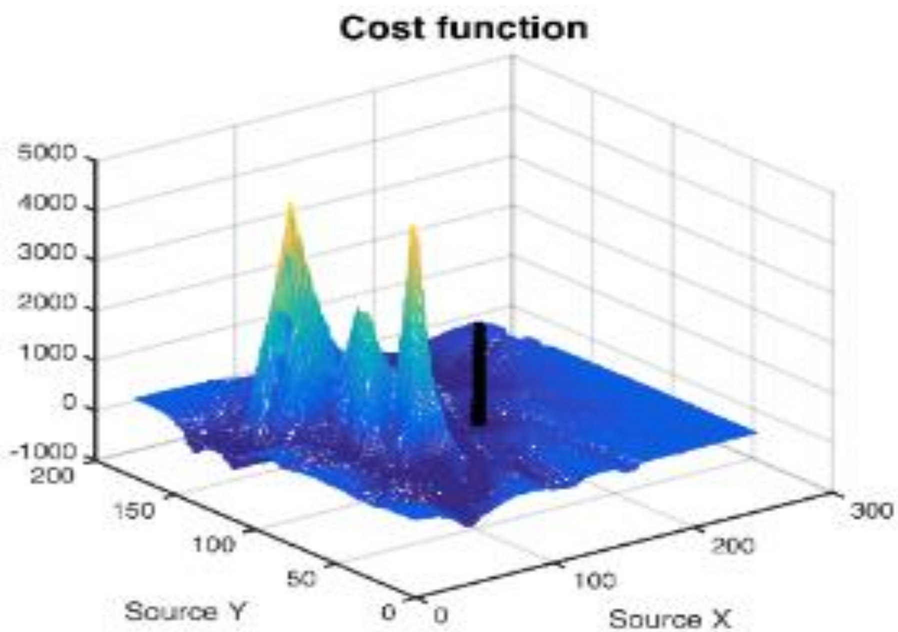
HYBRID VARIATIONAL DATA ASSIMILATION TECHNIQUES

- Maximize the likelihood function

$$\text{Normal: } f(\theta) = \frac{1}{2}\sum_{i=1}^{N_s}\sum_{j=1}^{N_m}\left[\frac{v_{ij}-F_i(\theta)}{\sigma_i}\right]^2$$

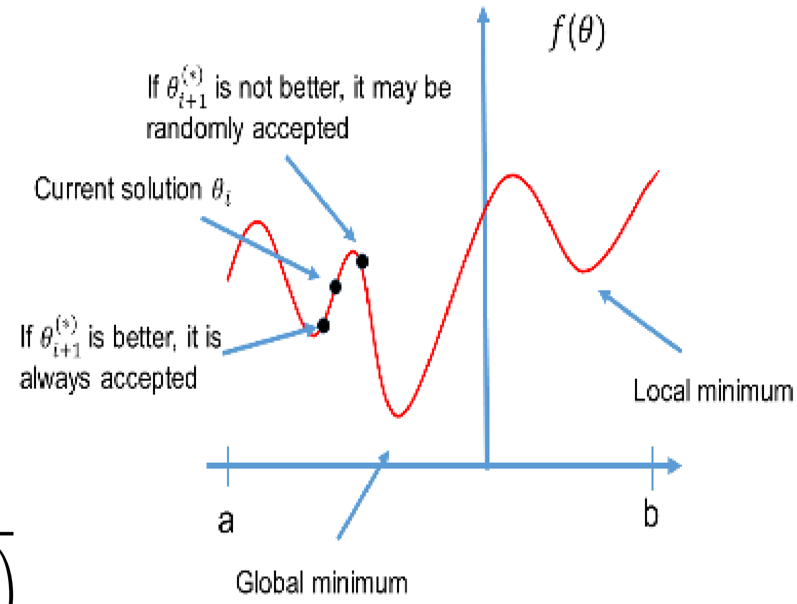
$$\text{Poisson: } f(\theta) = \frac{1}{2}\sum_{i=1}^{N_s}\sum_{j=1}^{N_m}\left[-v_{ij}\cdot\ln\big(F_i(\theta)\big)+F_i(\theta)\right]$$

- Issues with non-smooth radiation model



SA

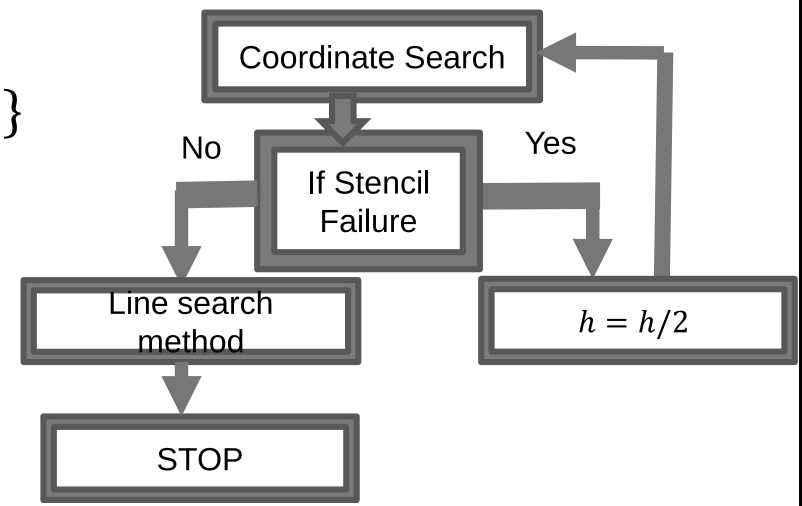
1. Annealing scheme: $T_i = T_0 \cdot 0.95^{\bar{i}}$
2. $\theta_{i+1}^{(*)} = \theta_i + r \cdot T_i,\ r \sim U(-1,1)$
3. If $f\left(\theta_{i+1}^{(*)}\right) < f(\theta_i)$, set $\theta_{i+1} = \theta_{i+1}^{(*)}$ else
4. $\theta_{i+1} = \theta_{i+1}^{(*)}$, prob $\alpha_i = \dfrac{1}{1+\exp\left(\dfrac{f\left(\theta_{i+1}^{(*)}\right)-f(\theta_i)}{\max(T_i)}\right)}$



IF

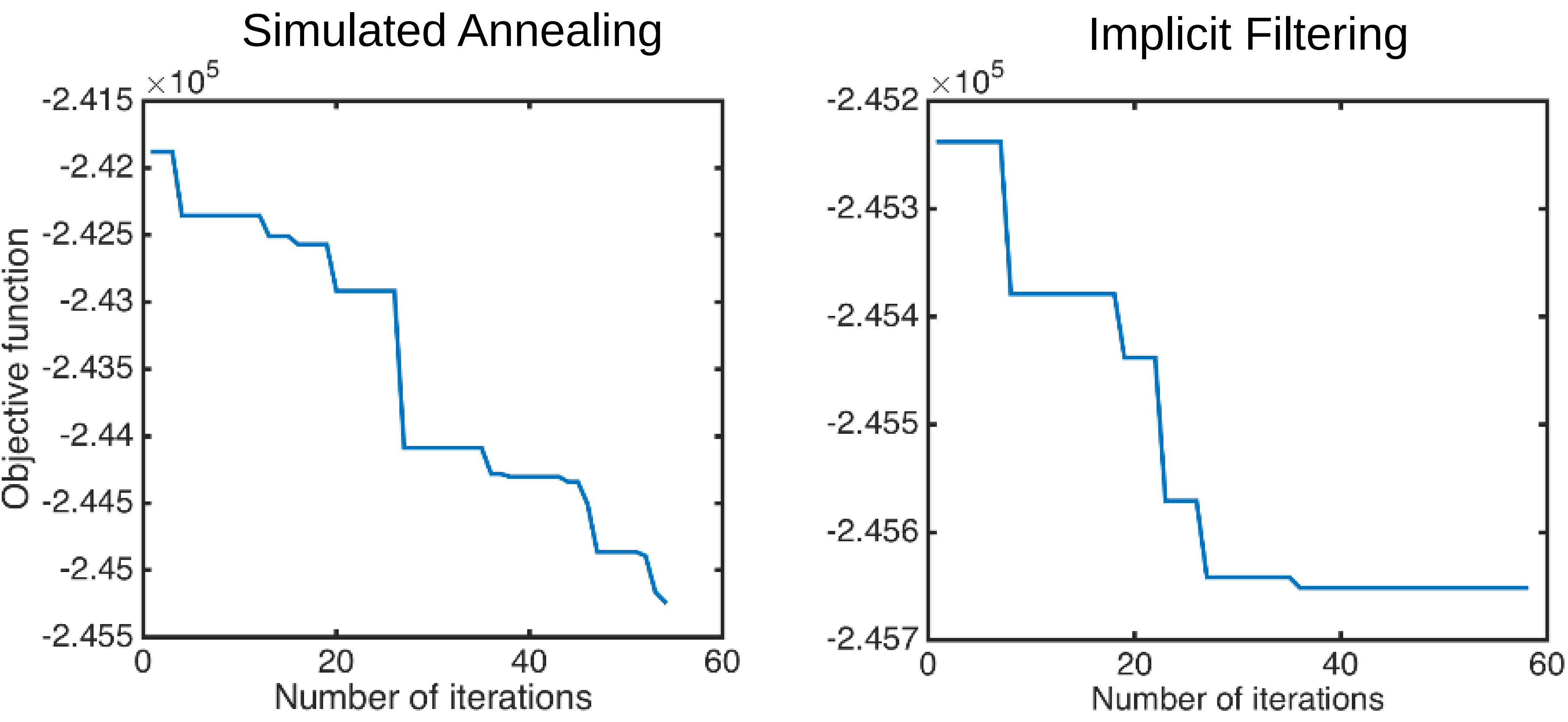
1. Stencil points $S(\theta, h, V) = \{z; z = \theta \pm h v_i\}$
2. Quasi-Newton direction
3. Modified Line Search method
4. Stopping criteria - gradient

   - iterations number



SA+IF RESULTS

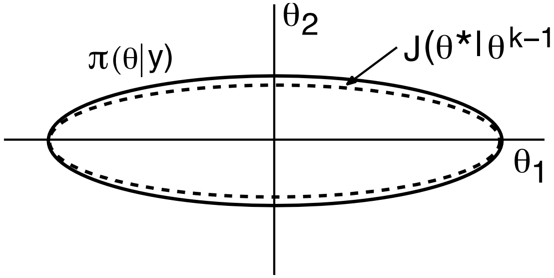
Poisson Likelihood



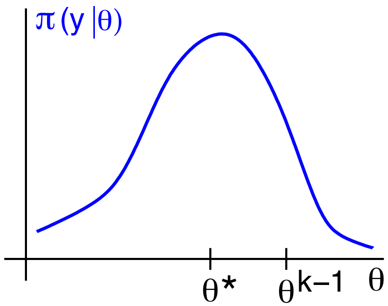
DELAYED REJECTION ADAPTIVE METROPOLIS (DRAM)

Proposal Distribution

$$J(\theta^*|\theta^{k-1}) = N(\theta^{k-1}, V)$$

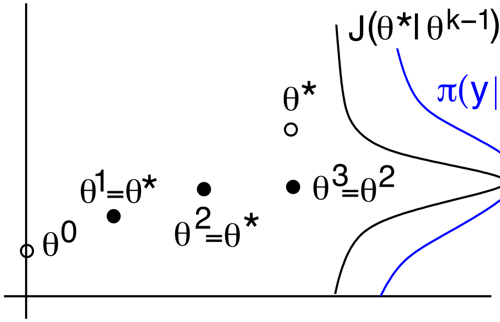


1. Determine $\theta^0 = \arg\min_{\theta} \sum_{i=1}^{N} [\nu_i - F_i(\theta)]^2$.
2. Construct covariance estimate V.
3. For $k = 1, ..., M$
   (a) Construct candidate $\theta^* \sim N(\theta^{k-1}, V)$.
   (b) Compute $\pi(y|\theta^*)$ and $\pi_0(\theta^*)$.
   (c) Compute $\alpha(\theta^*|\theta^{k-1}) = \min\left(1, \frac{\pi(y|\theta^*)\pi_0(\theta^*)}{\pi(y|\theta^{k-1})\pi_0(\theta^{k-1})}\right)$.
   (d) Accept θ^* with probability α.



Adaptation: Update V using accepted candidates

Delayed Rejection: Improves efficiency



SA+IF AND DRAM RESULTS

Normal Likelihood

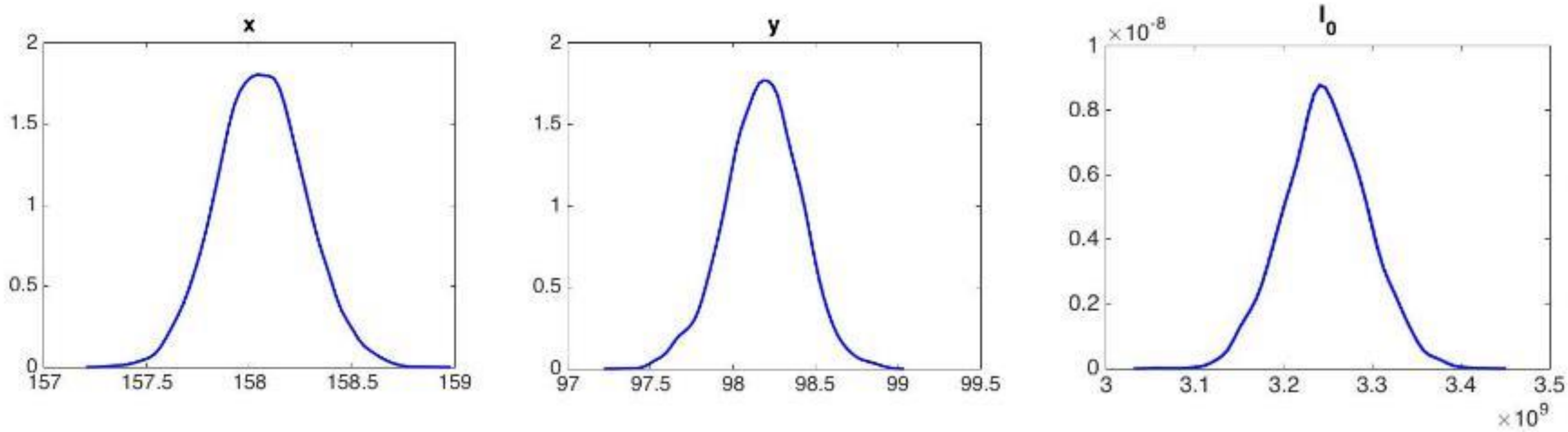
	True	SA + IF	DRAM
$\hat{x}$	**158**	158.092	158.06
$\hat{y}$	**98**	98.208	98.179
$\hat{I}_0$	$\mathbf{3.214 \times 10^9}$	3.2348×10^9	3.2474×10^9
CPU time (sec)	-	794.274	13327.98

Poisson Likelihood

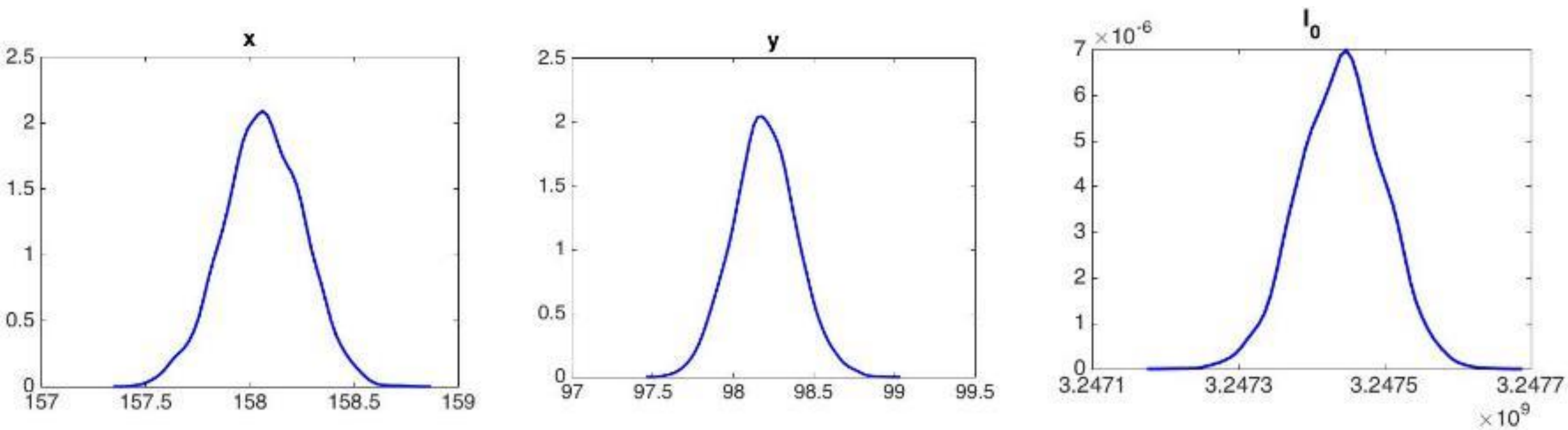
	True	SA + IF	DRAM
$\hat{x}$	**158**	158.067	158.07
$\hat{y}$	**98**	98.199	98.193
$\hat{I}_0$	$\mathbf{3.214 \times 10^9}$	3.2514×10^9	3.2474×10^9
CPU time (sec)	-	269.579	8665.89

DRAM RESULTS

Normal Likelihood

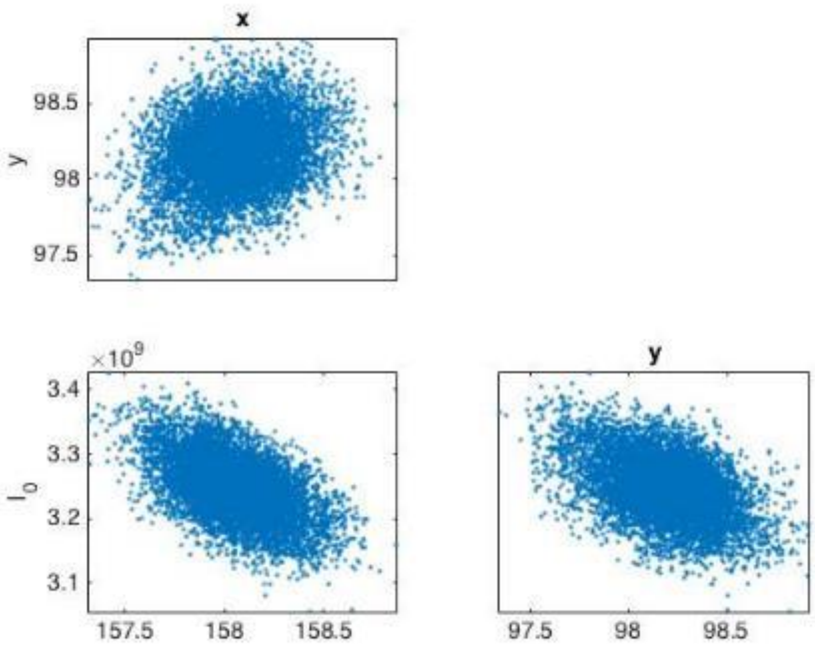


Poisson Likelihood

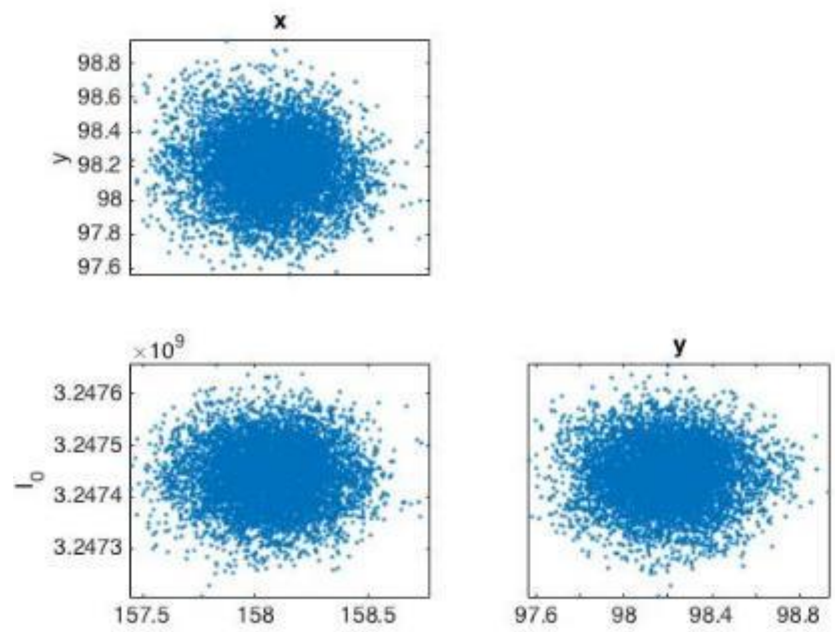


DRAM RESULTS

Normal Likelihood



Poisson Likelihood



FUTURE WORK

Hybrid Variational Data Assimilation

- Multiple sources identification
- Adaptive sensor locations

Stochastic Bayesian

- Employ energy statistics to quantify similarity between posterior distributions inferred using DRAM and DREAM
- Explore the lower limit of source intensity for accurately determining source location

Deterministic Variational Data Assimilation

- Source identification using coupled nuclear particle transport model and numerical weather prediction model